Gravitational Wave Detection

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Preface

TAMA Workshop on Gravitational Wave Detection was held at Saitama, Japan, from 12 to 14 November, 1996. The main subject of the Conference was the development of laser interferometric gravitational wave detectors. The Conference was attended by 87 scientists from 11 countries. We were very pleased to welcome many leading researchers in this field. There were 59 papers presented comprising 40 oral presentations and 19 posters. This Conference was very timely since the LIGO, VIRGO, GEO and TAMA projects have already started to be constructed. At this stage we have common technical problems concerning the development of large-scale detectors; this is the first experience for all of us. We hope that these proceedings are useful for understanding the present status of gravitational wave detection being carried out around the world.

As editors of the Proceedings, we acknowledge the sponsorship of the Ministry of Education and National Astronomical Observatory. TAMA project is supported by a Grant-in-Aid for Creative Basic Research of the Ministry of Education.

Editors
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M.-K. Fujimoto
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ABSTRACT

Coalescing binary neutron stars is one of the most important sources of the gravitational waves. In the workshop I argued four aspects of coalescing binary neutron stars: 1) event rate, 2) the last three minutes, 3) the last three millisecond, 4) Intermediate Binary Neutron Star (IBNS) problem. Since I did not intend to give a full review talk but gave my personal comments, I shall not refer the relevant references.

1. Event Rate

Usually from the observed binary neutron stars such as PSR1534+12, PSR1913+16 and PSR2127+11C, the event rate of the coalescing binary neutron stars is estimated. The coalescing time of these binary pulsars are $3 \times 10^9$y, $4 \times 10^8$y and $3.2 \times 10^8$y, respectively. Among these, PSR1534+12 contributes to the most of the event rate since the distance to PSR1534+12 is the nearest (500pc). That is, we should expect the large number of the system like PSR1534+12 in the unobserved region of our Galaxy. If the existence of PSR1534+12 is by chance, the event rate will be smaller by a factor 10. Phinney (1991) as well as Narayan, Piran and Shemi (1991) estimated the event rate first. The rate depends on various factors such as the cone angle of pulsar beam and the enhancement of the pulsar number density in the central part of the galaxy. In the best estimate the event rate is a few per year within 200Mpc while in the conservative one it is a few per year within 1Gpc. In any case the upper limit is given from the rate of Type Ib and Type Ic supernova rate as a few per year within 30Mpc.

Here I like to make two comments. The first comment is related to the distance to the pulsar. What is used to estimate the distance to the pulsar is the dispersion measure which is determined from the difference of the arrival time of the pulsar in different frequency. The dispersion measure is proportional to the distance times the mean electron density of the interstellar matter so that to obtain the accurate distance we need accurate estimate of the electron density. This is important. If the electron density is a factor 2 smaller than the estimated value, the real distance to the pulsar, say, PSR1534+12, is 2 times larger. Then the total number of the pulsar similar to PSR1534+12 in our Galaxy should be 8 times small. This means that the event rate of coalescing binary neutron stars will be 1/8 times smaller so that for the fixed event rate, say, a few per year, the mean distance to the source of the gravitational wave becomes 2 times larger. Therefore as a whole a factor 2 decreases of the electron density of the interstellar matter causes the factor 2 decreases of the amplitude of gravitational waves from a coalescing binary neutron star for a fixed event rate. A factor 2 is not so important to theorists but it is crucial.
to experiments. Taylor and Cordes (1993) gave us the catalogue of revised pulsar distance based on the revised distribution of the electron density in our Galaxy. Due to the decrease of the electron density by a factor $\sim 2$, the revised distance to the pulsar is $\sim 2$ times larger than the previous one. Therefore the event rate of the coalescing binary neutron stars will be smaller than the previous estimate.

Another way to estimate the event rate is to use the formation theory of the binary neutron stars. There are several ways to form the binary pulsars. Firstly, start with massive binary stars of mass greater than $10M_\odot$. The more massive star (star 1) evolves first and loses mass or exchanges mass with the companion (star 2). After the supernova explosion we have a neutron star 1 and a massive main sequence star 2 binary. Then star 2 exhausts hydrogen nuclear fuel and becomes a red giant. If the initial separation of the binary is small enough, neutron star 1 becomes inside a red giant 2. A neutron star spirals in as well as the hydrogen rich envelope of star 2 may be stripped off. In a certain case it is possible to form a neutron star 1 with a helium star 2 of mass smaller than $4.2M_\odot$ and the separation $a \sim R_\odot$. Then we can expect the formation of the binary pulsar like PSR1913+16 after the supernova of helium star 2. If, however, the mass of star 2 is greater than $4.2M_\odot$, the binary disrupts after the supernova since the binding energy of the binary becomes positive due to the rapid mass loss. In another case the separation is not small enough so that the coalescing time is much larger than the age of the universe. PSR2303+46 and PSRJ1518+4904 seem to belong such binary neutron stars. Therefore the formation rate of the binary pulsar like PSR1913+16 depends on many not-well-known factors such as the history of the star formation rate, the distribution of the initial mass of the binary and the initial separation, the mass loss rate and mass exchange rate before the first supernova, the mass loss and angular momentum loss rates in the common envelope phase and so on. One more question exists on the formation theory. PSR2127+11C and PSR1913+16 have similar binary parameters. They look like a twin. However the formation history of the twin should be completely different since PSR2127+11C is in the globular cluster so that there are no massive young stars.

Tutukov and Yungelson (1993), for example, computed the merger rate of the coalescing binary neutron stars from the formation theory adopting various assumptions on not-well-known factors I quoted above. As a result they obtain the merger rate of a binary neutron star will be $\sim 100$ per year within $200\text{Mpc}$. The main reason for this number comes from the increase of the formation of very short period neutron star binary with the orbital period smaller than $\sim 8\text{hours}$. In the phase of neutron star 1 and helium star 2, it is possible in a certain case that neutron star 1 enters inside the helium envelope of star 2. In this case the helium envelope of star 2 is stripped off and a neutron star 1 and C-O star 2 with smaller separation is formed. After the supernova explosion of star 2 we have a binary neutron star with coalescing time much shorter than the age of the universe since the coalescing time is proportional to the fourth power of the separation. In the observed binary neutron star there are no such system. However this is consistent because of the very short life time of the system. What we should compare is the rate of Type
Ic supernova since the supernova event of a C-O star will be observed as Type Ic. Nomoto et al. estimated the rate as $6 \times 10^{-5} \sim 1.2 \times 10^{-4}$ Type Ic/galaxy/year. This suggests that if all the Type Ic supernova form coalescing binary neutron star, the event rate will be $\sim 10$ per year within 200Mpc.

2. Last Three Minutes

The merger rate of binary neutron star may be high or may be low. The rate may be crucial for experiments on the detection of the gravitational waves. However there may be quite new sources of gravitational waves with extremely high event rates. So I suggest people to be optimistic. For a theorist, irrespective of the merger rate, it is interesting to know the final phase of coalescing binary neutron star. The problem is as follows;

Problem: **Given a neutron star-neutron star binary of mass $m_1$, $m_2$ and the separation $a$. Predict the wave amplitude and the polarization as a function of time up to the final merging phase as accurate as possible. The same problem applies for neutron star-black hole and black hole-black hole binary also.**

This problem is well posed and many people contribute many aspects of the problem, which will be divided into three parts: 1) the last three minutes, 2) the last three millisecond, 3) Intermediate Binary Neutron Star (IBNS). In this section I comment on the first part: the last three minutes.

The merging time of coalescing neutron star is given by

$$t_{\text{mrg}} = 3 \text{min} \left( \frac{m_1}{1.4M_\odot} \right)^{-1} \left( \frac{m_2}{1.4M_\odot} \right)^{-1} \left( \frac{m_1 + m_2}{2.8M_\odot} \right)^{-1} \left( \frac{a}{470 \text{km}} \right)^4, \quad (1)$$

and the frequency of the gravitational waves is given by

$$f = 19 \text{Hz} \left( \frac{m_1 + m_2}{2.8M_\odot} \right)^{\frac{3}{2}} \left( \frac{a}{470 \text{km}} \right)^{-\frac{3}{2}}. \quad (2)$$

The number of the rotation before the coalescence is estimated as

$$N = 2635 \left( \frac{m_1}{1.4M_\odot} \right)^{-1} \left( \frac{m_2}{1.4M_\odot} \right)^{-1} \left( \frac{m_1 + m_2}{2.8M_\odot} \right)^{-\frac{1}{2}} \left( \frac{a}{470 \text{km}} \right)^{\frac{5}{2}}. \quad (3)$$

For definiteness by **the last three minutes** I mean that the separation is $\sim 500\text{km}$, the frequency of the wave is more than 20Hz and the binary rotates $\sim 3000$ times. In this region the rotation velocity of the binary is $\sim 0.1c$ and the post Newtonian expansion seems to operate. For example the luminosity of the gravitational waves is expressed by

$$L_{GW} = L_q \sum_{n=0}^{\infty} a_n \left( \frac{v}{c} \right)^n, \quad (4)$$
where
\[ L_q = \frac{32}{5} \frac{G^4 \mu^2 m_t^3}{c^5 a^5}, \tag{5} \]
and
\[ v = \sqrt{\frac{Gm_t}{a}}. \tag{6} \]

In the above equation \( \mu \) and \( m_t \) are the reduced mass and the total mass of the system, respectively. In the stage of 1993, only first three \( a_i \) (\( a_0 = 1, a_1, a_2 \) and \( a_3 \)) are known. No one knew the higher terms and convergence of the expansion. However people noticed that if, for example, we omit the term which contributes only 0.1\% of \( L_{GW} \), the decrease or increase of number of rotation is expected to be more than 3 because of \( \sim 3000 \) rotations as a whole. This means that the phase of the wave will be totally unpredicted. We need the accurate luminosity with uncertainty below 0.1\%. No one knew up to which term is needed: \( n=6?,7?,8?..... \)

Caltech group first tried to know accurate \( L_{GW} \) in the limit of \( m_2/m_1 \to 0 \). In this case we can use the perturbation theory of a black hole, which means that we replace \( m_1 \) as a black hole of mass \( M \) and \( m_2 \) as a test particle of mass \( \mu \ll M \) in a circular orbit of radius \( a \). Even in this case it was difficult to obtain the higher terms. Caltech group used the numerical integration of the perturbation equation around the black hole and found \( a_4, a_5 \) and \( a_6 \) with the estimated error of 2\%, 10\% and 50\% , respectively. This approach was followed by Kyoto group. They first obtained the term up to \( a_9 \) semi-analytically and found the existence of the logarithmic term at \( n=6 \) and \( 8 \). They next obtained the results analytically and confirmed the semi-analytical results as

\[
L_{GW} = L_q \times \left[ 1 - \frac{1247}{336} \left( \frac{v}{c} \right)^2 + 4\pi \left( \frac{v}{c} \right)^3 
- 4.928461199295258 \left( \frac{v}{c} \right)^4 - 38.29283545329089 \left( \frac{v}{c} \right)^5 
+ 115.73172132 \left( \frac{v}{c} \right)^6 - 16.304761151 \left( \frac{v}{c} \right)^7 \log \left( \frac{v}{c} \right) 
- 101.509987 \left( \frac{v}{c} \right)^7 - 117.787 \left( \frac{v}{c} \right)^8 + 52.69901 \left( \frac{v}{c} \right)^8 
\times \log \left( \frac{v}{c} \right) + 700.45 \left( \frac{v}{c} \right)^9 - 209.78 \left( \frac{v}{c} \right)^9 \log \left( \frac{v}{c} \right) \right]. \tag{7} \]

From Eq.\( (7) \) it is concluded that to predict the phase of the wave accurately, at least the terms up to \( a_6 \) is needed.

Probably encouraged by these perturbation calculations, French-Us group calculated \( a_4 \) in 1995 and Blanchet calculated \( a_5 \) in 1996. Blanchet is now computing \( a_6 \). In all these calculations the results should agree with the perturbation calculations in the limit of \( m_2/m_1 \to 0 \), which is in reality confirmed. Therefore at present there is a big hope that we will obtain the higher terms in the post-Newtonian expansion needed to analyze the observed data. The perturbation calculation is now
going up to \( n = 11 \) and is given as

\[
L_{GW} = L_q [1 - 3.711309523809524x^2 + 12.56637061435917x^3 - 4.928461199294533x^4 - 38.2928354569344x^5 + (115.7317166756113 - 16.3047619047619 \ln x)x^6 - 101.5095959597416x^7 + (-117.5043907226773 + 52.7430839022676 \ln x)x^8 + (719.1283422334299 - 204.8916808741229 \ln x)x^9 + (-1216.906991317042 + 116.6398765941094 \ln x)x^{10} + (958.934970119567 + 473.6244781742307 \ln x)x^{11} + \cdots],
\]  

(8)

where \( x = v/c \).

3. Last Three Millisecond

Now consider the coalescing binary neutron star of mass \( 1.4M_\odot \) each. Since the total mass is bigger than the maximum mass of the neutron star, the final product will be a black hole. Such a process can not be studied by the post Newtonian approach. Numerical relativity is needed. In 1989 Oohara and Nakamura started numerical simulations of coalescing binary neutron stars. In their first simulations general relativity is included only as a radiation reaction potential. Since then so many people performed extensive numerical simulations using various models and approximations. However as for the fully general relativistic simulations on coalescing neutron stars, we have only test simulations by Nakamura in 1994. He showed basic equations and numerical methods of a 3D numerical relativity code using a conformal time slicing and pseudo-minimal shear condition. He performed preliminary numerical simulations for coalescing binary neutron stars using \((80)^3\) Cartesian grids. He also showed results of spherically symmetric collapse to black hole, the formation of rotating black hole and collision of two dust spheres as a check of the code. Qualitatively the code works well without serious difficulties. However the number of grids is too small to say something quantitatively so that the full scale simulations with at least \((400)^3\) grids are required. I feel that within a few years we will have the results of fully general relativistic simulations of the final phase of the coalescing binary neutron stars. As for coalescing binary black holes Us-grand challenge group has been developing 3D numerical code.

4. IBNS Problem

In the previous two sections we noticed that there is a big hope in post-Newtonian expansion as well as numerical relativity. Then one may think that we can solve the problem raised in section 2, that is, \textbf{We may predict the wave amplitude and the polarization as a function of time up to the final merging phase as accurate as possible soon}. However there is one big unsolved problem. The accuracy of the post-Newtonian expansion becomes worse with the decrease of \( a \). For \( a \lessapprox 20m_t \), the error is unacceptably large unless we have higher terms up to \( n=10 \) or so. But this will be difficult. From \( a \sim 20m_t \) the binary rotates \( \sim 30 \) times or so. In principle we can switch post-Newtonian expansion to numerical relativity
code at $a \sim 20m_t$. However in practice we need too long computing time and numerical errors will be accumulated in such a long integration. This means that there is a certain region where neither post-Newtonian expansion nor numerical relativity may give accurate wave form. We like to call this problem as Intermediate Binary Neutron Star (IBNS) problem. In IBNS region there are three time scales; $t_{mrg}$, $t_K$ (Kepler time) and $t_{ff}$ (free fall time of the neutron star). The relation among three time scales is $t_{mrg} \gg t_K \gg t_{ff}$. This relation is similar to the quasi-static evolution of the star where $t_n \gg t_{KH} \gg t_{ff}$ with $t_n$ and $t_{KH}$ being the nuclear time scale and the Kelvin Helmholtz time scale, respectively.

The evolution of the star is a time dependent phenomenon. However except for the formation phase of the star and the supernova explosion, the dynamical code is not used but the quasi-static code is used to evolve the star. This strongly suggests that IBNS should be solved by the quasi-stationary method. This is the same as asking how to define the quasi-Killing vector, which is an unsolved problem in general relativity. The formulation of the problem will be difficult but physically there should be an answer so that there should be a way. After the solution of IBNS problem we will finally solve the problem raised in section 2 and wait for the arrival of the gravitational waves from the coalescing binary neutron stars.
The gravitational-wave signals that can be detected by the laser interferometers currently under construction are briefly reviewed. The basic formulae determining the detectability of the signals and accuracy of estimation of their parameters are given. The numerical formulae for the signal-to-noise ratios and root-mean-square errors are calculated for single detectors.

1. Introduction

A number of projects to build long arm laser interferometric detectors of gravitational waves has been funded. These are American LIGO project to build two 4km detectors, French-Italian VIRGO project to build a 3km detector, British-German GEO600 project to build a 600m detector and Japanese TAMA project to build a 300m detector. As we have heard in this workshop the construction of all the above detectors is rapidly progressing. Because of their wide band and projected high sensitivity these detectors should achieve the following aims:

1. Detect directly gravitational waves.

2. Provide a wealth of astrophysical information complementary to that provided by optical and radio telescopes.

3. Verify Einstein’s general relativity to an extraordinary accuracy, perhaps to $(v/c)^6$ order beyond the quadrupole approximation.

The aim of this presentation is to review the potential of the constructed laser interferometers to detect gravitational wave sources. To this end we introduce simple models of the noise in the detectors (Section 2), we present fiducial gravitational-wave signals (Section 3), and we quantitatively describe detectability of these sources and accuracy of estimation of their parameters that can be achieved using optimal data analysis techniques (Section 4).

2. Models of Noise

A large effort is put by the experimentalists to understand and reduce sources of noise in the detectors. Impressive achievements in the area of low-loss optics and seismic isolation have been reported in this meeting.
For simplicity we assume that the noise in the interferometer is a stationary, Gaussian, zero mean random process. By $S_h(f)$ we shall denote its one-sided spectral density.

For the case of large-sized interferometers (LIGO and VIRGO) we introduce two simple models of noise: the model of noise for initial configuration when the detectors will start operating and a model for the advanced configuration when the performance of the detectors will be improved to their ultimate sensitivity.

1. Noise of the initial detectors.

$$S_h(f) = \begin{cases} \infty & f < 40\text{Hz} \\ S_1 [(f_1/f)^4 + 2(1 + (f/f_1)^2)]/5 & 40\text{Hz} \leq f \leq 750\text{Hz} \\ \infty & f > 750\text{Hz}, \end{cases}$$

where $S_1 = 1.5 \times 10^{-46}\text{Hz}^{-1}$ and $f_1 = 200\text{Hz}$.

2. Noise of the advanced detectors.

$$S_h(f) = \begin{cases} \infty & f < 10\text{Hz} \\ S_1 [(f_1/f)^4 + 2(1 + (f/f_1)^2)]/5 & 10\text{Hz} \leq f \leq 750\text{Hz} \\ \infty & f > 750\text{Hz}, \end{cases}$$

where $S_1 = 3 \times 10^{-48}\text{Hz}^{-1}$ and $f_1 = 70\text{Hz}$.

The above formulae are only indicative and reflect neither the ingenuity of the experimentalists to reduce various sources of noise to achieve the initial sensitivity nor their efforts needed to achieve the final sensitivity of the laser interferometers.

We use the above formulae in Section 4 to calculate the performance of the LIGO and VIRGO detectors. For the case of GEO600 detector we used their projected spectral density of noise. The GEO600 detector will introduce signal recycling which for frequencies above several 100Hz enables to increase sensitivity of the detector for a given frequency $f_o$ (at the expense of reducing sensitivity outside a certain bandwidth around $f_o$). The noise curve given has a minimum of around 215Hz however it is not optimized for that particular frequency.

For the case of TAMA detector we used a set of standard formulae for the shot, thermal and quantum noises giving their projected spectral density of $4 \times 10^{-42}\text{Hz}^{-1}$ at 300Hz.

The noise curves are drawn in Figure 1.

3. Simple Models of Gravitational-Wave Signals

In this section we present simple models of basic gravitational-wave signals. The amplitudes of the signals correspond to the appropriately averaged detector response over the source positions.$^1$
3.1. Models of Deterministic Gravitational-Wave Signals

Supernova Signal: Impulse

The gravitational-wave signal from a supernova can be modelled as an impulse. The precise form of the impulse is not known. It can be modelled in various ways e.g by a Gaussian function, box function, sinc function, half-cycles of the sine function.

Here we consider a model of half-cycles of the sine function.

Let $W$ be the window function defined as

$$ W(t; t_1, t_2) = \begin{cases} 
1 & \text{for } t_1 < t < t_2 \\
0 & \text{otherwise.} 
\end{cases} $$

(3)

The gravitational-wave signal $h_{SN}$ from the supernova is then given by

$$ h_{SN}(t) = h_o \sin(2\pi f_g t) W[t; 0, \frac{N_{hc}}{2f_g}], $$

(4)

where $h_o$ is a constant amplitude, $f_g$ is the characteristic frequency, and $N_{hc}$ is the number of half cycles. Typical characteristic frequency is 1kHz and typical number of cycles is 1 to 4. Thus the duration of the signal is a few milliseconds. The amplitude is estimated as

$$ h_o = 7.7 \times 10^{-21} \left( \frac{\Delta M_{GW\odot}}{10^{-2}} \right)^{1/2} \left( \frac{1\text{kHz}}{f_g} \right)^{1/2} \left( \frac{10\text{Mpc}}{r} \right) \left( \frac{1}{N_{hc}} \right)^{1/2}, $$

(5)

where $\Delta M_{GW\odot}$ is the fraction of solar masses emitted in gravitational waves as a result of explosion of the supernova and $r$ is the distance to the supernova.
Pulsar Signal: Quasi-Periodic

The signal from pulsar is basically periodic. However the estimates (see below) show that to extract it from noise we may have to integrate the data for several months! Thus a small correction due to the motion of the Earth around the Sun and the spin-down of the pulsar will need to be taken into account. Let \( R \) be 1 astronomical unit (AU), \( \Omega = 2\pi/1 \text{year} \) and let the position of the pulsar on the sky be \((\theta, \phi)\) in the coordinate system based on the ecliptic (i.e., \( \theta = \pi/2 \) is Earth-Sun plane, and \( \phi = 0 \) is position of Earth at \( t = 0 \)). Let \( f \) be the gravitational-wave frequency from the pulsar and let \( \dot{f}, \ddot{f} \) be the first and the second derivative of the frequency w.r.t. time respectively (spin-down parameters). We can approximate the frequency modulation law by its Taylor expansion. The number of terms needed in the expansion depends on the observation time and the expected values of the frequency derivatives. Here we consider terms up to the second derivative of the frequency.

Then we have the following model of the gravitational-wave signal.

\[
    h_{P}(t) = h_{o} \sin\left(2\pi\left[ft + \frac{1}{2}\dot{f}t^{2} + \frac{1}{6}\ddot{f}t^{3} + R/c \sin \theta \cos(\Omega t - \phi) f + \phi_{o}\right]\right),
\]

where \( h_{o} \) is the constant amplitude and \( \phi_{o} \) is the phase of the signal. The amplitude \( h_{o} \) is estimated as

\[
    h_{o} = 7.7 \times 10^{-26} \left(\frac{I_{zz}}{10^{45} \text{g cm}^{3}}\right) \left(\frac{1 \text{kpc}}{r}\right) \left(\frac{f}{100 \text{Hz}}\right)^{2} \left(\frac{\delta}{10^{-5}}\right),
\]

where \( I_{zz} \) is the moment of inertia of the pulsar about its rotation axis, \( r \) is the distance, \( f_{g} \) is the gravitational wave frequency and \( \delta \) is the ellipticity of the pulsar. The ellipticity of \( 10^{-5} \) corresponds to the maximum strain that the neutron star crust may support.\(^1\) In the realistic model a number of other small corrections will need to be taken into account.

Coalescing Binary Signal: Chirp

This signal arises from a binary system like Hulse-Taylor binary pulsar. For circularized orbits the gravitational-wave signal from a binary system consisting of two point masses \( m_{1} \) and \( m_{2} \) is given by

\[
    h_{CB}(t) = Af(t)^{2/3} \sin \left(2\pi \int_{t_{a}}^{t} f(t') dt' + \phi_{o}\right),
\]

where \( \phi_{o} \) is the phase of the signal at time \( t_{a} \) and the amplitude \( A \) is given by

\[
    A = \frac{8}{5} \pi^{2/3} G \mu (Gm)^{2/3} \left(\frac{r c^{4}}{Gm}\right),
\]

\( \mu \) and \( m \) are the reduced and the total mass respectively, \( R \) is the distance to the binary. A typical binary consists of 1.4 solar mass neutron stars. It is expected that
a few coalescences per year will occur at the the distance of 200Mpc. This gives the following numerical value of the amplitude \( h_0 \).

\[
h_0 = Af^{2/3} = 7 \times 10^{-24} \left( \frac{200 \text{Mpc}}{r} \right) \left( \frac{\mu_{\odot}}{0.7} \right) \left( \frac{m_{\odot}}{2.8} \right)^{2/3} \left( \frac{f}{100 \text{Hz}} \right)^{2/3}
\]

(10)

The characteristic time for the evolution of the binary to the currently known 5/2 post-Newtonian order is given by

\[
\tau_{5/2PN} = \frac{f}{df/dt} = \frac{5}{96 \mu m^{2/3}} \left( \frac{\pi f}{3} \right)^{8/3} \times [1 + \ldots]
\]

(11)

In the formula above we have neglected all contributions due to spin effects. This is a good approximation for the observed neutron star binaries like Hulse-Taylor pulsar.

3.2. Model of Stochastic Signals

Stochastic gravitational wave signals can arise in the early universe during the inflation era or from the cosmic strings. We assume that the signal is a stationary random process. It is convenient to express the strength of the stochastic gravitational wave signals in terms of the quantity \( \Omega_g(f) \) defined as

\[
\Omega_g(f) = \frac{1}{\rho_c} \frac{dE}{dx^3 d(\ln f)},
\]

(12)

where \( \rho_c \) is the energy density to close the universe

\[
\rho_c = \frac{3c^2 H_0^2}{8\pi G} \approx 1.6 \times 10^{30} h_0^2 [J/m^3],
\]

(13)

where \( h_0 \) is the Hubble constant \( H_0 \) expresses in 100 [km/(Mpc sec)]. The one-sided spectral density \( S_s(f) \) of the stochastic signal is given by

\[
S_s(f) = \frac{8G\rho_c \Omega_g(f)}{f^3}.
\]

(14)

4. Optimal Signal Detection and Parameter Estimation

To detect the signal and to estimate its parameters we apply the method of maximum likelihood detection i.e. we maximize the likelihood function \( \Lambda \) with respect to the parameters of the signal. If the maximum of \( \Lambda \) exceed a certain threshold
calculated from the false alarm probability that we can afford we say that the signal is detected. The values of the parameters that maximize $\Lambda$ are said to be maximum likelihood estimators of the parameters of the signal. The magnitude of the maximum of $\Lambda$ determines the probability of detection of the signal.

Another way of finding an optimal method of detection is to find a linear filter that maximizes the signal-to-noise ratio after filtering.

Suppose that the signal is additive, i.e. the data $x$ are given as $s + n$, where $s$ is the signal and $n$ is the noise. In the case of the deterministic signal the output of the linear filter $F$ is denoted by $(x|F)$ and in the case of the stochastic signal the output of linear filter $Q$ is $(x|Qx)$ where

$$ (x|F) = \int_0^T x(t')F(t')dt', \quad (x|Qx) = \int_0^T \int_0^T x(t')Q(t', t'')x(t'')dt'dt''. $$

(15) (16)

Let us assume that both the noise in the detector and the stochastic signal are zero mean value stationary random processes of one-sided spectral densities $S_s(f)$ and $S_h(f)$ respectively.

The expectation and the variance in the optimal functional for the deterministic signal and the stochastic signal in the case of observation time $T$ much longer than the correlation time of both the signal and the noise are given by

$$ E(x|F) = 2\int_{-\infty}^{\infty} \frac{\tilde{s}(f)\tilde{F}^*(f)}{S_h(f)} df, \quad \text{(17)} $$

$$ Var(x|F) = 2\int_{-\infty}^{\infty} \frac{\tilde{F}(f)\tilde{F}^*(f)}{S_h(f)} df, \quad \text{(18)} $$

$$ E(x|Qx) = T/2 \int_{-\infty}^{\infty} S_x(f)\tilde{Q}^*(f)df, \quad \text{(19)} $$

$$ Var(x|Qx) = T/2 \int_{-\infty}^{\infty} S^2_x(f)\tilde{Q}'^2(f)df, \quad \text{(20)} $$

where $S_x(f)$ is the spectral density of the data ($S_x(f) = S_s(f) + S_h(f)$). Signal-to-noise ratios after filtering are defined as

$$ d^2_{\text{det}} = \frac{(E(s|F))^2}{Var(x|F)}, \quad \text{(21)} $$

$$ d^2_{\text{stoch}} = \frac{(E(s|Qs))^2}{Var(x|Qx)}. \quad \text{(22)} $$

Assuming that $S_h \gg S_s$ and using the Schwarz inequality one easily gets that the optimum filters and signal to noise ratios in both cases are given by

$$ F^*(f) = A \frac{\tilde{s}(f)}{S_h(f)}, \quad \text{(23)} $$
\[
d^2_{\text{opt}} = 2 \int_{-\infty}^{\infty} \frac{\tilde{s}(f)^2}{S_h(f)} df, \tag{24}
\]
\[
Q(f) = B \frac{S_s(f)}{S_0^2(f)}, \tag{25}
\]
\[
d^2_{\text{opt}} = T/2 \int_{-\infty}^{\infty} \frac{S_s(f)^2}{S_h(f)^2} df, \tag{26}
\]
\[
d^2_{\text{opt}} = T = 2 \int_{-\infty}^{\infty} \frac{\tilde{s}(f)}{S_h(f)} df; \tag{27}
\]

where \(A\) and \(B\) are arbitrary constants. The same optimal filters are obtained form study of likelihood ratio under assumption that the noise and the stochastic signal are Gaussian random processes. The rms errors of the parameters are approximately given by the square roots of the diagonal elements of the inverses of the Fisher information matrices given by

\[
\Gamma_{ij}^{\text{det}} = 2 \int_{-\infty}^{\infty} \frac{\partial \tilde{s}(f)}{\partial \theta_i} \frac{\partial \tilde{s}^*(f)}{\partial \theta_j} \frac{1}{S_h(f)} df, \tag{28}
\]
\[
\Gamma_{ij}^{\text{stoch}} = T/2 \int_{-\infty}^{\infty} \frac{\partial S_s(f)}{\partial \theta_i} \frac{\partial S_s^*(f)}{\partial \theta_j} \frac{1}{S_h^2(f)} df, \tag{29}
\]

where prime denotes the filter. The commonly accepted procedure to detect the stochastic signal\(^3\) is to pass the data from two detectors through the optimal filter. However the analysis with only one detector is also possible.\(^4\)

Here we are not addressing the highly non-trivial practical issues of the gravitational-wave data analysis.\(^5\)

We have used the above formulae to calculate the signal-to-noise ratios and the rms errors in the parameters in the tables given below. For the case of supernova and stochastic signals we have given the rms error in the amplitude, for the coalescing binary we have given the rms error in the chirp mass \((M = \mu^{3/5} m_2^{2/5})\), and for the case of pulsars, rms error in the frequency. We have taken into account the correlations with the other parameters. We have taken the pulsar frequency to be 215Hz and the time of integration both for pulsar and the stochastic signal to be 10\(^7\)sec.

\begin{center}
\begin{tabular}{|l|c|c|}
\hline
\textbf{Signal} & \textbf{Signal-to-noise} & \textbf{Rms error (\%)} \\
\hline
Supernova, \(r = 10\text{Mpc}\) & 7.3 & amplitude: 14 \\
Pulsar, \(r = 1\text{kpc}\) & 88 & frequency: \(5.5 \times 10^{-9}\) \\
Binary, \(r = 10\text{Mpc}\) & 51 & chirp mass: 0.01 \\
Stochastic, \(\Omega = 10^{-7}\) & 3.5 & amplitude: 29 \\
\hline
\end{tabular}
\end{center}
ADVANCED DETECTORS

<table>
<thead>
<tr>
<th>Signal</th>
<th>Signal-to-noise</th>
<th>Rms error (%)</th>
</tr>
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<td>Supernova, r = 10Mpc</td>
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<td>320</td>
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<tr>
<td>Binary, r = 200Mpc</td>
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<tr>
<td>Stochastic, $\Omega = 10^{-10}$</td>
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GEO600

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TAMA

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</tr>
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</table>

Acknowledgements

I would like to thank Prof. A. Brillet and Prof. K. Danzmann for helpful remarks and Dr. K. Strain for providing the formulae for the projected noise curve of the GEO600 project. This work was supported in part by Polish Science Committee grant KBN 2 P303D 021 11.

References

DATA ACQUISITION SYSTEM FOR TAMA GRAVITATIONAL WAVE INTERFEROMETER

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and

The data working group of the TAMA collaboration

ABSTRACT
The design of an online system for TAMA gravitational wave interferometer is presented at the workshop. The online system consists from a high rate data acquisition part with VXI bus, a low rate data acquisition part and online control part which are distributed along long arms of the interferometer using fiber-optic network. A total amount of data is up to 600 kByte/s. The construction had been started and will be complete at the end of 1997.

1. Introduction
Since the TAMA interferometric gravitational wave detector will have a continuous observation phase for the last two years of the project, we must prepare the data acquisition and monitoring system in 1997. Moreover, some control loops in the interferometer operation will need the support of computers. We report a design for, and progress towards, the data acquisition and on-line control system for TAMA interferometer.

1.1. Signals from the Interferometer
Signals to and from the interferometer can be categorized into three groups:

• high sampling-rate (20 kHz or more) signals as the main interferometer signal, which will dominate the amount of data storage,

• signals with lower rate (0.1 -10 Hz) but many channels, such as environmental monitoring distributed along the interferometer arms,

• online feedback loops (0.1 -10 Hz) in the low frequency region, such as for correction of drift of optical elements.

Table 1 shows signals which will be record or will be output.

1.2. Strategy of the Online System
The aim of the online system is to acquire above signals, to storage them completely and to achieve feedback loops for a robust operation of an interferometer. The total amount of signals is evaluated as 600 kByte/s. The high sampling-rate
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<td>12</td>
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Total 600 kByte/sec
signals are dominant in the data quantity. On the other hand, low rate signals and control loops are distributed along 300 m arms of the interferometer. Therefore, we need integrated online system which can record and monitor both high rate data and distributed signals.

2. Design of the Online System

2.1. High Rate DA Part

The high rate signals which are most important for gravitational wave detection include the main interferometric signal, the feedback or error signals from the optical elements and the laser intensity. A sampling rate must be greater than about 20 kHz for these signals. The capacity of the data transfer and recording system should be even faster to allow a safety margin.

We have chosen a VXI system with the DSP engine,\(^1\) which allows a higher data transfer rate through a VXI bus, bypassing the master control module of VXI crate. After the data transport from ADCs to an intermediate spooling disk by this “throughput” process, another process records the data into magnetic tapes (8mm or DAT) for archival.

2.2. Low Rate DA Part and Online Feedback Part

Low rate signal sources such as vacuum status exist along the arms of the interferometer. As well as the environment, some optical elements and vibration isolation system will require remote monitoring.

Moreover These are closely related to the online control system. For example, the tilt of the X-pendulum will need to be monitored and corrected. Similarly, the arm length will be monitored for correction of drift due to thermal expansion of the ground and of the components of the vacuum and vibration isolation systems. Some of the online feedback is closed in local area, and others require feedback between remote places via network.

Software of the System

To process these distributed signals and control loops, one or both of the following two software packages will be used:

- **EPICS**
  
  EPICS (Experimental Physics and Industrial Control System)\(^2\) is a collaboration of the control groups of many research organizations that use the EPICS tool-kit. It has a distributed architecture, which consists of UNIX workstations, VME systems, and GPIB etc. inter-connected with a network. It already has a GUI to display data.

- **HP VEE**
  
  HP VEE\(^3\) is a GUI environment for controlling VXI crates and GPIB peripherals. The GUI makes it easy to configure sequence and display data. It is particularly strong at handling GPIB peripherals.
2.3. Network

Since we want to operate the system remotely, a network is required. We have constructed a fiber-optic system throughout the TAMA experimental building. It consists of 16 fibers, 300 m in length and 8 fibers of 150 m length at each arm (48 cables in total). The computer network uses 20 of 48 fibers. The network is structured as three independent segment. Each room is connected as a star topology so that any trouble will be localized.

The remaining fibers will transmit raw signals using an analog-digital-analog method. The topology of the online system with the network is shown in figure 1.

![Figure 1: Topology of the online system with the network](image)

2.4. Online Trigger for GW

To reduce the amount of data which we must analyze in offline, the online system must include trigger system for gravitational wave events. This trigger
system consists of various type of logics for expected phenomenon as a coalescence of binary stars and supernova explosions. The data will be checked by these logics after the event builder in a data flow. Triggers will be done by software.

We start to study of a resampling$^{4,5}$ for chirp waveform from a coalescence recently, and need to prepare also burst wave. We will try various trigger logics in observation phase of TAMA after 1998.

2.5. Data Flow

The data flow in the TAMA online system is shown in figure 2. High rate signals are sampled and first stored in spooling disk. Low rate signals are sampled by VME/VXI hosts. Local and global online control systems are implemented in VME/VXI computers. Low rate data and online control status are collected via the
network using EPICS/VEE. These are merged with the high rate data and stored together. The online trigger will be attached after data flame building.

3. Conclusion and Schedule

Finally we outline the installation schedule. The network has been ready since the summer of 1996. The platform for the low rate data system will be decided by the end of 1996. The outline of the high rate system was decided and more details are the current discussion topics. It will be installed at the end of 1997.

References

1. Software on VXI system, Hewlett-Packard Co.
5. R.Flaminio et al. LAPP-EXP-93.13
SIESTA
A GENERAL PURPOSE SIMULATION PROGRAM
FOR THE VIRGO EXPERIMENT

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ABSTRACT
The simulation program developed for the VIRGO collaboration is designed as a general purpose tool for detector design, detector commissioning and data analysis. It allows to simulate in an integrated way some expected gravitational wave signals as well as many aspects of the detector response. The general structure and contents of the simulation program are described, as well as a typical application example.

1. Introduction

The development of an integrated, general purpose simulation program for the VIRGO experiment has been going on for several years. One of the goals of the SIESTA software was to provide a tool for the detector design and commissioning, especially as far as the control of the interferometer is concerned. To reach this goal, a global simulation of the interferometer operation is needed, not only for the case when the interferometer is around its working point but also for the lock acquisition phase. The other major goal of SIESTA is to provide simulated data for data analysis, in order to develop search and trigger algorithms. Moreover, attention has been paid to make the software modular enough so that some pieces may be used in connection with other VIRGO softwares. These motivations have led to the main features of the software design.

One of the choices has been to perform the simulation in the time domain. This is important for data analysis since it allows to produce the same type of simulated data as real data. It is also useful when non-linear effects have to be taken into account, such as non-linearities in the interferometer optical response or in the electronics.

SIESTA is written in the C language, and is based on an object-oriented structure that is most suitable to build an integrated simulation involving many different aspects. The implemented framework allows to integrate the generation of gravitational wave events with the simulation of the most relevant aspects of the detector operation: behavior of suspended mirrors, interferometer optical response, control, DAQ (see Figure 1). Since the level of accuracy needed for some aspects of the simulation may vary according to the issue investigated, various models are made available as options whenever possible.

Not all the simulation developments carried out in VIRGO have been integrated into SIESTA. Very specific issues such as the problem of scattered light or the needed requirements on the mirror surface quality have been addressed through dedicated simulation tools that need not be integrated in the general software.

2. Contents of the Simulation

In this section, we review briefly the contents of the different modules building up SIESTA.
2.1. Gravitational Wave Event Generator

This module simulates the signal emitted by a source and translates it into a signal at the detector level taking into account Doppler effect as well as amplitude and phase modulation due to Earth rotation. As far as sources are concerned, a basic generator is available for pulsars and binary coalescences (up to first post-Newtonian order).

2.2. Simulation of Suspended Mirrors

This module allows to compute the position of a suspended mirror as a function of time taking into account the action of seismic noise, thermal noise, and feedback forces that may be applied on the mirror from its reference mass or from the marionette. The simulation is based on a numerical resolution of the motion equation of a chain of massive pendula in one dimension. The angular degrees of freedom are not yet taken into account. The injection of seismic and thermal noise is made by generating random sequences with white frequency distribution and filtering them in order to get the proper frequency spectrum.

2.3. Optics Module

The part of SIESTA that deals with optical simulation is designed as a toolkit that provides some general methods to simulate any user defined configuration to-
together with more specific models dedicated to the case of a VIRGO-like interferometer. The generic simulation may be performed using either a wavefront computation technique using FFT for propagation, or a method based on an eigenmode expansion of the field and using simple algebra for propagation and reflection.

The specific models provide a variety of tools to perform fast quasistatic simulations as well as dynamic simulations both in the linear and in the non-linear domain (i.e. when cavities are close to resonance or not) and taking into account misalignment effects. As a matter of example, this allows us to extract the transfer functions between the vibration modes of the interferometer and the different signals collected on the photodiodes, or to model the signals observed in case a cavity sweeps through resonance with high velocity.

2.4. Signal Processing

A SIESTA module is dedicated to signal processing, namely digital filtering and simulation of analog to digital conversion. This allows us to simulate control loops explicitly in the time domain. A typical application example is shown in more details in the next section.

3. An Application Example

A typical application of SIESTA involving most of its contents is the simulation of the interferometer longitudinal control, as summarized on Figure 2. This kind of

![Figure 2: Principle of a typical simulation involving the longitudinal control of the interferometer.](image)

simulation involves the simulation of the suspended mirrors, the simulation of the interferometer response accounting for non-linearities and frequency dependence, the conversion of the signals collected on the photodiodes, the (frequency dependant) extraction of the mirror position error signals, and the processing to extract correction forces fed back to the mirrors. Figure 3 shows the spectrum obtained from a FFT of an observed sequence of values of the main signal at the antisymmetric port of the interferometer for a given choice of control strategy. To translate this spectrum into a sensitivity spectrum, a pseudo calibration can be performed by measuring the transfer function between the differential vibration mode of the
interferometer and the observed signal. This transfer function can then be used to unfold the detector response in the spectrum considered. This spectrum is shown as a matter of example and illustrates how such a simulation gives us the possibility to evaluate the strategy choices for the control and their robustness, as well as the possible effect of the limited dynamics of the converters or the influence of residual mirror misalignments, etc.

![Figure 3: Spectrum of the antisymmetric port signal derived as the Fourier transform of a sequence of values obtained from a simulation in the time domain involving the longitudinal control of the interferometer.](image)

4. Conclusion

The SIESTA software provides a tool for an integrated simulation of the VIRGO interferometer. Its main current application fields are the design of the interferometer control system and the production of simulated data to develop data analysis algorithms.

The software developments are steadily going on. The mid term efforts should concern the simulation of the suspended mirrors - with the implementation of the angular degrees of freedom - and the development of the gravitational wave generator.

References

A COMMON DATA FORMAT FOR GRAVITATIONAL WAVES INTERFEROMETERS

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ABSTRACT

The requirements for a data format suitable for gravitational wave interferometers will be discussed. A prototype of such a format with the associated software will be presented. The LIGO and VIRGO joint study on this topic will be reported.

1. Introduction

Several large interferometers designed for the detection of Gravitational Waves (GW) are under construction around the world. They are designed for permanent operation and will produce very large data set. They will eventually compare their signals and do coincidences searches. Therefore we need not only to design a data format suitable for our type of data but we should also prepare this future work. A minimal level of compatibility would be to use the same data architecture. A better one would be to use the same data format even at the binary level.

In this paper, we first review the data format design issues, then we describe our selected solution and briefly discuss the current status and future developments.

2. Data Format Design Issues

Let us first remind that the interferometric detectors will provide a large amount of data mainly because a small number of channels are sampled at high frequency. This means that we can afford a little overhead for the data organization of all the channels, as long as we stay efficient for the high frequency channels. Therefore, the primarily idea is to group all the information necessary for the understanding of the interferometer behavior in units called frames. A frame contains high frequency and low frequency channels as shown by figure 1. The frame duration is not fixed a priori but will be typically of one or a few seconds. At this point we should notice that a GW event (binary coalescence, supernova,...) will last several frames.

The foreseen searches are of different types. There will be searches for burst events where we will need all the possible detector monitoring information for a few frames. There will be pulsar searches where we just want to integrate one signal but for a very long time. This implies that we need a flexible format where we can remove information without changing the top structure. We need also to be able to add information after the frame building done by the online data acquisition system. The information we want to add is for instance online trigger information, reconstructed h signal,... It could be also simulation information if the frames are produced by a Monte Carlo program. So, the flexibility of the format is one of the key issues and this will be achieved by using a tree like organization.

The format and its associated software (the frame library) has also to fulfill
several technical requirements. Among them, let us mention that it should contain the description of the data stored to simplify to distribution of information and to provide some mechanism for format evolution. The input/output functions have to be efficient to handle the large data flow of the online system (several MBytes/s). It should the simple and portable on a wide type of platforms.

3. The Frame Organization

To implement the tree like structure and to simplify its manipulation, a frame is organized as a set of C structures described by a header holding pointers to additional structures and values of parameters expected to be stable over the integrated time interval: the starting time of the frame, its duration, values produced by the slow monitoring. This header is followed by an arbitrary number of additional structures, each holding the values of a rapidly varying parameter like the main signal, the seismic noise. Each active element producing data at a rate higher than the frame rate is thus accumulated in a dedicated structure.

This frame structure is a standard which has to be conserved over the various stages of the analysis. Frame history, detector geometry, trigger results, monitoring data, reconstructed data, simulation results lead thus just to additional structures. It is always possible to add new structures or to drop old ones. The figure 2 presents a schematic view of this organization.

4. The Status

In 1996, a LIGO/VIRGO collaborative effort started to investigate the possibility of using a common format. It has recognized the need for a specific software which can handle efficiently our type of data. Starting from a software developed by Virgo, the two groups have identified the needed improvements and defined a set of tests. The software developments are still in progress and the format and associated software is being exercised on the LIGO 40m and on the VIRGO data
acquisition prototype. These efforts should converge on a common software which will be publicly available.

In addition, an interface/translation to other public formats will be studied as a possible future path for public distribution of gravitational data ”products” to complement the frame library software.

Acknowledgments

I would like to thank the LIGO team and especially K. Blackburn for stimulating discussions.

Figure 2: The tree like structure used to stored the data. Only a part of the structures and of their contents is shown in this figure.
ABSTRACT
With present techniques, thermal noise (governed by the Fluctuation-Dissipation Theorem) is expected to limit the sensitivity of the first generation of interferometric gravitational wave detectors only in a small range of frequencies. But if the expected reductions in optical noise and in seismic noise are achieved, then advanced detectors will also require substantial improvements in thermal noise. Both center of mass motion and excitation of internal modes of vibration are expected to contribute significantly. I review the present state of our understanding, and list the open questions that will require answers before we can achieve our goals. I also outline an experimental research program that will help find the answers to these questions.

1. Why Thermal Noise Is Important
The strongest gravitational waves, those from astrophysical sources such as neutron star binary coalescence, supernovae, or black hole formation, are expected to produce strains of $10^{-21}$ or smaller. This means that gravitational wave detectors must meet daunting performance specifications if success is to be achieved. Simply to register such small spatial perturbations requires measurements of unprecedented precision. Simultaneously, the test masses, which must be free to respond to the gravitational wave, must also be isolated to an unprecedented degree from other disturbing influences.

Measurement precision will be achieved by making the arms of the Michelson interferometer from Fabry-Perot cavities (allowing the signal to accumulate for a time comparable to half the period of the wave), and by illuminating the interferometer with sufficient optical power to make the fundamental limit to strain measurement precision, shot noise in the power measurement at the interferometer output port, small enough.

The two chief disturbances that compete with the gravitational wave effect are seismic noise and thermal noise. External vibrations from natural or man-made sources will be isolated by mechanical filters of many poles, including the pendulum suspensions of the test masses themselves. Minimization of thermal noise requires designs that reduce to the lowest possible levels the amount of mechanical dissipation in the test masses themselves, and in their suspensions.

Estimates of the noise spectra of interferometric gravitational wave detectors follow a basic pattern: the high frequency band will be dominated by the shot noise in the precision of the interferometric readout, while the low frequency band will be dominated by insufficiently filtered seismic noise. In between, thermal noise is likely to be the strongest source of noise. In the earliest versions of these devices, seismic noise and shot noise may be high enough that thermal noise will dominate.
over only a very small band. An estimate of the noise budget for the initial LIGO interferometer is shown in Figure 1.

There are well-developed strategies for reducing shot noise (with high power lasers and various forms of the optical techniques known as recycling), and for reducing seismic noise (by constructing vibration filters with lower resonant frequencies.) Unless there is comparable progress in reducing thermal noise from the levels we know how to achieve now, future improvements in gravitational wave sensitivity may come slowly. Without such progress, sensitivity in the entire band from 10 Hz to 1 kHz may be limited by thermal noise.

1.1. Physics of Thermal Noise

Although it has roots in the study of Brownian motion by Einstein in the early years of the 20th century, the fundamental theoretical underpinning of our understanding of thermal noise is the Fluctuation-Dissipation Theorem. The first clear statement of the theorem in its general form is usually credited to Callen and his collaborators. This powerful theorem describes the fluctuations of any linear system in thermal equilibrium at a temperature $T$. For our purposes, the most useful statement the theorem is the expression for the displacement power spectrum $x^2(f)$ at the point of interest,

$$x^2_{\text{therm}}(f) = \frac{k_B T}{\pi^2 f^2} \Re(Y(f)),$$

where $Y(f)$ is the admittance of the system, $v/F$. The real part of the admittance, $\Re(Y(f))$, is a measure of the amount of dissipation in the system.

In an interferometric gravitational wave detector, the most important point at which thermal noise will appear is at the front surface of a test mass. The thermal noise power spectrum at that point is dominated by motion in three kinds of modes: the fundamental pendulum mode, modes of the pendulum wires, and internal modes of the test mass itself. For any single normal mode, the thermal noise power spectrum can be found by solving for the admittance $Y$. It is given by

$$x^2(f) = \frac{4k_B T k \phi}{2\pi f][(k - m(2\pi f)^2)^2 + k^2 \phi^2],$$

where $k_B$ is Boltzmann’s constant, $T$ is the temperature, $k$ is the spring constant, $m$ is the mass, and $\phi = \phi(f)$ is the loss angle, representing the fractional part of the spring constant associated with a dissipative (out-of-phase) response as opposed to the elastic (in-phase) response of the spring.

The integral of the thermal noise over all frequencies corresponds to an energy per degree of freedom of $k_B T/2$, as one would expect from the Equipartition Theorem. When the level of dissipation is very low, almost all of that noise power is concentrated at the resonant frequency. A fundamental part of the design strategy of gravitational wave interferometers is to place as many of the mechanical resonances as possible outside of the signal band of interest. The pendulum mode will
Figure 1: Estimated noise budget of the initial LIGO interferometer.
have a frequency of about 1 Hz, while the test mass internal modes will all be in excess of 10 kHz. In this circumstance, only a tiny fraction of the noise remains at frequencies far from the resonance. The smaller the dissipation, the lower the spectrum of off-resonance thermal noise.

(The one exception to the design strategy is the “violin modes” of the pendulum wires. But, since the vibrating parts have small mass compared to the test masses themselves, the net motion of the test masses is very small, and will only be visible in very narrow bands right at the resonant frequencies.)

The Fluctuation-Dissipation Theorem itself is an established part of the field of non-equilibrium statistical mechanics. But the linkage of the theorem with the phenomenology of internal friction in materials has some surprising features. Most striking is the common case in which damping has the so-called “structural” form, in which the loss angle $\phi(f)$ is a constant. When the frequencies of interest are low compared to the resonant frequency of an oscillator (such as for a test mass mode) the thermal noise power spectrum has a frequency dependence proportional to $1/f$. This spectrum is formally divergent if extended to arbitrarily low frequency. Damping with this form would be troubling from a practical as well as theoretical point of view if it applied to the fused silica test masses of LIGO; it could lead to thermal noise being the dominant (and nearly white) noise term in the 100 Hz band in any advanced receiver, unless dissipation levels were reduced substantially from today’s $\phi \sim 10^{-6}$.

2. Recent Results at Syracuse

2.1. Dissipation in Pendulum Wires

In order for the mirrors in a gravitational wave interferometer to approximate freely-falling test masses, they must be suspended with a resonant frequency that is low compared with the signal frequencies of interest. The universal choice of a pendulum for this suspension comes from a special feature of pendulums that should make thermal noise lower than in a suspended purely elastic restoring forces. The dominant restoring force in a pendulum comes from the horizontal component of the constant tension in the wires due to the weight of the test mass, with only a weak additional elastic restoring force from flexing of the wires. While there is internal friction associated with the latter elastic effect, there is none (to first order) associated with the tension restoring force; its horizontal component arises purely from the change in angle of the wire as it moves. Thus the dissipation from internal friction is diluted by a factor representing the latter’s fractional contribution to the total restoring force.

The validity of this “dissipation dilution” effect has been assumed in all models of the strength of thermal noise in gravitational wave interferometers. Since in typical pendulum designs the elastic restoring force is of order $10^{-3}$ times the tension restoring force, this feature of a pendulum suspension is responsible for a factor of $\sqrt{10^{-3}} \approx 1/30$ reduction in thermal noise amplitude, compared with the case of a purely elastic suspension.
While pendulums are in fact generally seen to exhibit high quality factors, there has been little careful checking of the validity of the model discussed above. Part of the reason is that recoil of the structure that supports the pendulum may diminish the quality factor (or $Q$) that it will exhibit, without invalidating the model or thermal noise predictions based on it.

The 1996 Ph.D. thesis of Yinglei Huang at Syracuse was devoted to the sort of detailed checks that this central feature of interferometer designs deserves. The motivations of his work were twofold: to obtain as clean a check of the theory as is possible, and also to use detailed comparisons of theory and experiments as a tool to hunt for excess loss mechanisms in addition to internal friction in the wires.

Huang primarily measured quality factors of the violin modes of wires in a variety of styles of pendulums. Violin modes exhibit the same dissipation dilution effect as the pendulum mode itself. (This has been known for some time, but the most thorough theoretical treatment of the problem is a paper published by González and Saulson in 1994.) The advantages of study of violin modes come from our ability to 1) isolate the modes from recoil in the structure by interposing a compact upper mass in a double pendulum arrangement, 2) check the frequency dependence of various effects by comparisons of the $Q$’s of various modes, and 3) compare $Q$’s under tension with zero-tension dissipation measured in the same band of frequencies, in the manner of Kovalik and Saulson.

The last element of the list given above is the key to the precise test of theory Huang carried out, and is responsible for several of his discoveries. A measurement series began by hanging a sample wire by means of a clamp at its top, under no tension other than the weight of the wire itself. (Spooled wire can require straightening by stretching or annealing; some of Huang’s latest measurements used wires that have never been spooled.) Except for the lowest mode of such a wire, the spectrum of transverse modes accurately matches that expected from a cantilever clamped at one end; in that case the modal quality factors $Q_n$ are related to the loss factor of the wire by the simple relation

$$Q_n^{-1} = \phi(f_n).$$

Since these modes are closely spaced, we obtain excellent information on the frequency dependence of the internal friction of the wire.

Because this measurement of the internal friction of the wire is obtained from the very same sample about to be placed under tension in a pendulum, held in the same way and excited into the same sort of transverse oscillation, we have the right to expect that a correct theory of the “dissipation dilution” effect would give a close match to the quality factors measured under tension. Repeatability of such measurements is typically at the 10% level or better; this sets the quality of our confrontation between theory and experiment.

Experiments were carried out primarily on wires of stainless steel and of tungsten, in thicknesses close to those appropriate for suspending 10 kg masses in single or double slings. Wires were subjected to wide ranges of tension up to their break-
ing strength, attached at their ends in several different ways. The result of a large number of measurements is summarized below.

1. “Dissipation dilution” works as advertised in a large number of cases. Many modes in many wires, under tensions close to the breaking strength, show the improvement in $Q$ predicted by the theory. The highest $Q$ observed in agreement with theory is $4 \times 10^5$.

2. Typically, the best agreement between theory and measurement is seen at the highest tension. The only indication of internal friction enhancement at high tensile stress is a 40% degradation of $Q$ in tungsten wires loaded to 160% of the tabulated breaking stress. Larger disagreements with theory at low tension appear to be an indication of sliding friction, since they occurred only with prism contacts, not in bolted clamps (see next item).

3. When wires are attached by being held by tension against sharp prisms, sliding friction can be an important source of excess loss. Sliding friction has an intrinsically non-linear dependence on the tangential force between the wire and its contact, so can be diagnosed by measurements at large amplitude. It can take on a linear dependence at small amplitudes, in the presence of a bias force, such as that caused by misalignment of wires, or when low frequency modes (e.g., pendulum, rocking, torsional) are excited to large levels. Sliding friction can be substantially reduced when wires are held between machined steel plates clamped together by screws; under these circumstances, we almost always see losses within a factor of two of the theoretically predicted level. Sliding friction is large when wires are held over glass prisms.

4. Thermoelastic damping, the dominant internal friction mechanism in steel wires of the appropriate thickness, has a different frequency dependence when the wires are under high tension than that predicted by the classic theory of Zener. The change is caused by non-negligible conduction of heat along the length of the wire, instead of purely transverse conduction. This occurs when tension is large enough that the length scale of elastic bending becomes almost as short as the diameter of the wire. The effect has been seen in steel wires, although comparison with theory at better than the factor of two level has not been carried out; accurate calculation will require thermal analysis using finite element techniques.

5. No wire yet tested has shown $Q$ in agreement with theory at the 10% level for all modes, so we suspect at least one undiagnosed loss mechanism is present. Mechanisms tested or calculated and shown to be negligible at the $Q \sim 10^6$ level include: recoil in support structure, excitation of test mass internal motion, eddy currents, and residual gas damping.

In a real sense, even our discrepant measurements represent a substantial accomplishment. Previously, we have only been able to trust comparisons between
theory and experiment at the factor of two level. At this looser tolerance, all of our careful measurements agree with theoretical predictions.

2.2. Time-Domain Measurements of Test Mass Dissipation

Once interferometers perform beyond the level of the seismic noise and the shot noise in the “initial LIGO” goals, thermal noise from the internal modes of the test masses may dominate the noise budget over a wide band near 100 Hz. This would be the case for test masses with the presently-achieved $Q$ values, if their dissipation is approximately of the “structural” form.

At present, the evidence for structural damping in fused silica comes only from combining $Q$ measurements at different frequencies made in different oscillators. What is really desired for such an important noise source is the ability to determine the level of dissipation, as a function of frequency, for actual test masses. The obstacle to obtaining this kind of information has been our reliance, up until now, on determining dissipation by measuring quality factors of resonances; test masses are specifically designed to have no resonances in the frequency band in which LIGO will search for signals.

This obstacle could be overcome if we replaced measurements at resonant frequencies with measurements of a test mass’s response to an impulse or step function in stress. There are well-developed techniques in anelasticity theory to interpret such time-domain measurements in the more familiar frequency-domain language of $\phi(f)$. On various occasions this technique has proven the most useful way to interrogate physical systems.

An especially clean version of this kind of measurement is the one called the anelastic aftereffect. The system under test is prepared by being subjected to a stress for a long time. Then the stress is released and the strain of the system is measured as it evolves toward the relaxed state. If the time evolution of the system’s strain is given by $J(\tau)$, then the loss function can be shown to be

$$\phi(f = 1/2\pi\tau) \approx \frac{\pi}{2} \frac{d \ln J(\tau)}{d \ln \tau}.$$

A system governed by structural damping will exhibit an anelastic aftereffect with a time dependence proportional to $\ln \tau$.

The anelastic aftereffect is the response function of choice for two reasons. Firstly, system preparation is easy; simply squeeze the test mass in some kind of vise that can be released rapidly. The squeezing needs to last for a time long compared with the longest time scale at which one wants to measure $J(\tau)$. Secondly, anelastic aftereffect measurements are made only after the vise is no longer in contact with the sample. This means that internal dissipation in the squeezing mechanism can make no contribution to the measurement, and need not be a worry.

Since one can apply a large stress to the test mass, in principle one need not use an especially sensitive sensing system to measure $J(\tau)$. But there are several aspects of the problem that make important demands on this measurement. Firstly, low dissipation translates into small values of $J$; $Q \approx 10^6$ means fractional length
changes of order $10^{-6}$. Secondly, a suspended test mass will respond with pendulum, torsional, rocking, and bouncing motions at some level; it is necessary to have a sensing system that strongly rejects rigid body motions in favor of sensing strain itself.

Some version of an interferometer might very well be arranged to meet these demands. But a very simple (and thus superior) proposal was made by Alex Abramovici of the LIGO group at Caltech: shine polarized light through the bulk of the test mass, and determine $J(\tau)$ by measuring the rotation of polarization caused by the photoelastic effect. As with so many good ideas, this one had already been anticipated by others.\textsuperscript{10}

At Syracuse, postdoctoral research associate Mark Beilby has designed and assembled an apparatus to make these measurements. He now has a functioning apparatus able to resolve loss factors below the $10^{-5}$ level. The obstacle is not raw sensitivity; the shot noise limit in a single measurement is about $10^{-6}$ with the low power (10 mW) laser we are using. Instead, Beilby’s efforts have mainly been aimed at diagnosing and removing the sensitivity of the apparatus to residual excitation of low frequency modes in the test mass suspension and in the optics. Both the excitation of such modes and the sensitivity of the optics to them have been dramatically reduced, and it looks hopeful that sensitivity at or near the shot noise level can soon be achieved. Figure 2 shows a schematic diagram of the apparatus he has constructed, and shows an example of $J(\tau)$ measured for BK7 glass.

3. Future Research Topics at Syracuse

3.1. Study of Old LIGO 40-M Test Masses with Time-Domain Technique

One early scientific goal of our development of the time-domain dissipation measurement scheme is to check an important hypothesis concerning the noise model of the LIGO 40-meter interferometer. As described in LIGO Technical Report TR94-7,\textsuperscript{11} a dramatic change was achieved in the 40-meter’s noise spectrum after a change made in October, 1994. The noise in a band from 200 Hz to about 1 kHz, which had remained constant during several years of design changes, dropped by factor of 3 when the composite test masses were replaced by monolithic fused silica test masses. This was taken as confirmation of an hypothesis that the noise had been due to thermal noise in the original test masses. Previous investigation had shown that the $Q$’s of the internal modes of these masses were surprisingly low, some as low as 800. If the dissipation were of the “structural” form (loss angle $\phi =$ constant), then the old noise level would have been consistent with a thermal noise origin.

If true, then this would represent the first example of the observation of broadband thermal noise (away from resonant peaks) in the noise spectrum of a gravitational wave interferometer, as well as an indication that the structural damping picture could be important in such instruments. But there has been heretofore no way of confirming the thermal noise explanation, because we lacked a procedure for determining the level of dissipation away from the resonant frequencies. (The
Figure 2: a) Schematic diagram of apparatus for time-domain dissipation measurements. b) Anelastic aftereffect \( J(t) \) for sample of BK7 glass, in air. The sample is a cylinder (0.10 m in diameter by 0.089 m in length), and was resting on a 4-point rigid mount.
lowest resonance in these masses is at about 25 kHz.)

With the new time-domain dissipation measurement apparatus, we should be able to check the structural damping hypothesis for these masses. It represents an excellent test case for the technique, for several reasons. Firstly, an important scientific hypothesis (that the 100 Hz to 1 kHz noise level was dominated by thermal noise until 10/94) can be unambiguously tested. Secondly, the level of dissipation is large enough that we should be able to see the signal very clearly. Thirdly, this case will demonstrate another key advantage of the technique, namely that it is applicable to complete test masses as assembled and installed; this means it can diagnose problems whether they arise from intrinsic loss processes in the body of the fused silica test mass or instead from technical details of the suspension.

The LIGO Project has loaned us one of the test masses that were removed from the 40-meter interferometer in October, 1994. We have made a preliminary attempt at the measurement of the internal dissipation in this mass. At present, we can set an upper limit on the dissipation just above the level at which it would explain the LIGO noise results. The difficulty is that this mass has a very large degree of “frozen in” stress birefringence, associated with the attachment of the small mirror to the front of the test mass. This gives the measurement a much larger parasitic sensitivity to rigid body modes than for more typical samples. We will attempt to reduce our excitation of these modes, and if successful will be able to test the model that thermal noise was responsible for the change in the 40-m noise spectrum in October 1994.

3.2. Tests of Dissipation in Monolithic Fused Silica Test Masses

As interesting as the above measurement is, it only represents a warm-up to the most important application of the technique. Whatever we learn about the old 40-meter test masses, it will be telling us only about a test mass design known to be flawed by excess losses. The real payoff will come from studying test masses whose performance is expected to meet or exceed the initial LIGO standards. For this purpose, we will need to ensure that the apparatus will work at or near its shot noise limit, since fused silica loss factors are of order $10^{-6}$ or lower. (To push to sensitivities below $10^{-6}$, we can use more passes of the optical path through the test mass, and can also average multiple measurements.)

The crucial issue to be tested is whether velocity damping is an approximately correct model for fused silica, whether the newer structural damping model (with its larger thermal noise) is a better representation, or indeed whether Nature is unkind in terms of the dissipation in fused silica at low frequencies. Hanging in the balance are the prospects of early achievement of the advanced LIGO sensitivity goals. The greater the departure from velocity damping, the greater will be the improvement in dissipation levels required to bring down thermal noise in the crucial 100 Hz band.
3.3. Dissipation in Alternative Fused Silica Recipes

The already low level of dissipation in fused silica (no matter the answer to question of the frequency dependence of the dissipation) is one of the greatest undeserved gifts from Nature to the gravitational wave detection effort. Great, because if we did not possess this one material with excellent optical properties and dissipation of $10^{-6}$ or better, it is doubtful that any of our present designs for interferometers would work well enough to detect gravitational waves. Undeserved, because we have no firm physical understanding of the level of dissipation seen in fused silica at room temperature in the acoustic frequency band.

Suggestive models exist to explain the low temperature dissipation peak ($\phi \sim 10^{-3}$ at $T \sim 40$ K), involving “tunneling states”; while the identity of the relaxing degrees of freedom has not been identified, it is known that they must be generic features of amorphous solids, since the behavior of the low temperature dissipation and the specific heat is nearly universal. But the characteristic energy scale of this mechanism is far from what would be required to explain acoustic frequency dissipation at room temperature.

The closest this model comes to explaining dissipation in our range of interest is as the leftover remnant far out on the tail of the distribution of individual relaxation sites. But there are not strong reasons for choosing any particular distribution of site properties. This is actually a hopeful state of affairs; it suggests that the room temperature dissipation might not be universal, but instead might be manipulated, if only we knew what to manipulate.

This point of view may be consistent with the idea that dissipation levels could be controlled by the chemical state of the fused silica sample. Fraser found a correlation between room temperature acoustic loss and the oxidation state of various kinds of silica. Braginsky has noted that the highest $Q$ values ever measured in fused silica ($Q \approx 3 \times 10^7$) were obtained only in ultra-high purity samples. Perhaps it is the case that chemical impurities have an influence on the distribution of properties of the relaxing degrees of freedom.

These chemical hypotheses are worth pursuing farther. We have already had preliminary discussions with researchers at Corning, who had previously investigated manipulating impurity levels to study the effect on optical properties of fused silica. They have supplied us with small samples of fused silica with the following sets of properties: standard 7940 fused silica, low Cl fused silica, silica with low OH but high Cl, samples of 7940 modified by two different annealing procedures, and one more produced in an oxidizing flame. We will test these samples for variations in dissipation, both by measurements of resonant $Q$’s (as a function of temperature) and by the time-domain method. The Corning group has expressed an interest in supplying additional custom-made samples, if our preliminary investigation warrants it.

3.4. Integrated Suspension/Test Mass Design

In collaboration with the LIGO Project, the GEO project, and the Stanford group, we intend to continue our own research program on dissipation in pendulum
suspensions. The new focus will be the study of designs proposed for the LIGO “enhanced” interferometers. We need to revisit our experiment that cast doubt on the utility of small sharp fused silica prisms as the contact between wire and test mass. Perhaps it can be rescued by redesign of the prism, such as by adding a vertical groove as appears in many designs, or by securing the wire by glue as others have done. It may turn out that another idea entirely works better than any of these. Even if the best design has already been sitting right in front of us, it is still important to obtain a more detailed understanding of the underlying dissipation mechanisms and of the critical design parameters than anyone has obtained to date, so that actual components can be designed with confidence and debugged with insight.

Acknowledgements

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References

14. V.B. Braginsky, private communication.
ABSTRACT
Thermal noise in mechanical suspension systems is presently the most severe limitation to the sensitivity of the new generation of interferometric gravitational wave detectors, like VIRGO, in the frequency range between 4 and 500 Hz.

1. Introduction
Thermal noise is the physical source of the fluctuations affecting any measurable quantity of a macroscopic system in thermal equilibrium with its environment. As such, it is ubiquitous and is one of the unavoidable limits to the precision of mechanical measurements.

Thermal noise is a manifestation of the internal energy of a macroscopic apparatus at thermal equilibrium. This energy is shared between all its degrees of freedom or, equivalently, between all its normal modes each carrying an average energy $kT$, where $k$ is the Boltzmann constant and $T$ the equilibrium temperature. This is true also for such modes as the oscillations of springs, pendula, needles, etc. Such an energy manifests itself as a random fluctuation of the relevant observable and it is experimentally perceived as the noise affecting its measured value.

2. The Thermal Noise Limit to the Antenna Sensitivity
Thermal noise poses a severe limit to the sensitivity of interferometric gravitational wave detectors due to the fluctuations in the position of the suspended elements (test masses and optics) of the interferometer and to the internal modes of the mirrors. The whole suspending structure (called super attenuator in the case of VIRGO) can be treated as a multi-stage pendulum whose element positions will fluctuate in time. Such fluctuations combine with the gravitational wave induced displacement, thus setting a lower limit to the antenna sensitivity.

The detector sensitivity curve can be estimated by measuring the dissipation properties of the suspension structure or, to be more precise, the imaginary part of the response function of the observable of interest $x(t)$ (i.e. the position of the mirror center of mass) to the conjugate force $f(t)$. The response function

$$H(\omega) = \frac{X(\omega)}{F(\omega)}$$

is obtained by recording the time series of the force and the relevant displacement and, then, taking their Fourier transforms (capital letters). Finally, the fluc-
tuation spectral density (thermal noise) is computed from the $H(\omega)$ imaginary part, $H''(\omega)$, by using the Fluctuation-Dissipation theorem (FDT).\footnote{1,2}

\begin{equation}
<x(\omega)^2> = -4kT\frac{H''(\omega)}{\omega} \tag{2}
\end{equation}

The quantity $H''(\omega)$ can be accessed experimentally and the fluctuation power spectral density $<x(\omega)^2>$ can be obtained accordingly.

In fig. 1 we show the thermal noise limit to the VIRGO sensitivity curve, as estimated with current mathematical models,\footnote{2,4} with the help of experimental measurements\footnote{5} at fixed frequencies to determine the noise absolute level.

![Virgo Thermal Noise](image)

Figure 1: Thermal noise contribution to the VIRGO sensitivity curve. On the vertical axes we have the expected $h$ (in $Hz^{-1}$), while on the horizontal axes we have the frequency (in $Hz^{-1}$). The major contributions arise from the longitudinal pendulum mode of the test mass and the test mass (drum) normal modes (global effect). The low frequency (below 10 Hz) peaks arise from the coupling between the vertical and horizontal motion, due to the geometric curvature of the earth surface.

### 3. Mechanical Shot Noise Induced by Creep

Another thermal noise related source which can play a role in limiting the interferometer sensitivity is represented by the mechanical shot noise induced by stationary creep in heavily loaded mechanical suspensions (wires, spring blades, etc.).

On long time scales the static relaxation properties of a mechanical device under tensile stress can show the so called creep phenomenon which becomes appreciable
at high temperature and for heavy loads\textsuperscript{7}). As a result, the mechanical losses consist of two independent contributions: the conventional internal friction (independent of load), and the mechanical shot noise related to a stationary creep mechanism.

Preliminary estimates based on a simple dynamical model\textsuperscript{8} and recent measurements taken in Perugia show that this effect can be significantly reduced by pre-heating the suspension wires at a temperature of about 150°C for one week. Under this condition the estimated shot noise is much below the pendulum thermal noise, for all the frequencies of interest.\textsuperscript{8}

4. How to Reduce Thermal Noise Effects

Once the expected spectral noise properties of the thermal noise affecting the system are known, a crucial task is the reduction of thermal noise effects in the frequency band of interest. This is usually accomplished by acting on the dissipation properties of the mechanical structures. After reducing external losses, one has to address the internal friction effects and creep mechanisms. An effective quenching strategy is mainly based on a proper choice of the material and on a careful characterization of the intrinsic dissipation mechanism.

Such a characterization is performed phenomenologically through the study of the material loss angle $\phi$. This empirical quantity which should account for a number of different dissipation sources resists a general theoretical treatment.\textsuperscript{3,5} Moreover, simple "ad hoc models" generally fail to reproduce experimental data.

The detailed knowledge of the dependence of the loss angle on load and frequency is still an open problem, a pre-condition for the realization of highly sensitive mechanical devices. Such a strategy for the reduction of thermal noise effects is commonly implemented by designing mechanical oscillators with high quality factor.

The effectiveness of such a strategy relies much on the assumption that the loss angle is independent from the frequency. While this assumption is far from being generally verified by experiments, we still miss a viable approach to frequency dependent losses. In particular, the problem of measuring losses in the low frequency regime, for a wide frequency band, is still unsolved.

References

8. G. Cagnoli et al. (1996) - to be published.
Q MEASUREMENTS OF FULL SCALE PENDULUM PROTOTYPES

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ABSTRACT

A series of measurements were undertaken to measure the quality factor of a full scale VIRGO pendulum. The measurements were performed on a structure whose recoil losses placed an upper limit of $7.6 \pm 0.7 \times 10^6$ for the measurable $Q$ of a pendulum with a mass of 21 Kg. The suspension wires consisted of two loops of C85 harmonic steel with a diameter of 200\textmu m that formed a pendulum with a length of 0.70 m. The tops of the wires were held by two pieces of tool steel inside an aluminum clamp. The theoretical value for the pendulum $Q$ was calculated to be $3 \times 10^6$ for a wire loss of $\delta_{\text{wire}} = 5 \times 10^{-4}$. The measured pendulum $Q$ varied from $1 \times 10^5$ for the wire simply cradling the mass to $5 \times 10^5$ for the wire being offset from the mass with a cylindrical spacer. The best measured $Q$ was $1.3 \times 10^6$ for a 20.5 Kg polyethylene mass with metal clamps rigidly attached to it. The clamp that holds the top of the wire performs satisfactorily. The main factor that limits the pendulum $Q$ is the way the wire is attached to the test mass.

1. Introduction

The sensitivity of VIRGO will be limited by the pendulum thermal noise between the frequencies of 4 Hz and 60 Hz. A great deal of effort has already been undertaken to study the wire material and the type of clamp for the pendulum suspension in order to achieve a high $Q$.\textsuperscript{1} The final test of the research and development for the VIRGO pendulum suspension is the measurement of the pendulum $Q$ on a full scale prototype.

The first part of this discussion looks at the results from earlier tests and then scales them to predict the $Q$ for a full size pendulum. The experimental technique and apparatus are then discussed. Finally, the results of the experiments are presented.

2. Background Parameters and Pendulum Q Calculation

The mirrors next to the beam splitter of VIRGO will be 21.2 Kg cylinders of fused silica with a diameter of 350 mm and a thickness of 100 mm. For the purpose of this discussion, we only refer to these mirrors. (The end mirrors will be of the same diameter, but with a thickness of 200 mm instead.)

Each mirror will be hung by cradling it with two wire loops that are 50 mm apart and centered on the cylinder. The length of the pendulum will be 700 mm. The wire material will be C85 harmonic steel with a diameter of 0.20 mm. This

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material was chosen for its low loss (see figure 1) and its high breaking strength. The wire is loaded at 65% of its breaking strength of 2.6 GigaPascals. The elastic limit of this wire is at 90% of its breaking point. The standard calculation of the

\[
\phi(f) = \phi_0 + \Delta \left( \frac{f}{f_0} \right) \left( 1 + \left( \frac{f}{f_0} \right)^2 \right)
\]

\[
\phi_0 = 0.00019
\]

\[
f_0 = 474.6
\]

\[
\Delta = 0.00186
\]

Figure 1: A plot of the wire loss angle \( \phi \) as a function of frequency for a C85 harmonic steel wire of diameter 0.20 mm. The test was performed by clamping the top of the wire between two pieces of tool steel and letting it hang freely. Various modes of the wire were excited electrostatically and the decay measured using a shadow meter. The fit represents a function that consists of a constant loss angle \( \phi_0 \) plus a thermoelastic damping term.

The pendulum Q comes from the formula

\[
Q_{\text{pend}} = \frac{L}{\phi_{\text{wire}}} \sqrt{\frac{Mg}{EIn}}
\]

where \( \phi_{\text{wire}} \) is the wire internal loss, \( L \) is the length of the pendulum wire, \( M \) is the mass of the pendulum, \( g \) is the gravitational acceleration, \( E \) is the Young’s modulus, \( n \) is the number of wires and \( I = \frac{\pi r^4}{4} \) (\( r \) = wire diameter) is the moment of inertia of the wire cross section. The Young’s modulus for steel used in this calculation is \( E = 2.0 \times 10^{11} \text{N/m}^2 \). The \( \phi_{\text{wire}} \) is \( \sim 5 \times 10^{-4} \) which is the value usually found for small (500 g) pendulum prototypes at 1 Hz. Assuming the wire loss does not increase with tension, the pendulum Q should be \( 2.5 \times 10^6 \)
3. Experiment

The pendulum Q is limited by a number of different effects including residual
gas damping, clamp losses and recoil of the structure that supports the pendulum.
The gas damping is made negligible by doing the experiment under in vacuum at a
pressure of $\sim 10^{-5}$ torr which would give a pendulum Q better than $10^8$.

The clamp design was studied in order to minimize the losses produces by stick-
slip friction effects. The wire is squeezed between two pieces of tool steel that are
imbedded inside two aluminum pieces. The clamping pressure is exerted by two
M6 screws that are tightened to a torque of 14 Nm which is near the point where
the screw breaks or the thread inside the aluminum is ruined. One of the pieces
of tool steel has a 200 $\mu$m groove on its surface for aiding in the alignment of the
wire. Squeezing the wire at a high pressure is necessary in order to reduce the losses
induced by the clamps. While this deforms the wire considerably, it does not seem
to have much of an effect on the the wire breaking point.

The pendulum was suspended from an “A” frame type steel structure that was
designed to be very stiff at about 1 Hz. The structure is bolted directly to the
bottom of a vacuum tank that is about 1 metre in diameter. The vacuum tank is
clamped by six 20 mm bolts that are imbedded in a block of cement that is 1.5 m
by 1.5 m wide and 0.5 m deep. The recoil losses of the structure place an upper
limit on the pendulum Q given by

$$Q = \frac{Mg}{kl\phi}$$  

where $k$ is the spring constant of the structure (at the pendulum frequency) and $\phi$ is
the phase angle between the force applied to the structure and its displacement. The
measured values for these paramters are $k = 3.5 \pm 0.1 \times 10^7 N/m$ and $\phi = 0.94 \pm 0.08^\circ$.
This sets an upper limit to the measurable Q (for a 20 kg mass) of $7.6 \pm 0.7 \times 10^6$.

The motion of the pendulum was measured with a “shadow meter” that con-
sisted of an LED and a bi-cell photodiode. The shadow was cast by the pendulum
suspension wire.

The pendulum was excited electrostatically by placing an aluminum plate ap-
proximately 30 cm in diamter a few millimeteres from the mass. The plate was
biased to about 1 kilovolt and the a sinususoidal signal from the pendulum motion
was superposed on this with a typical amplitude of almost a kilovolt. The pendu-
lum signal was phase shifted until it produced a positive feedback and increased the
amplitude of the pendulum motion.

The pendulum was then left to swing freely and the amplitude of the oscillation
was then recorded using a PC.

Since the the pendulum is symmetric in its two degrees of freedom, the two
pendulum frequencies are very close. This leads to a signal that consists of two
beating sine waves that are decaying in amplitude. The amplitude of the pendulum
motion then needs to be modelled by a function of the form

$$A(t) = A_1 e^{-\gamma t} + A_2 e^{-\gamma t} \sin(\delta \omega t + \phi)$$  (3)
where $\delta \omega$ is the difference in frequency between the two pendulum modes. The pendulum $Q$ is then

$$Q = \frac{\omega_0}{\gamma_1}$$

(4)

4. Results

The first configuration tested was the simplest possible, namely a 21 Kg herasil mirror with a diameter of 350mm and a thickness of 100 mm cradled by the two wire loops. The Q was very highly amplitude dependent and was $\sim 10^5$ at best.

The next configuration added various sets of spacers between the wire and the mirror surface. A cylinder 100 mm long with diameter varying between 5 and 10 mm was placed on each side of the mirror. The material of the spacer was either aluminum or stainless steel. One set of spacers that was tested had narrow grooves cut into them in order to trap the wire. In all cases with the spacers, the Q improved to $\sim 6 \times 10^5$ and was much less amplitude dependent (see figure 2). One set of spacers actually had clamps attached to it. This did not improve the Q which suggests that there is rubbing between the spacer and the mirror surface. The next designs tried to attach a clamp directly to the mirror surface. Since the herasil mass is quite expensive and needed for other tests, other masses were used. The wires were epoxied to an aluminum mass of the same dimensions of the VIRGO mirror (but a mass of 26 Kg). The Q was amplitude dependent (although less than that for the wire simply cradling the mirror) and the Q was at best $\sim 10^5$. Clamps were the screwed into the mass, but the pendulum Q only improved to about $Q \sim 6 \times 10^5$. In this case, the Q could be limited by eddy current damping of the Aluminum mass moving through the earth’s magnetic field.

A dummy mirror that consisted of circular glass plates that were glued together was then used. Aluminum clamps were attached to the mirror using vacuum epoxy and also a metal–glass bonding cyanoacrylate. In both cases, the Q was $\sim 5 \times 10^5$. The clamps were also attached by a large hose clamp that was tightened around the diameter of the mass. Again, the Q was about $\sim 5 \times 10^5$, although slightly more amplitude dependent than the glued case.

Since it seemed that Q’s were consistently about $Q \sim 5 \times 10^5$, a new set of experiments were made using single wire pendula and steel masses that varied from 1 to 5 Kg. The wire was attached to the mass by a pin vise type clamp. The Q’s were on the order of a few $\times 10^6$ which is consistent with the prediction and suggests that the losses in the wire (and clamp) are not strongly tension dependent.

One more attempt was made using a 20.5 Kg polyethylene mass and 4 wires. The wires were clamped to metal rods that were tightly fitted in the mass and then further held by a set screw. The pendulum Q was measured to be $\sim 1.3 \times 10^6$ which is much closer to the predicted value (see figure 3). As part of ongoing research to look at new materials, an informal collaboration with GEO 600 (Jim Hough, Sheila Rowan and Sharon Twyford at the University of Glasgow) was begun to test fused silica suspensions. The best result obtained was a $Q = 1.4 \times 10^7$ for a 1.9 Kg pendulum made from a brass mass and two suspension wires ( the flexural part of
Figure 2: This graph plots the amplitude of the pendulum motion versus time for a variety of different configurations. The wire simply cradling the mirror shows a very strong amplitude dependence while the different configurations with spacers all show approximately the same behaviour. Two straight lines representing $Q$'s of $10^5$ and $10^6$ are also plotted for reference.
Figure 3: A plot of amplitude versus time for a 20.5 Kg polyethylene mass suspended by four wires. The clamps were attached to rods that were tightly fitted into the mass and held by set screws. The dashed straight line is a fit corresponding to a $Q$ of $1.3 \times 10^6$. 

$Q=1.3 \times 10^6$
Table 1: Q measurements for different configurations

<table>
<thead>
<tr>
<th>Material</th>
<th>Q (\sim) Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Herasil</td>
<td>(1 \times 10^5)</td>
<td>highly amplitude dependent</td>
</tr>
<tr>
<td>Herasil with spacers</td>
<td>(6 \times 10^5)</td>
<td>independent of spacer configuration</td>
</tr>
<tr>
<td>Aluminum with wires</td>
<td>(1 \times 10^6)</td>
<td>amplitude dependent</td>
</tr>
<tr>
<td>Aluminum with clamps</td>
<td>(6 \times 10^5)</td>
<td>limited by eddy current damping</td>
</tr>
<tr>
<td>Glass with clamps</td>
<td>(5 \times 10^5)</td>
<td>limited by loss in glue</td>
</tr>
<tr>
<td>Polyethylene with clamps</td>
<td>(1.3 \times 10^6)</td>
<td>not suitable configuration for VIRGO</td>
</tr>
</tbody>
</table>

the rod was approximately 0.2 mm in diameter). This implies the material loss is \(\approx 3 \times 10^{-5}\) which is too large for fused silica. This is probably due to the clamps which were “v” grooves that held a monolithic 3 mm rod end of the wire.

5. Conclusions

Although theoretical estimates give a pendulum Q of \(2.5 \times 10^6\), the best attainable Q for a wire suspension with spacers is \(Q \sim 6 \times 10^5\). This is limited by the clamping between wire and mirror surface. A clamping technique at the top using high pressure to squeeze the wire between two hard materials works well.

Further research investigating other mechanical means to attach the wire to the mirror are needed if high Q pendula are to be achieved. Since this will probably affect the internal Q of the mirror, the overall thermal noise spectrum must be then optimised.

References

DESIGN OF A MIRROR SUSPENSION SYSTEM USED IN THE TAMA 300M INTERFEROMETER

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ABSTRACT
We are designing a suspension system with considerations of several properties necessary to attain the designed performance of the TAMA interferometer: vibration isolation, controllability of the mirror position and angle, thermal noise of the pendulum and the mirror, the vacuum compatibility, and the handling for installation and initial setup. Vibration isolation tests of a preliminary model using a shaking table shows the performance was sufficient for TAMA interferometer in combination with other two isolation systems, stacks and X-pendulums. We are trying to estimate controllability and other performances using the mode cleaner chamber. From these studies, final suspension system will be designed, and at first installed as mode cleaner mirror suspensions.

1. Introduction

The main functions of a mirror suspension system are to isolate the mirror from external vibration and to support it freely to work as a test mass in the gravitational wave detection. In addition, test masses are precisely controlled to form a long-baseline interferometer without applying any external noise that may be degrade the sensitivity. Therefore in designing a mirror suspension system, we especially considered the following several factors: vibration isolation, controllability, thermal noise, and vacuum compatibility.

The current design of the suspension system used in the TAMA interferometer is shown in Figure 1. From a point of view of vibration isolation, the suspension system will be used with stacks and X-pendulums to suppress ground vibration above ∼50Hz and below several hertz, respectively.

The following sections describe details of several factors considered in designing the suspension system, and experiments with a preliminary model.

2. Vibration Isolation

Seismic noise should be reduced by a mirror suspension below aiming sensitivity in the observation band. To attain TAMA sensitivity of $h_{rms} = 3 \times 10^{-21}$ around 300 Hz, the mirror displacement noise should be less than $2.5 \times 10^{-20} \text{ m/ Hz}$. Using typical seismic noise spectrum, $10^{-7}/f^2 \text{ m/Hz}$ ($f[Hz]$ is a frequency of vibration), vibration isolation ratio of better than -150 dB is needed.

To realize such a small isolation ratio, we adopted two-stage pendulum of mirror suspension, since in general multi-stage pendulums have better isolation ratio above
Figure 1: Design of the mirror suspension system used in the TAMA interferometer.
their resonant frequencies. Since pendulum motion of the mirrors needs to be damped to form a stable operation of the interferometer, a passive damping using eddy current with strong permanent magnets is applied for simplicity and vacuum compatibility. Simple damping with magnets which are rigidly fixed to the ground, however, will degrade the vibration isolation at high frequencies, due to damping force excited by the ground vibration. To avoid these effects of ground vibration at high frequencies, damping magnets are flexibly supported with thin rods and blade springs (Figure 1).

Figure 2 shows theoretical vibration isolation ratio for a case of a single pendulum without damping and three cases of a double pendulum with no damping, rigid damping, and flexible damping. At high frequencies, the double pendulum with flexible damping has enough isolation ratio as much as that of double pendulum without damping, while the pendulum motion is damped at its resonances. The isolation ratio (-200 dB at 300Hz) is also enough for our purpose.
3. Controllability

For a long-term stable operation, the mirror position should be controlled precisely at the lock point of the interferometer and aligned properly.

First we considered the dynamic range of mirror control in the direction of optical axis to operate the interferometer. To attain the designed sensitivity of the interferometer, actuators to control a 1-kg mirror must have noise level of less than $2.5 \times 10^{-20} \times (2\pi f)^2 \text{ N/} \sqrt{\text{Hz}}$ which corresponds to $1 \times 10^{-13} \text{ N/} \sqrt{\text{Hz}}$ at 300Hz.

The length of the interferometer with arm length of 300m will change basically by earth tides of $\sim 10^{-5}$m and may be more by diurnal temperature change especially for a shallow tunnel like TAMA (about 3m below the ground surface). Using measured data of a simillar shallow tunnel at the National Research Laboratory of Metrology, we can estimate the diurnal change of $2 \times 10^{-5}$ m and annual change of $3 \times 10^{-3}$ m. To compensate the displacement, the actuator need to apply dc force of $8 \times 10^{-4}$ N (diurnal) and $1 \times 10^{-1}$ N (annual) to the mirror, assuming the pendulum of a 1-Hz resonance and a 1-kg mirror. One actuator is hard to attain such low noise and dynamic range simultaneously.

So we use a combination of two types of actuators: coils and picomotors. A coil and magnet actuator is used to lock interferometer at the operation point mainly against seismic noise. This actuator has sufficiently low noise and will have control range of $\lesssim 50 \mu$m to attain low noise requirement. The diurnal and annual changes of mirror position are compensated by the picomotor (New Focus, Inc.) which is a motorized micrometer. The diurnal or annual position change of the mirror suspended from linear stages with picomotors will intermittently be cancelled by driving the picomotor, probably several times a day; the stroke of the picomotor is designed about 6mm.

Next we considered the control of mirror angles. Using a coil and magnet actuator with the mirror suspended by a loop of wire, to compensate possible long-term drift of the stack inclination, say $10^{-3}$rad, the force of $3 \times 10^{-3}$N will be necessary, assuming the 3-Hz resonance of pitch motion and differential force applied by two actuators at the opposite end of the mirror; it is beyond the limit of the dynamic range if we use the same actuator as used for locking interferometer. Even smaller compensation involves with dc offset current of the coil. So we control alignment by moving uppermost suspension point of the pendulum with a mirror suspended by two loops of wires. Horizontal or vertical movement of the suspension point corresponds to yaw or pitch control of the mirror, respectively.

To control the suspension point, we applied piezo-electric transducers (PZTs) in combination with picomotors. Large inclination is intermittently compensated by the picomotor, while small inclination is controlled by the PZT with a precision of less than $10^{-7}$ rad which will be necessary to attain the designed recycling gain stably. The inclination larger than $\sim 10^{-4}$rad, within which PZTs can compensate, will be compensated by picomotors.
4. Pendulum Thermal Noise

A mirror suspension should be a highly low-loss system in order to reduce the thermal pendulum motion. Loss of final suspension of the mirror and passive damping may limit the sensitivity of the interferometer.

First, we have estimated the effect of thermal noise of the eddy current damping. Owing to the upper-mass damping, the thermally excited displacement of the upper mass is reduced at the mirror whose displacement noise in the observation band is calculated to be below $2.5 \times 10^{-20} \text{m/} \sqrt{\text{Hz}}$ in the direction of optical axis (Figure 3). For the yaw and pitch motion, we have estimated the mirror angle fluctuation of less than $1 \times 10^{-18} \text{rad/} \sqrt{\text{Hz}}$ and $1 \times 10^{-17} \text{rad/} \sqrt{\text{Hz}}$, respectively in the observation band. If we assume the precision of beam centering to be less than 1mm, we can conclude the thermal noise of the magnet damping will not affect the sensitivity as shown in Figures 4 and 5 (less than $3 \times 10^{-24} \text{/} \sqrt{\text{Hz}}$ for yaw motion and $3 \times 10^{-23} \text{/} \sqrt{\text{Hz}}$ for pitch motion in the strain sensitivity).

As for the final suspension of the mirror, we need the quality factor (Q) of more than $5 \times 10^5$ and $2.5 \times 10^5$ for pendulum and wire resonance modes, respectively (Figures 3–5). Currently we are estimating the Q of final suspension experimentally in several methods: measurement of Q using two identical pendulums to avoid recoil loss of the system other than the pendulum to be measured, a similar measurement using a torsion pendulum, and estimation of the loss of wire material from the resonance Q of the wire. From these experiments, we could obtain the prospect to
Figure 4: Theoretical thermal noise of the yaw rotation originating from passive damping or loss of final suspension.

Figure 5: Theoretical thermal noise of the pitch rotation originating from passive damping or loss of final suspension.
realize Q of more than necessary value for the final mirror suspension.

5. Internal Thermal Noise

The loss of test mass itself also produces the thermally-excited internal motion of the mirror. The mirror of TAMA has a diameter of 100mm and thickness of 60mm. Thermal noise of the mirror could be calculated by summing up the contribution of each internal mode. Since the wave equation of a 3-dimensional elastic body is hard to solve analytically, we calculated the vibration mode of the mirror using Hutchinson’s method. We found we should attain Q of $2 \times 10^7$ to realize the designed strain sensitivity of $3 \times 10^{-22}/\sqrt{\text{Hz}}$. Although it may be hard to realize such a high-Q mirror considering our current experimental results, we still continue to investigate in various measurement conditions and to estimate the possibility to realize as high Q as possible.

6. Vacuum Compatibility

Since suspension systems including mirrors are installed in the high vacuum ($p \sim 10^{-8}\text{Pa}$), they should be made of low-outgas materials. Especially components which have friction mechanism with lubricant, such as linear stages and picomotors, will produce large amount of outgas. Therefore we applied special-ordered linear stages without any lubricant and picomotors of high-vacuum-compatible type.

PZTs for alignment control are canned to reduce outgas. Coil bobbins and coil wires used for actuators of mirrors are made of ceramics and ceramic-insulated wires, respectively. As mirror safety clamps we used as little Teflon(R) as possible. The other components are based on stainless steel (frame), aluminum (upper mass, mirror safety cage, etc.), or other kind of metal (screws made of copper, tungsten wire for suspension, etc.).

7. Construction and Installation

In this suspension system, the mirror is installed in the safety cage with clamps which serve to fix the mirror during transportation, as positioner for initial adjustment of the mirror, and as accepters of mirror in case of wire breakings. After installation of the mirror into the safety cage, the fine adjustment of the wire length seems to be hard to be made. So we will perform the following installation procedure.

The wire clamping block of the final suspension is removable from the upper mass so as to adjust length of two turns of wires on the adjustment bench. The adjustment will be carried out with a dummy mirror having the same dimension as the real mirror and with adjustable wire clamps by micrometers. The wire length is adjusted so as to align the angle and position of the dummy mirror. Subsequently the wires are clamped to the block, and is returned to the upper mass. In this situation, the wire length is adjusted, so we only have to suspend the real mirror in the cage and align the angle of the mirror.

After fixing the mirror in the safety cage, the suspension system is transported
and placed to the proper position in the interferometer. After releasing the mirror, the coarse alignment of the mirror angle will be realized referring to the clamps in the safety cage. The fine alignment will be adjusted by the picomotor remotely. In case of a wire breaking or a large earthquake, the shock to the mirror will be relaxed by Teflons at the end of mirror clamps.

8. Current Experiments and Schedule

Before deciding the final suspension design, we constructed a preliminary model to measure fundamental performances of the suspension system and to estimate other practical problems.

Measurements of vibration isolation ratio using a shaking table showed theoretical performance at low frequencies. Although we could not estimate the isolation ratio of less than -80dB at high frequencies, mainly due to the ambient acoustic noise, the calculation shows that the suspension system combined with the other two isolation system (stacks and X-pendulums) can realize sufficient isolation ratio for the TAMA sensitivity, even assuming measured isolation ratio.

In these experiments, we also estimated the magnitude of cross coupling between perpendicular directions and coupling of mirror rotation from horizontal or vertical ground motion. From these coupling estimations, we could explain them by rigid-body model with some asymmetry and obtained the precision requirements in fabricating the suspension system.

Other problems such as alignment control using picomotors and PZTs, vacuum components, installation procedure are under investigation using vacuum chambers of mode cleaner and its 10-m optical path.

The final suspension system will be firstly installed in the mode cleaner, and the basic performance of suspension systems and stacks will be tested in the ring cavity with round-trip length of about 20m. After minor improvements based on the mode cleaner experiments, a Fabry-Perot cavity with 300m arm length will be constructed with the suspension systems as well as stacks and X-pendulums.

References

5. N. Ohishi, *ibid*.
LOW FREQUENCY GEOPHYSICAL PERTURBATIONS
OF A FREE MASS GRAVITATIONAL ANTENNA

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ABSTRACT

Ultra low frequency perturbations of free mass gravitational antenna produced by surroundings can be considered from the two points of view: a possibility of its utilization for to estimate some characteristics of the antenna and a possibility of it measuring to extract a usefull geophysical information as by-product in respect of the main goal of the set up. In particular in the low frequency region $10^{-4} - 10^{-5}$ Hz a long base gravitational -wave interferometer can be presented as a differential tiltmeter insensitive to perturbations of a local normal to the land. The effect of variations of reciprocal deflection of two pendulum-mirrors is estimated for three conceivable scenarios of the earth core movements. The possibility of measurement of this effect is discussed according to the noise background estimate typical for the modern gravitational antenna projects.

1. Introduction

One of the principal problem for a free mass gravitational antenna is to keep a stable tuning, i.e. a fixed operational point position to provide permanently desirable sensitivity during of a long duty cycle of observations. The correspondent technique as active feed back control systems for adjustment of length of interferometer arms and alignment of light beam with respect to interferometer optical axis was developed and tested on table top modeles and intermediate scale prototipes by the all groups involved in the correspondent projects. Under this development an attention was paid more to the accuracy with which the operational point position could be controlled. The other essential characteristic of feed back such as it dynamical range was remained in shade. Meanwhile the last one has to depend on a maximum value of low frequency (quasistatic) perturbations of the interferometer base produced by variations of surrounding nature as pressure and temperature daily changes, gravity tidal deformations, movements of tectonic plates ect. These perturbations generally could be prognosticated but in fact it is better to measure it directly at the place where a set up is planned to be installed. A typical longitudinal crust tidal deformation has an order of magnitude $10^{-7} - 10^{-8}$ for the latitude 45 grad. That means a relative displacement of two mirrors separated by the distance 100 m must be at the level $(1 - 10) \mu$m, depending on a specific geological structure of the place. As for tectonic drift it could be estimated only very conventionally by taken into account a value of average relative displacement for a definite time interval which has to be long enough for to be representative. For example for 100 m Baksan Laser Deformograph the typical drift formed during of one year observation had the average magnitude less then 500$\mu$m. This value could be increased remarkably if there is a break off in geological structure among
the interferometer base line. Besides the all estimations above tends to at least ten times larger absolute values for interferometer base 3-4 km as it was planned in LIGO and VIRGO projects. Thus the ambient relative displacements between the points of mirror’s location could achieve 1 mm for daily variations and several centimeters for the period from three months to one year.

Meanwhile the accuracy with which the operational point position has to be kept is estimated in the best case as one tenth part of the cavity resonance width. From here a minimal acceptable variation of the cavity length with finesse \( F \) should be:

\[
\Delta L_{\text{min}} = 0.1c/4\pi F f_0 \approx 10^{-10} \text{ m under } F = 200, \ f_0 = 1.5 \times 10^{14} \text{ Hz.}
\]

One can conclude that the tidal variations \( \Delta L_t \approx 5 \times 10^{-8} L_0 \) should require for its compensation a feed back system with dynamical range \( D = \Delta L_t / \Delta L_{\text{min}} = 1.5 \times 10^5 (L_0/300\text{m}). \)

So for \( L_0 = 300 \text{ m} \) set up the estimation \( D = 1.5 \times 10^5 \) does not look abnormal as well as \( D = 1.5 \times 10^6 \) required for VIRGO set up. At any case it could be provided by conventional electronics. But a tectonic drift on the level of several centimeters and more, which corresponds to requirement \( D = 10^5 \) would demand a special technique to be compensated. It seems to be one of the problem of big projects LIGO and VIRGO.

The same problem however might be considered from the other point of view. It is clear that trying to avoid ambient perturbations on the way of it compensation one introduces in set up construction some elements and circuits which provide a possibility to monitor and to measure these effects in a relatively wide frequency range from few hundred herth up to zero. It is clear that just an error signal in feedback circuits is the carrier of the desirable information. This idea was discussed briefly in the papers.\(^1\),\(^2\) It was proposed use a gravitational wave interferometer on free masses for observations and measurements of global geodynamical processes induced by internal energy sources and external tidal potential as well. In this case the gravitational wave interferometer was considered as a differential tiltmeter in the very low frequency range \( (10^{-4} - 10^{-5}) \text{ Hz.} \) This region is placed far out zone of gravitational wave interests \( (10 - 10^3) \text{ Hz.} \) where a project sensitivity has to achieve \( (10^{-21} - 10^{-23}) \) in terms of metric perturbations.\(^3\),\(^4\) At the low frequencies mechanical displacements of mass-mirrors on a pendulum suspension are supposed to be compensated by feedback system producing of ”operational point keeping” in the middle of dark strip of interferometrical fringe.\(^3\),\(^4\) The correspondent voltage compensation, so called ”error signal” could be a carrier of desirable information about slow geodynamical variations of the gravity force vector. The idea looks very attractive in many aspects and first of all for the problems of global geodynamics and astrometry because movements on the Earth’s core have influence upon the precession and nutation of the Earth’s axis.\(^5\),\(^6\)

It was noted in the papers\(^1\),\(^2\) that a potential (instrumental) sensitivity of the new device might exceed considerably (two-four orders of value) the sensitivity of conventional geophysical instruments like a gravimeter, seismometer and strainmeter. This property would have a practical use if one could solve the main problem how to extract a small desirable signal from predominant geophysical perturbations. In the same papers the differential character of the new long based tiltmeter was
not enough emphasized. Meanwhile this quality defines a relevant opportunity of
the filtering of global geodynamical effects on a background of coherent hindrances.
Besides in papers\(^1,2\) there were no any estimations of real or hypothetical geody-
namical phenomena to forecast a feasibility of its measurement by means of the new
instrument. In the present paper we consider these questions further in detail.

2. A Differential Tiltmeter.

Traditionally tiltmeters give an angular direction of plumb line with respect to
the local normal to the land or a platform where the device is placed. Thus any local
perturbations of the normal due to atmospheric or geophysical variations produce a
predominant noise background from which an extraction of small global geodynam-
nical signals is not always possible. For this reason tiltmeter’s data are considered
sometimes as non representative ones for a testing of the global geodynamics.

The differential tiltmeter composed by two separated mass-mirrors of the gra-
vitational wave interferometer in principle has to be independent upon movements
of the local normal. What this device measures is a reciprocal angular position
between two plumb lines of the soft suspended mirrors. Variations of the local
normal can change an orientation of the support but it has nothing common with
mutual orientations of the plumb lines (a light beam space position is supposed to
be fixed).

A value of signal in the differential tiltmeter of course must be proportional to
the length of base. One could say the device is a kind of gravitational gradiometer
which gives the meaning of angular gradient of gravity force vector where con-
ventional gradiometers provide a gradient of its modules. A coherent hindrance
might be produced only by considerable mass shifts in near-by regions, but such
phenomena are very rare and also have to be very slow (we don’t consider at this
point disasters like earthquakes, explosions etc.).

To be more concrete in the following discussion we shall give rough estimations
of differential tilt for three conceivable scenarios of Earth’s core perturbations: dis-
placements of the inner (solid) core along rotation axis, quadruple oscillations of the
core’s fundamental modes, deflections of (inner) core axis from the general Earth’s
rotation axis.

As the Earth model we shall take a simplified triple-stratum mechanical system
consists of a rigid spherical cover (crust-mantle), ellipsoidal cavity with ideal homo-
ogeneous liquid outer (or liquid) core and inner (solid) core located in equilibrium in
the center of the cavity. Also we neglect of the centrifugal inertial force and do not
take into account corrections related with the Coriolis and Lorentz forces.
Hereafter we accept the following notations: \(M_0, R\) - mass and radius of Earth;
\(r_l, r_n\) - radiuses of the outer and inner cores; \(\rho_l, \rho_n\) - densities of the outer and inner
cores; \(\epsilon\) - ellipticity and hemiaxises: \((a - c)/a = \epsilon, \ a \simeq c \simeq r_i, \ (i = l, n)\), \(\vec{r} = \vec{r}_r\) -
radius-vector to the point of observation.
2.1. Polar Displacements of the Inner Core (PIC)

The problem of polar inner core oscillations was solved for example in papers\textsuperscript{8,9} where a fundamental frequency was defined with accuracy of a factor depending on the inner core structure. In principle the Coriolis and Lorentz forces do not any influence on frequencies of polar oscillations because this are displacements along the rotation axis. All is defined by gravitational and hydrostatic forces under a condition of slow motions. Our goal at this point is only a calculation of gravitational perturbations on Earth’s surface produced by small shift of the inner core on the vector \( \vec{r}_0 \) from equilibrium position along the direction of rotation axis. It should be supposed also that \( r_0 \ll R \).

The gravity force acceleration \( \ddot{g} \) in observation point in the our model could be presented as

\[
\ddot{g} = -(GM/R^2)e - (Gm/|\vec{r} - \vec{r}_0|^3)(\vec{r} - \vec{r}_0)
\]

where \( e \) - the unit vector to the observation point: \( \vec{r} = r\vec{e} \simeq R\vec{e} \); \( M = M_0 - m; \)

\( m = (4/3)\pi r_0^3(\rho_n - \rho_i) \) - the mass excess of the inner core. Expanding the dominator in the second term of formula (1) in power series of \( r_0/R \)

\[
|\vec{r} - \vec{r}_0|^3 = R^{-3}[1 + 3(\vec{e}r_0)/R + O(r_0^2/R^2)],
\]

one can get in the same approximation formulae for disturbed modules of the gravity force \( |\ddot{g}| \) and changed direction of the plumb line \( \vec{e}_g = \ddot{g}/|\ddot{g}| \)

\[
|\ddot{g}| = (GM/R^2)[1 + 2(m/MR)(\vec{e}r_0) + O(r_0^2/R^2)],
\]

\[
\vec{e}_g = -\vec{e}[1 + (m/MR)(\vec{e}r_0)] + (m/M)(\vec{r}_0/R) + O(r_0^2/R^2).
\]

A differential effect, a small angle \( \alpha \) between two plumb lines on the base \( L \ll R \), can be easy found through the scalar product of two unit vectors \( (\vec{e}_1\vec{e}_2) = \cos \alpha : \)

\[
\cos \alpha = (\vec{e}_1\vec{e}_2) + (m/MR)[(\vec{e}_1\vec{r}_0)(\vec{e}_1\vec{e}_2) - (\vec{r}_0\vec{e}_2) + (\vec{e}_2\vec{r}_0)(\vec{e}_1\vec{e}_2) - (\vec{r}_0\vec{e}_1)] + O(r_0^2/R^2).
\]

From here through the substitution \( \sin \alpha_0 = (\vec{e}_1\vec{e}_2) = L/R + O(L^3/R^3) \) one comes to the expression

\[
\Delta\alpha = \alpha - \alpha_0 = (m/M)(L/R^2)|\vec{r}_0|\sin \phi + O(r_0^2/R^2).
\]

In the last expression \( \phi \) is the averaged latitude of the two plumb lines. Besides it is usefull to extract from (3) the absolute value of gravity force variation.

\[
\Delta g \simeq (GM/R^2)(m/M)(\vec{e}r_0)/R
\]

This formula was derived already in the papers\textsuperscript{8,9} but we give it here to compare the opportunities of ”gravimetric” and ”tiltmetric” detection of inner core movements.
2.2. Quadruple Oscillations of the Inner Core (QIC)

In this scenario the core is considered to be an oblate axial ellipsoid (for example as a result of rotation) with oscillating eccentricity $\epsilon = \epsilon(t)$. To calculate a gravity anomaly on the earth surface one can use a well known expression for the force function of axial body.\(^{10}\)

$$V = (Gm/r)[1 - (r_0/r)^2]P_2(z/r)I_2 + ...,$$

where $I_2$ - the normalized moment of inertia : $I_2 = a^2(2 - \epsilon)/5r_0^2$ and $P_2(z/r) = (1/2)[3(z/r)^2 - 1]$- the Legendre polynomial of second order; the parameter $a$ in that model is some average size of the ellipsoid: $a \simeq c \simeq r_0$

Besides the unit vector $\vec{e} = \vec{r}/|\vec{r}|$ it is useful to introduce the unit vector in direction of $z$ axis: $\vec{e}_z$. Then calculating a vector derivative of the force function (8), one can find a formula for the gravity force $\vec{g}$ in a surface point on the latitude $\phi$ :

$$\vec{g} = -(GM/R^2)[1 - (k/2r^2)(5\sin^2\phi - 1)]\vec{e} + (k/R^2)\sin\phi\vec{e}_z + O(1/R^3),$$

where $k = (3/5)(m/M)\epsilon(2 - \epsilon)a^2$. From now through the known routine procedure it's possible to get the unit vector in a gravity force direction

$$\vec{e}_g = -[1 - (k/R^2)\sin^2\phi]\vec{e} - \sin\phi\vec{e}_z + O(1/R^3),$$

as well as absolute value of the gravity force acceleration

$$|g| = (GM/R^2)[1 - (k/2R^2)(3\sin^2\phi - 1)] + O(1/R^5).$$

The angular difference of two plumb lines separated on the distance $L \ll R$ one can get in the same manner like in subsection 2.1.(Eq.(5))). through the scalar product $(e_{g1}e_{g2})$. The final formula results in

$$\Delta\alpha_\epsilon \simeq -(6/5)(m/M)(a/R^2)(L/R)\Delta\epsilon\sin^2\phi.$$  \hspace{1cm} (12)

A correspondent value of the gravity force variation versus of eccentricity might be estimated as

$$\Delta g_\epsilon \simeq -(3/5)(Gma^2/R^4)\Delta\epsilon[3\sin^2\phi - 1].$$  \hspace{1cm} (13)

One might neglect the sign in formulae (12),(13)because only amplitude of the effects is the subject of interest.

2.3. Angular Deflections of the Inner Core’s Axis (AIC)

This type of perturbation is presented by small angular oscillations of the inner core’s axis in respect of the Earth’s rotation axis.\(^{11}\) Essentially the inner core is supposed to be ellipsoidal. Recently some new arguments in favour of it were obtained from a deep seismological ranging of the earth interior.\(^{11}\) An axial anisotropy of the inner core was established which permits one to suppose its ellipticity; besides
the remaining part of the planet rotates faster around the axis which has an angle about 10 grad to the nors-south axis of the earth\textsuperscript{11,12}.

We will consider small angular deflections of the inner core’s axis as a turn of core ellipsoid on small angle $\Delta \theta$ around the coordinate line $X'X'$ formed an angle $\lambda_0$ with $XX$ axis of the geocentric coordinate system. Thus $\lambda_0$ is the longitude of $X'X'$ line. The obvious method for calculation a value of gravitational perturbation due to such deflections is to employ the formula (8) for the force function in the coordinate system fixed in ellipsoid principal axes so that the line $X'X'$ is one of them. Then through the formulae of coordinate angular transformations one can come back in geocentric system. Following this procedure one can find the expression for the perturbed force function.

$$V = (GM/r)[(m/M) - (k/6r^2)[(3z^2/r^2) - 1] - (k/r^2)(\vec{e}_\theta \vec{r})(z/r^2)],$$

(14)

where the deflexion vector $\vec{e}_\theta$ was introduced on the line $X'X'$ with value proportional to small perturbation $\Delta \theta$:

$$\vec{e}_\theta = (\sin \lambda_0, -\cos \lambda_0, 0) \Delta \theta.$$  

(15)

Then repeating the scheme of calculations in subsections 2.1,2.2 one can find a perturbed acceleration of gravity force in the earth surface point with coordinates $(\lambda, \phi)$, where $\phi$ as early is latitude.

$$|\vec{g}| = (GM/R^2)[1 - (k/2R^2)(3\sin^2 \phi - 1) - (3k/R^2)(\vec{e}_\theta \vec{r})\sin \phi] + O(1/R^3),$$

(16)

here again $\vec{e}$ -the unit vector in $\vec{r}$ direction, besides

$$(\vec{e}_\theta \vec{e}) \simeq -\cos \phi \sin(\lambda - \lambda_0) \Delta \theta.$$ 

(17)

A variation of gravity force produced by deflexion $\Delta \theta$ of hard core axis presents by the formula

$$\Delta g_\theta \simeq (9/5)(Gm/R^2)(a/R)^2 \epsilon \sin 2\phi \sin(\lambda - \lambda_0) \Delta \theta.$$ 

(18)

After calculations in analagical manner one has for perturbed plumb line vector $e_{\vec{g} \theta}$

$$e_{\vec{g} \theta} = e_{\vec{g} \theta} + (k/R^2) \left[2(\vec{e}_\theta \vec{e}) \sin \phi \vec{e} - (\vec{e}_\theta \vec{e})\vec{e}_z - \sin \phi \vec{e}_\theta\right] + O(1/R^2).$$ 

(19)

Finally for desirable value of differential angle between two plumb lines separated the distance $L$ one can yield

$$\Delta \alpha_\theta \simeq (6/5)(m/M)(a/R)^2(L/R) \epsilon \sin 2\phi \sin(\lambda - \lambda_0) \Delta \theta$$

(20)
3. Numerical Estimations

To get numerical value of the effects of interest we need numerical characteristics of the Earth which are collected in the List of Main Parameters (see Table 1). Not all values in this list are defined with good accuracy. The first four positions (1-4) are used very often and reliable with precision no worse then 10%. One could take for checking the old reference book or relatively modern presentation of Standard Earth Model (PREM). As for the densities and effective oscillating mass, positions (5-8), the situation here is more vague due to density variations along the core radius. Detailed diagrams one can find in paper and also in the monograph. In the (Table 1) we pointed out for the liquid outer core the value corresponding to a border region of outer-inner cores. For the inner core we gave the range of density variations, position (6), up to the center of the Earth. Thus the density difference 0.6 kg/m³ belongs to the outer-inner core border and results in the effective oscillating mass $m = 4.5 \times 10^{21}$ kg and effective mass ratio $(m/M) = 7.5 \times 10^{-4}$, position (8,9). It is obvious however these estimations could be increased in 2-2.5 times for the average density difference $(1 - 1.5)$ kg/m³.

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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Earth’s mass</td>
<td>$M_0$</td>
<td>$6 \cdot 10^{24}$ kg</td>
</tr>
<tr>
<td>2</td>
<td>Earth’s radius</td>
<td>$R$</td>
<td>$6.38 \cdot 10^6$ m</td>
</tr>
<tr>
<td>3</td>
<td>O-core radius</td>
<td>$r_l$</td>
<td>$3.48 \cdot 10^6$ m</td>
</tr>
<tr>
<td>4</td>
<td>I-core radius</td>
<td>$r_n$</td>
<td>$1.22 \cdot 10^6$ m</td>
</tr>
<tr>
<td>5</td>
<td>O-core density</td>
<td>$\rho_l$</td>
<td>$12 \cdot 10^3$ kg/m³</td>
</tr>
<tr>
<td>6</td>
<td>I-core density</td>
<td>$\rho_n$</td>
<td>$(12.6 - 13.5) \cdot 10^3$ kg/m³</td>
</tr>
<tr>
<td>7</td>
<td>Density difference</td>
<td>$\rho_n - \rho_l$</td>
<td>$(0.6 - 1.5) \cdot 10^3$ kg/m³</td>
</tr>
<tr>
<td>8</td>
<td>Effective mass</td>
<td>$m$</td>
<td>$4.5 \cdot 10^{21}$ kg</td>
</tr>
<tr>
<td>9</td>
<td>Mass ratio</td>
<td>$m/M$</td>
<td>$7.5 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>10</td>
<td>I-core mj. axis</td>
<td>$a$</td>
<td>order of $r_n$</td>
</tr>
<tr>
<td>11</td>
<td>Axis ratio</td>
<td>$a/R$</td>
<td>$4 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>12</td>
<td>O-core el.</td>
<td>$\epsilon_l$</td>
<td>$2.6 \cdot 10^{-3}$</td>
</tr>
<tr>
<td>13</td>
<td>I-core el.</td>
<td>$\epsilon_n$</td>
<td>$10^{-4} - 10^{-7}$</td>
</tr>
<tr>
<td>14</td>
<td>Base line</td>
<td>$L$</td>
<td>$10^5$ m</td>
</tr>
<tr>
<td>15</td>
<td>I-core shift</td>
<td>$r_0$</td>
<td>1 m</td>
</tr>
</tbody>
</table>

$\sin \phi = \sin \pi/4 = 0.7$

The most difficult positions in (Table 1) are the ellipticity values. For the outer core there is a theoretical calculations of a hydrodynamical compression supported by astrometrical measurements of the Earth axis nutation. With accuracy 10% the recommended value is $\epsilon_l = 2.6 \times 10^{-3}$, position (10). We could not find in a literature numerical estimations of the inner core ellipticity so we were free in our speculation with the restriction this value has to be less then the outer core ellipticity i.e. one
could take $\epsilon_n \leq 10^{-4}$, position (11).

The last four positions in (Table 1) contain the obvious estimations of the inner core major axis as well as the inner core-Earth axis ratio (accuracy 10%) and besides two free parameters of our problem: the interferometer base line and hypothetical amplitude of inner core oscillations. We took initial reference values $L = 1$ km, $r_0 = 1$ m for a reader could easy change a final estimations through a variation of these parameters. As for other perturbation values,- the inner core ellipticity and it’s axis orientation,-we point out them in parallel with giving numerical results.

Then according to formulae (1),(4),(7) for the angular perturbation effects and the data of (Table 1) one can collect the following numerical estimations.

PIC – effect : $\Delta \alpha_p \simeq 1.3 \cdot 10^{-14}$ rad, $[\rho_n - \rho_l = 0.6$ kg/m$^3]$;

QIC – effect : $\Delta \alpha_\epsilon \simeq 5.6 \cdot 10^{-14}$ rad, $[\Delta \epsilon_n = 10^{-5}]$; (21)

AIC – effect : $\Delta \alpha_\theta \simeq 3.10^{-15}$ rad, $[\epsilon_n = 0.1 \epsilon_l, \Delta \theta = 2 \cdot 10^{-3}$ rad].

AIC-effect looks as the weakest one in spite of the fact we took for the estimation the optimistic value of inner core ellipticity $\epsilon_n = 2.6 \cdot 10^{-4}$ and the amplitude of deflection $\Delta \theta = 2 \cdot 10^{-3}$ rad $\simeq$ one degree ! In a more realistic picture the both values must be much smaller. It means the AIC- effect has to be considered as negligible one.

As for the PIC and QIC effects the estimations could be increased up to the value $\sim 10^{-12}$ rad if one will set the interferometer base 4 km, paper$^3$ and supposed to take $r_0 = 3$ m (then one could yield for PIC $\sim 1.5 \cdot 10^{-13}$ rad and QIC $\sim 2.2 \cdot 10^{-13}$ rad) ; besides there is an increasing factor 2 in the evaluations of the density difference, position (6), and amplitude of the inner core ellipticity oscillations $\Delta \epsilon_n$. We realize however that our very simplified models of the phenomena can provide numerical estimation of the effects only on the order of value. So finally we might infer that in the best case inner core oscillations could produce mutual angular perturbations of the 4 km free mass antenna mirrors on the level $10^{-13} - 10^{-12}$ rad. Periods of PIC and QIC oscillations cover the interval from 100 minutes up to 10 hours.$^7$-$^9$,15

It means the interesting frequency range $3 \cdot 10^{-4} - 3 \cdot 10^{-5}$ can be separated from the predominant tidal effects.

Numerical data for the gravity force variations according to formulae (2), PIC-effect and (5), QIC-effect, results in the value $\delta g/g \simeq (0.5 - 1.2)10^{-10}$. That is in agreement with previous estimations$^8$,9 but a little behind of the threshold of reliable registration for conventional gravimetrical devices.$^{18}$ Contents of this section sea at paper.$^{32}$

4. Quadruple Oscillations of the Outer Core (QOC)

Contents of this section sea at paper.$^{32}$
5. **Specifics of the Set Up Construction**

Some remarks must be done concerning of the specific suspension of mass-mirrors and manner of operational point keeping. It was supposed in our consideration above that mirrors repeat angular deflections of the pendulum filament (a direction of the plumb line). But it is true only for a trivial suspension when a filament is attached to the top of mirror body above its center of gravity. At present construction however designers use two filaments overlooped the mirror body. Points where the filaments touch a mirror’s back side surface last time are fixed by the friction force but its have a bending degree of freedom. Due to this variations of the filament deflection is not accompanied by inclinations of the mirror surface. During of pendulum movements a mirror surface remains to be parallel to itself. Thus a ”tilt error signal” in alignment circuit disappears. On this subject one can note that ”tilt perturbations of the filament” in this case have to be converted into ”base line variations” and a ”lateral misalignment” of the mirrors with respect on FP-cavity axis. A value of the lateral shift might be estimated as \( \Delta a \simeq l \sin \alpha_0 \Delta \alpha \) where \( l \) is the length of filament and \( \alpha_0 \) is initial angle between ”plumb lines” separated to the distance \( L \). For \( l=100\text{cm}, \ L=4\text{km}, \ \alpha_0 \sim 7 \cdot 10^{-4} \text{rad} \) one has the estimation \( \Delta a \simeq 0.07 \cdot \Delta \alpha \text{ cm} \).

A second remark has a deal with a manner of keeping lengths of interferometer arms to provide a location of operation point inside an interferometer fringe. In section 3. it was supposed that lengths of both arms were fixed by feed back drivers with accuracy \( \Delta L \sim \lambda \). Under this condition a ”geometrical noise” produced by tidal and tectonical perturbations remains to be small enough. However at present designers of the interferometer use another type of the operational point control system. They do not fix a size of individual arm but instead they try to equalize the lengths of arms with accuracy \( \lambda \). Under this a desirable position of operational point is provided but the length of individual arm can change a size significantly under geophysical purturbations. As a consequence the ”geometrical noise” might become much larger a value of ”geodynamical signal”. By chance a ”geometrical noise” can be calculated precisely if variations of the arm length is measured. The ”tidal noise” can be calculated precisely with the help of tidal theory and then can be removed from the observational data. Only residuals must be studied for a searching of any global geodinamical perturbations.

6. **Discussion**

We tried to develop the idea of adapting a new large scale set up, free mass gravitational wave antenna, for another fundamental problem,- the test of global geodynamics. We have found no principle obstacles to the realization of this idea. At least at the level of liquid core movements it should be the adequate measuring device. However a jump from available angular sensitivity \( 10^{-10}\text{rad}/\text{Hz}^{1/2} \) to desirable one \( 10^{-12}\text{rad}/\text{Hz}^{1/2} \) in the very low frequency region \((10^{-4} - 10^{-5})\text{Hz} \) to provide a measurement of inner core perturbations probably will be not a simple problem in practice.
It is obvious that there are two approaches to the problem: the first from the side of technical state of art and second from the side of scientific purposes concerning applications.

As we have shown the present understanding of GW-interferometer as an angular gravity gradiometer provides a favorable ground for successful signal off noise filtration. To improve it two desirable modifications in general construction of GW-free mass antenna were pointed out: a) the pendulum suspension for a laser source device to avoid variations of the local normal and b) the parallel deformograph optical line with mirrors fixed to the land for to control land perturbations independent from gravity. The both modifications will be connected with some complification of the general set up although it does not present any principal technical problem.

Having discussed of tilt signal measurements in papers\textsuperscript{1,2} we addressed to the Anderson alignment technic.\textsuperscript{19} It was clear how to distinguish the tilt signal for a simple FP-cavity by this method. But a nontrivial character of the tilt signal in the complete device composed by two FP-cavities in Michelson arms with recycling mirror could make a procedure of signal extracting too complex if not impossible. So this point deserves a special consideration. It is not enough clear also what is a lowest signal frequency which could be read out through feed back circuits. A limitation here depends on a specific feed back design.

Of course the main point which could be decisive for a practical realization of the idea of gravity gradiometer is the level of low noise background. Above we tried to make some estimations but a last word has to belong to direct measurements of it.

Our estimations of the hypothetical geodynamic phenomena are far from being exhaustive but they have to be valid at least on the order of magnitude. The arbitrary element in our calculations was of course the amplitude of core oscillations. There is the obvious restriction of it from the side of energetic limitation. A fundamental frequency of the inner core oscillations along the Earth axis could be found from balance of gravitational and hydrostatic forces which in the first approximation govern of core behavior.\textsuperscript{9}

\[
\omega_0^2 = (4/3)\pi G (\rho_i/\rho_n)(\rho_n - \rho_i) \simeq 1.5 \cdot 10^{-7} \text{ 1/sec}^2,
\]

(we took here the density value: \((\rho_n - \rho_i) = 0.6 \cdot 10^3 \text{ kg/m}^3\); it corresponds to period of oscillations \(\sim 4.5\) hours and value of effective mass \(m \simeq 4.5 \cdot 10^{21}\text{kg}\)). Then it is easy to estimate the energy of inner core oscillations with amplitude \(r_0 = 1\text{m}\):

\[
E_p \leq (1/2) m \omega_0^2 r^2 \simeq 3.4 \cdot 10^{21}(r_0/100\text{cm})^2 \text{ erg}.
\]

According to the papers\textsuperscript{7-9} a reasonable short time energy variation for the inner core just has the order \((10^{21} - 10^{22})\text{erg}\). (due to large earthquakes and other sources). The next reason is that a power of oscillations \(P = \omega_0 E \leq 1.3 \cdot 10^{18}(r_0/100\text{cm})^2 \text{ erg/sec}\) does not exceed the total thermal flux of the globe \(dE/dt \simeq 3.4 \cdot 10^{19} \text{ erg/sec}\).
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References

PROGRESS IN THE DEVELOPMENT OF TECHNOLOGY
FOR ADVANCED LASER INTERFEROMETRIC
GRAVITATIONAL WAVE DETECTORS

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ABSTRACT

This paper presents recent progress in the development of technology for an advanced laser interferometer gravitational wave detector in Australia. Progress reported includes bench top confirmation of recycling interferometer performance, high power laser development, and development of techniques for pre-isolation at frequencies between 20 - 50 mHz in 3 dimensions, including demonstrated isolation of 90 dB at 7 Hz. The 8 m interferometer at UWA has been successfully locked using frontal modulation and operated with 50 mW of power injected from an in-line mode cleaner. The mode cleaner itself demonstrates the expected reduction of beam jitter noise. Favourable measurements of the properties of sapphire test masses are also described, including the recent demonstration of a Q-factor of $2.6 \times 10^7$ in a mass supported by a niobium Catherine wheel.

1. Introduction

The Australian Consortium for Interferometric Gravitational Astronomy (ACIGA) is developing technology for advanced gravitational wave detection with a view to construction of the southern hemisphere element in a world wide array of detectors. Such an array will allow unambiguous and relatively accurate determination of the location of gravitational wave sources. An Australian detector will provide near maximal possible baselines to the northern hemisphere detectors: LIGO Hanford, LIGO Livingston, VIRGO, GEO and TAMA.

ACIGA consists of the Australian National University (ANU) Quantum Optics group, the University of Adelaide Laser Physics Group, the CSIRO Optical Technology Centre, the Monash University General Relativity Group and the Australian International Gravitational Research Centre at the University of Western Australia. This paper presents some recent advances in the areas of optics, lasers, test masses.
2. Advanced Interferometer Requirements and Sensitivity

The goal of ACIGA is the construction of a dual recycling interferometer with very high power recycling gain, high signal recycling gain, seismic isolation down to a few Hz, and very low thermal noise through the use of sapphire test masses. The instrument will initially have a baseline of a few hundred meters but will be extendible to a 3-5 km sized instrument.

To achieve astrophysically interesting sensitivity in an initially short baseline instrument requires exceptionally low levels of both seismic noise and thermal noise. Recently it has been shown that the use of sapphire test masses and niobium flexure compound pendulum suspensions can, in principle, allow the thermal noise amplitude to be reduced by a factor of 16 compared with conventional wire hung silica test masses.\(^1,2\) In addition, very low frequency suspension systems can drastically reduce the forces required to be applied by control systems, thus minimising the problem of noise injection and simplifying the control problem. For maximum simplicity and low cost we will avoid using Fabry-Perot cavities in the arms and instead aim for maximum power and signal recycling. The recent achievement of power recycling factors \(\sim 500\) give us confidence in this strategy.

Figure 1 shows a comparison of the achievable sensitivity of a 400 m interferometer with advanced optics, isolation and suspension, compared with the LIGO Phase 1 sensitivity goal. Model parameters are given in the figure caption. The performance required puts a high demand on the beamsplitter which must carry the full recirculating power of the interferometer. Initially we propose to use a sapphire beamsplitter, but this could be replaced by a diffractive element.

![Figure 1: Comparison of interferometer sensitivities. The curves show the projected sensitivity of LIGO phase 1 (---), VIRGO (---) and AIGO-400 (---) based on sapphire test masses and narrow band dual recycling. The flat region between 200 Hz and 1 kHz of AIGO-400 sensitivity is the locus of narrow band sensitivity (....) achievable by tuning within this range.](fig01.ps)
3. Bench Top Interferometer Experiments

There are several perceived advantages in using the dual recycled configuration. The total interferometer is relatively simple as it avoids the need for three mirror, coupled Fabry-Perot cavities. The dual recycling configuration therefore has relatively few optical surfaces and a simpler control system compared to the power recycled, Fabry-Perot-Michelson interferometer. This simplicity should result in a lower construction cost and quicker implementation.

Figure 2 shows a simple schematic of the interferometer and control system used in this bench top experiment. Introducing an arm length difference of $\Delta l = \pi c / 2 \omega_m$ in the Michelson interferometer produces a relatively simple, independent control system. Full details on this control system and discussion of the results obtained can be found in reference.3

Another advantage of the dual recycling-Michelson interferometer is that the signal frequency response can be changed from broadband to resonant at a particular signal frequency, by small (fraction of an optical wavelength) changes in one interferometer dimension. The frequency response can therefore be reconfigured on a time scale of a few seconds. Figure 3 shows the experimental signal-to-noise ratio (SNR) response of the interferometer as the signal recycling cavity is tuned from broadband (trace(a)) to detuned and then to antiresonant (trace (h)). The response is scaled with respect to that of the simple Michelson under identical conditions. As can be seen, broadband recycling results in an increase of approximately 11 dB for this experiment, while tuned operation produces an improvement of approximately 5 dB. It should be noted that the improvement in SNR due to signal recycling is limited by the losses of the signal recycling cavity. For a large scale interferometer, losses are expected to be extremely low and so the predicted improvement in signal-to-noise ratio should be far greater than that achieved here.
Figure 3: The differential phase frequency response of the signal recycling interferometer for various detuning from broadband (trace (a)) to anti resonant (trace (h)).

4. Laser Injection Locking: Theory and Experiment

To control the noise properties of the high power solid state laser to be used in the gravitational wave detector, laser injection locking will be used. Here we address the transfer of noise through the injection locked system and discuss the implications of this noise transfer on the design of the high power laser source. Full details of the quantum optical theory and the experimental test of the predictions can be found in references $^4$Ha96.

Free running solid state laser systems, such as diode pumped monolithic Nd:YAG ring lasers used in this work, have intensity noise associated with their output that is due to an interaction between the atoms in the lasing medium; the cavity storage rate; variations in the intra-cavity photon number introduced from the laser’s pump source; and vacuum fluctuations. This noise can be reduced by injection locking.

Intensity noise filters through to the output of the injection locked laser from the slave’s pump lasers and from the master laser. We can view this intensity noise as being similar to a spectrum of amplitude modulation sidebands. The transfer of intensity noise from these noise sources can be summarised as follows (see Figure 4).

Figure 4: The differential phase frequency response of the signal recycling interferometer for various detuning from broadband (trace (a)) to anti resonant (trace (h)).

The slave laser acts like a low pass filter to the noise of its pump source, as is shown in Figure 4(i) and (ii). The transfer of modulation from the slave’s pump source through to the injection locked slave output is characterised by minimal attenuation at DC (zero modulation frequency), followed by continuously increasing attenuation with the modulation frequency until the signals fall below the QNL of the slave laser. The rate of change of the attenuation, which determines the bandwidth of the lowpass filter, can be as high as 20 dB per decade. The corresponding corner frequency for this attenuation is determined by the characteristics of the slave laser as well as the intensity and detuning of the injected master radiation.

The transfer of modulation of the master laser through the injection locked system is more complicated. It can best be described by discussing three distinct
frequency regions in the output noise spectrum of the injection locked laser system.

Let us start with the frequency region numbered (A). The output signal is an amplified version of the spectrum of the master laser. This is called the amplification region. In this region the slave acts like a linear optical amplifier to any modulation signals on the master laser. Both the modulation and the coherent fields are amplified and consequently beating between them produces an output signal which is amplified by the square of the power ratio of the master to the slave. Signals on the master retain their signal to noise ratio in the output of the locked slave if they are large compared to the QNL. However, for signals that are close to the QNL the signal to noise is reduced. It should be noted that even if both the master and free running slave lasers were QNL then the noise in this region would still be larger than the QNL. This increased noise level is due to the amplification of the quantum noise of the master. The bandwidth of this region is determined by the characteristics of the slave laser as well as the intensity and detuning of the injected master radiation, as is discussed later in this paper.

In the frequency region numbered (B), there is neither attenuation nor amplification of the signal from the master laser. The modulation signal of the master laser is directly reflected off the cavity of the slave laser without entering. The reflected signal coherently beats with the strong slave beam and produces a modulation signal at the output which is greater in magnitude than the original modulation of the master by the power ratio. The noise floor is set by the noise level of the master laser and the signal to noise ratio at the output is identical to that of the master laser.

In the frequency region (C), the slave laser is very strongly coupled to its pump laser source and hence marginally affected by modulations of the master laser. That is, the total output power of the slave laser system is determined by its pump source

Figure 4: The injection locked transfer functions. (i) and (ii) are the pump noise transfer functions: theory and experiment respectively. (iii) and (iv) are the master noise transfer function: theory and experiment respectively.
and hence any forced oscillation from the external field cannot be amplified. This implies that the total output power of the injection locked system is the geometric sum of the master and slave free running optical powers, and hence near DC the response of the injection locked laser to master noise fluctuations is also the geometric sum.

These results show that the noise on the output of the injection locked laser is dependent on the noise contributed from both the slave’s pump and the master laser noise. Inside the amplification region the output noise is always greater than QNL. This region has to be avoided for any QNL measurement applications. Therefore, the laser source for the gravitational wave detector must be designed such that the modulation signals gravitational wave signal band are at frequencies outside the amplification region.

5. High Power Laser Development

Future gravitational wave interferometers will require high power lasers as an attractive means of reducing photon shot noise and relieving the requirements on reflective coatings, recycling efficiency, etc. We believe that a long term goal of laser sources in the 100 W-1 kW regime is appropriate for the next generation devices. To date the maximum power achieved in single mode cw Nd:YAG lasers is 62 W, and the stable resonator approach used is probably not scalable beyond a maximum of about 100 W. The required frequency stability is achieved by using injection locked oscillators. In a robust system the power gain per stage will be limited by the locking range and the characteristics of the servo system used to maintain lock. Starting with a typical non-planar ring oscillator (NPRO) of about 100 mW, it is unlikely that the required power levels can be reached in a single stage. For this reason we have chosen a two stage approach. The first stage uses an NPRO master oscillator (MO) to injection lock a first slave oscillator (SO1), producing an output power of 5-10 W. The output of this oscillator is in itself a useful laser for first generation interferometers, and it can be used as a source to injection lock a second slave oscillator (S02) that has an output power > 100 W.

The SO 1 is a novel design, making use of a side pumped slab, originally developed for pulsed operation. We have developed this concept as a stable ring laser, suitable for injection locking, as shown in Figure 5. It makes use of a slab, side pumped by a 20 W laser diode through an optical fibre that serves as a focussing cylindrical lens for the pump. We have characterised the frequency stability of this 5 W laser, and improved the design to produce a free running frequency stability of less than 1kHz/√Hz, suitable for injection locking. We have also demonstrated open-loop injection locking of the laser for short periods of time (minutes). At the present time we are investigating methods to increase the power of the ring laser to beyond 5 W, and plan to implement a servo loop to maintain lock.

Our approach to the second stage of amplification is to investigate a new, unstable resonator architecture for injection locked slab lasers. We have chosen an unstable resonator design because recent developments in graded reflectivity out-
couplers have made this a viable approach for lower gain lasers. Furthermore, our
design makes use of a resonator design which is unstable in one direction only,
and stable in the other, thus optimising the design for use on laser-diode pumped,
solid-state slab lasers. The main features are shown in Figure 6. In this case we
use fibre-coupled laser diodes to deposit the pump radiation efficiently in the mode
volume of the S02 gain medium, while removing the cooling requirements of the
laser diodes from the vicinity of the laser 'head'. We have numerically modelled the
resonator, and expect to be able to produce a single-mode output suitable for mode
matching into a high finesse stable TEM$_{00}$ mode of a typical interferometer, with a
mode matching efficiency in excess of 97%.

6. The 8m Interferometer

The 8 m single-pass Michelson interferometer developed at UWA (Figure 7)
is a testbed for the Australian Consortium to test the performance of ultra-low
frequency isolators, low frequency vibration isolators, compound test masses and
advanced optical techniques for gravitational wave detection. The beamsplitter
and end mirrors are suspended by multistage cantilever spring vibration isolator\(^9\)
developed at UWA. The schematic diagram of the isolator is shown in Figure 8.
The isolators are mounted on a steel frame which is supported by concrete pillars.
Vertically it is a four stage isolator with a single spring on the top and three pairs of
curved springs. Horizontally it is a five stage pendulum including the last compound
test mass pendulum which is suspended at the bottom of the control mass by a
thin foil.\(^10\) This arrangement allows very high pendulum Q-factors to be achieved
and tight orientation control of the mirror direction and facilitates one-dimensional
control of the test mass for global control of the interferometer. The isolator system
has four vertical modes (1.3, 3.2, 4.5 and 6.5 Hz), four Horizontal modes (0.47, 0.8,
1.9 and 2.9 Hz), four rocking modes (below 1.2 Hz) and four torsional modes (below
0.4 Hz). The high frequency performance of the isolator has been tested using a
sapphire transducer. The vertical and horizontal response of the isolator at high frequencies (above the transducer mechanical resonance of about 60 Hz) reaches the noise floor of the sapphire transducer $3 \times 10^{-15} \text{m/}{\sqrt{\text{Hz}}}$. But at low frequency high $Q$ normal modes can result in large motions of the test mass at the normal mode frequencies. This can lead to difficulties in the global control of the interferometer since the motion can exceed the dynamics range of the transducer or even makes the interference of the two beam impossible. A damping system is necessary to suppress the large scale motions and align the mirrors to the point where the interference fringes may be observed.

The conventional shadow sensor/magnetic-coil assemblies were used to sense the normal mode motion of the translation of the control mass and the tilt of the test mass. Rotational motion is monitored using an optical lever arm. In each case the signals are fed back to the coil-magnetic actuators which apply forces to the control mass and the test mass to damp the normal modes and control them.

Because the shadow sensors are mounted on the frame, the position sensing is done with respect to the frame. It is impossible to avoid seismic noise in the sensing. This sensing noise has components in the signal band which must be prevented from driving the mass and appearing in the signal output. The filtering is required to reduce this coupling seismic noise. The 4-pole low pass filters with cut-off of 10 Hz is used to reduce this noise below the electronic noise floor above 100 Hz.

Since demonstrating the performance of the individual isolators, we have concentrated on demonstrating that this isolator structure, along with membrane suspended compound pendulum test masses, can be used in a practical interferometer. We have established an 8m suspended Michelson interferometer illuminated by a Nd:YAG laser ($\lambda = 1.06\mu m$) with frontal modulation scheme. The arm difference of the interferometer is set to 400 mm to generate sidebands for a phase-sensitive error signal. The interferometer has been successfully locked to a dark fringe. The
Figure 7: The optical layout of the 8m suspended interferometer.

Figure 8: The schematic diagram of the cantilever spring isolator with compound pendulum test mass.
best contrast is 0.99. We are particularly pleased to have been able to show that there is no apparent problem in terms of control of the low frequency multistage cantilever spring isolators, nor the compound pendulum test mass suspension.

7. Laser Prestabilisation

When the two arms of the laser interferometer are not the same (as in the systems using frontal modulation), laser frequency noise causes phase shifts at the output of the interferometer. Laser frequency noise can also introduce noise to the output of the interferometer if the scattered light interferes with the main beam.

The level of laser frequency required for an interferometer is:

$$\frac{\delta \nu}{\nu} = h \frac{l}{\delta l}$$

where $h$ is the strain sensitivity, $l$ is the average arm length, $\delta \nu$ is the frequency noise, $\nu$ is the frequency of the laser and $\delta l$ is the static arm difference or the extra path length travelled by scattered light. This requirement is of order $10^{-3}$Hz/$\sqrt{\text{Hz}}$ in the signal band.

A Free running diode-laser-pumped Nd:YAG laser has typical linewidth of 30 kHz at 1ms, $\sim 1$kHz/$\sqrt{\text{Hz}}$. The reference should have the intrinsic stability required, the locking scheme should have adequate discriminator sensitivity and the servo sufficient gain to achieve this.

We use the Pound-Drever-Hall\textsuperscript{13} locking scheme and an all-sapphire Fabry-Perot cavity as our frequency reference (see Figure 9). When the laser is locked to the cavity, the frequency stability of the laser in the closed loop system is determined by the shot noise limit, the thermal expansion of the spacer and possibly the mechanical thermal noise of the cavity.\textsuperscript{14}

We have demonstrated shot noise limited performance of the servo loop to 10 kHz with a cavity of finesse 500 giving an estimated stability (not considering cavity
Figure 10: Allan deviation results in initial two-cavity beat experiments with one all sapphire Fabry-Perot cavity and a second low finesse cavity with silica mirrors.

frequency fluctuations) of $10^{-2}\text{Hz/}\sqrt{\text{Hz}}$. We are now addressing the stability of the cavity and have constructed two experiments schematically shown in the figure above. The output of the two systems is mixed on a photodiode and the Allan Deviation of the beat signal measured. The result is shown in Figure 10. In this part of the time domain the noise is limited by the expansion of the cavity due to temperature fluctuations

$$\frac{\delta \nu}{\nu} = \frac{\delta l}{l} = \alpha \delta T$$

(2)

where $l$ and $\delta l$ now refer to the length of the reference cavity, $\alpha$ is the coefficient of thermal expansion and $\delta T$ is the fluctuation of temperature. The limiting noise in this case is the stability of the error signal.

From the above equation, it is obvious that materials with low coefficient of thermal expansion are preferable. In this experiment we reduce $\alpha$ with the use of sapphire at cryogenic temperature. Ultra Low Expansion Glass (Corning 7971) is a suitable material as well because it has an $\alpha$ crossing near room temperature and superb results have been achieved using cavities made from it. We have investigated the cryogenic all-sapphire system and cryogenic composite system made of sapphire spacer and fused silica mirrors. Our system managed to reach frequency stability of $2 \times 10^{-13}$ for long integration time of $>1$ s. Since the first phase noise measurements inferred from loop noise measurements, we have increased the finesse of our reference cavity to 100 000. When two systems with high finesse are operational we will confirm that the performance of our system meets the requirements of a laser interferometer.
8. In Line Mode Cleaner

To reduce geometrical fluctuations in an input laser beam of a laser interferometric gravitational wave detector, a mode cleaner is necessary. The Garching group\textsuperscript{17} showed that an auxiliary Fabry-Perot cavity between the laser and the interferometer can be used as filter for laser beam geometrical fluctuations. Another method\textsuperscript{18} used a single mode optical fibre as a mode cleaner to reduce beam fluctuation. Most current gravity wave interferometers employ mode cleaning cavity or optical fibre to condition the input laser beam.

The mode cleaner developed at UWA is a suspended triangular cavity mode cleaner mounted in one arm of the 8 m interferometer vacuum system, as shown in Figure 11.

A triangular cavity design avoids problems caused by light reflected directly back from the interferometer to the laser.\textsuperscript{18} The in line configuration has three main advantages. First it can share the end mirror of the interferometer, reducing the number of vibration isolators and optical components. Secondly the 45 degree mirrors in the central tank can, in principle, share all or part of the beamsplitter vibration isolation. Thirdly there is no need for an additional vacuum system or external laser transport components. This design constrains the mode cleaner to the same length as the interferometer. However this length requirement is itself an advantage since the long cavity length helps avoid thermal damage to the mirror coating by increasing the beam diameter on the mirrors. Thus overall this configuration can provide an additional margin for reducing frequency noise while improving the thermal stability of the cavity.

The best power throughput and the finesse of the mode cleaner, assuming each of the two 45 degree mirrors have the nominal loss (\textasciitilde3000 ppm) can be calculated to be 44\% and 330 respectively. Ring down measurement showed the finesse to be around 180. This was attributed to the poor cleanliness conditions in the laboratory. The measured throughput was 29\% which, given the observed cavity fringe
contrast of 78% and the measured cavity losses, is consistent with theory. The overall suppression factor afforded to beam jitter noise near 100 Hz was measured as being 34 dB. The long term drift of the cavity alignment was also monitored. The output intensity fluctuation was within 10% over a 15 hour recording period.

In principle a laser interferometer gravitational wave detector can be improved and simplified by using one interferometer arm to support a mode cleaner. There is a possible problem from scattered light, but preliminary estimates are reassuring. The advantages, particularly in relation to cost and mechanical complexity, are particularly relevant to a proposed low cost high performance interferometer such as AIGO 400.

9. Ultra Low Frequency (ULF) Pre-isolation

It has been shown that a relatively simple ultra-low frequency pendulum-like structure can be cascaded in front of a multistage isolator to greatly improve the seismic isolation performance in the frequency range from tens of millihertz through tens of hertz. The internal modes of these ULF structures bypass their isolation above a few tens of hertz. However the multistage isolator following the ULF stage provides such enormous isolation at these frequencies that this is no disadvantage. The advantage provided by pre-isolation is a large reduction in the seismic motion driving the multistage isolator normal modes and also an extension to the low frequency end of the detection band. The multistage isolator normal mode motion is so reduced that damping of these resonances is no longer required, providing a major system simplification. The residual low-frequency disturbing motion which needs to be counteracted by the fringe locking control loop is vastly reduced. This provides many control system advantages including easier locking with reduced loop gain and reduced force actuation with consequent reduction in noise injection.

One geometry based ULF structure developed and tested at UWA\textsuperscript{20, 21} is the folded pendulum. It consists of two rigid vertical pendulum arms, one linked to each end of a horizontal platform by flexible foils. One arm hangs below a flexible support as a normal pendulum and the other stands above a flexible support as an inverted pendulum. By adjusting the mass load position on the platform to divide it evenly between the two pendulums we have tuned its resonant frequency down as far as 17 mHz. By arranging for the mounting frame of the folded pendulum to be vibrated horizontally, we have measured its horizontal isolation performance as a function of frequency and this is shown in Figure 12. 90 dB of isolation has been achieved at about 7 Hz demonstrating that this structure makes an excellent one dimensional preisolator.

In order to achieve 2-dimensional horizontal isolation it is possible to cascade two one-dimensional stages\textsuperscript{22} but this creates significant constructional problems and the isolation achieved is rather limited. Two-dimensional isolation may be achieved with a single stage by a conical pendulum. An inverted conical pendulum\textsuperscript{23} may be tuned to a very low resonance and is mechanically the simplest but has the disadvantage of cancelling a relatively large gravitational force with a material spring
constant. With the large cancellation required to achieve a very low resonant frequency, the remaining time-dependent, temperature-dependent and non-linear effects in the elastic material become dominant and this system is also inconveniently sensitive to mass loading. At UWA we have developed another design which mimics a very long conical pendulum achieving $xy$ isolation in a single stage with none of these disadvantages. We have termed it the Scott-Russel pre-isolator as it is based on the linkage by that name, generalised to cylindrical symmetry. A full sized pre-isolator has been designed and built to suit our existing multistage isolators. Its performance will be measured and reported on in the near future. A conceptual diagram is shown in Figure 13(a). We are currently developing a vertical pre-isolator stage to be cascaded after the $xy$ horizontal pre-isolation. In this case the cancellation of a large gravitational force with a material spring constant is almost unavoidable and a servo system will be required to control drifts with temperature.

In order to obtain a low resonant frequency without extravagant vertical extension under load, it is necessary to arrange a non-linear force vs displacement characteristic with a flat region around the operating point. The VIRGO group have achieved this to a moderate degree on their super-attenuators by summing a magnetic "anti-spring" component with a normal cantilever spring constant. We are investigating making use of the non-linearity produced by torsion sprung crank-arms and suspension links as shown in Figure 13(b). Analysis suggests (Figure 13(c)) that the flat region can be made quite wide allowing a good vertical operating range of almost 1/3 the length of the crank arm. Over this vertical distance the resonant period should stay within the useful bounds of 5-20 seconds. Although periods greater than 10 secs were difficult to achieve repeatably with our simple test rig, we readily achieved a dynamic operating range of 7 cm with the resonant period remaining between 6-10 seconds using a crank arm length of 25 cm and a load of around 1 kg.
10. Sapphire Test Masses and Beamsplitters

It is well known that the thermal and mechanical properties of sapphire make it an interesting material for use in test masses. The high thermal conductivity means that thermal lensing can be minimised, while the combination of its very high Young’s modulus and low acoustic loss ensure that the internal resonant modes have high frequency and low thermal noise.

According to the fluctuation-dissipation theorem, the thermal noise due to the internal resonant of the test mass can be expressed by

\[ \Delta x^2 = \frac{4k_B T}{\omega} \Phi(\omega) \sum_i \frac{1}{M_i \omega_i^2} = \frac{4k_B T}{\omega} \Phi(\omega) \frac{1}{M_{eq} \omega_{eq}^2} \]

The ratio of \( M_{eq} \omega_{eq}^2 \) for sapphire and silica test masses of the same dimensions (\( d = 200 \text{ mm}, \text{thickness of } H = 200 \text{ mm and a laser beam size of } 2 \text{ cm} \)), is 6.54. The highest internal mode Q-factor reported in silica is \( 7 \times 10^6 \). This compares with \( 3 \times 10^8 \) reported for sapphire by Braginsky et al. The reduced losses, combined with the increased values of \( M_{eq} \omega_{eq}^2 \) of sapphire means that the thermal noise amplitude of this sapphire test mass will be a factor of more than 16 times better than that of a silica test mass with the same dimensions.

In this section we show the recent results of Q-factor measurements with a sapphire sample (HEMEX sapphire by Crystal Systems) at room temperature. The results confirm the low acoustic losses of sapphire.

To observe a high Q-factor it is essential to develop a low-loss suspension system because the acoustic losses are mainly due to the suspension system coupling. Based on the successful titanium Catherine Wheel developed for the niobium resonant mass gravitational wave detector at the University of Western Australia, we used a small niobium Catherine Wheel spring suspension for the experiments. We chose niobium as the material because of its inherently low acoustic losses. The suspension
system was designed so as to minimise the energy coupling between the sapphire sample and the suspension system. A capacitive transducer was used to measure the ring down curve of the sapphire sample. The result for the sapphire sample with 100 mm long and 50 mm in diameter is shown in Figure 14 along with the result with a fused silica sample. We have observed the Q-factor of $2.6 \times 10^7$ with the sapphire sample and $1.3 \times 10^6$ with the fused silica sample of the same size.

To achieve the optimum sensitivity of a laser interferometer it is essential that the test mass combines the high internal mode Q-factor discussed above, with extremely high pendulum Q-factors. Figure 15 shows a typical target noise level.

We have proposed alternative suspension techniques which are capable of achieving the required low losses without the use of wires. The advantage of using a short thin membrane flexure is that it has very high violin string modes, a low flexural spring constant and very low thermoelastic loss. It has been shown previously that a membrane flexure allows enhanced pendulum Q-factors $29,30$ compared with wire suspension. Preliminary tests using Cusil brazing has resulted in a joint between sapphire and niobium with strength $> 150$ MPa. It remains to determine whether any of the above joints have sufficiently low acoustic losses.

In practice a test mass will be suspended from a vibration isolation stack. The internal modes can lose energy through the membrane to the large lossy isolation stack and, in addition, the imperfect clamping of mechanical components will cause excess acoustic losses. To assess this problem, the system was modelled using one dimension lumped mass and spring elements. $31$ Two models were considered. One consists of a directly coupled system using a moderately low loss membrane hinge. The second consists of a system with a small cantilever stage inserted above the membrane suspended test mass to provide some degree of vertical isolation. The results are shown in Figure 16. It can be seen that, within the frequency range of interest, membrane suspensions directly connected to the upper isolation stack are
Figure 15: Expected thermal noise performance of an interferometer with sapphire test masses.

Figure 16: Modelling results of the effect of test mass suspension on the Q-factor of the sapphire test mass internal mode.

The intrinsic birefringence property of sapphire is one disadvantage. Relative misalignment of the polarisation and crystal axes will cause the light to become elliptically polarised as it travels through the sapphire. The ellipticity introduced to the beam that double-passes the beamsplitter will lead to imperfect interference. The non-interfered light escapes from the interferometer out of each output port. In a power-recycled interferometer power loss is detrimental to the performance of the system as it limits the power enhancement by resonant reflection of the bright output. The fraction of lost power is:

$$\frac{P_{\text{lost}}}{P} = \sin^2 2\psi \approx 4\psi^2$$ (4)
where $\psi$ is the misalignment angle. For angles less than 100 $\mu$ rad, which is easy to realise in our suspension systems, this loss will be significantly less than other loss sources such as mirror losses and curvature mismatch. We conclude that the birefringence of sapphire is not a detrimental factor in using it as a beamsplitter substrate.

On the other hand, the inhomogeneity of sapphire and stress in it can cause birefringence. We have preliminarily investigated birefringence of a small $10 \times 10 \times 25\text{mm}^3$ sapphire sample. The observed phase shift between the ordinary ray and extraordinary ray due to birefringence is shown in Figure 17. The star shaped pattern observed is a characteristic of stress birefringence due to surface strain. In large sapphire samples such as 20 cm diameter and 10 cm thick, the stress at the centre is expected to be much smaller. For the loss due to inhomogeneous birefringence to be of the same order or less than other mirror losses, the total inhomogeneous birefringence phase shift should be smaller than $2^\circ$ in a beamsplitter. It is important that further more accurate measurements on large sapphire samples be made to obtain definitive results.

11. Conclusion and Future Plans

ACIGA plans to build an advanced dual recycling laser interferometer of 400-500 metre baseline in the years 1998 - 2000. We have shown that a broad base of advanced technology and new techniques is available which will make the instrument simple, tunable and of sensitivity comparable to the initial LIGO interferometer. In future years the interferometer will be extendable. It will make a strong contribution to the sensitivity and directionality of the global gravitational wave telescope, due to its maximal distance from all of the northern hemisphere detectors.
References
VIBRATION ISOLATION STACK FOR TAMA300

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ABSTRACT

Vibration isolation ratio of 150dB at 300Hz is required for TAMA300. In order to achieve this aim, it is effective to introduce stack constructed of rubbers and metal blocks, because of the isolation effect to all degree of freedom. We have been developing 3 stage stacks which have vibration isolation ratio of more than 60 dB at 300 Hz, even if cross coupling is included. Additionally this system is completely vacuum compatible.

1. Introduction

At present construction of the interferometer gravitational wave detector, TAMA-300 with 300 m long base lines, advances in Japan. Aimed sensitivity for gravitational waves is \( h_{\text{rms}} = 3 \times 10^{-21} \) at 300Hz (BW300 Hz) that is observation band. Therefore mirrors, which form the interferometer, should be isolated from ground motion very well. Vibration isolation ratio of 150dB at 300 Hz is necessary in ground vibration level at Mitaka that is establishment place of TAMA300. In order to achieve this aim, vibration isolation system is taken partial charge at three the following parts.

1. Vibration isolation below 10Hz by a X-pendulum\(^1\) (horizontal) and an active control (vertical).

2. Vibration isolation above 10Hz by a double pendulum.

3. Vibration isolation of optical bench (breadboard) above 10Hz by a stack.

Fig.1 shows total vibration isolation system. It is established optical bench inside eight vacuum chambers and vibration isolation unit of 1 and 2 is composed upward. Because of this, not only vibration isolation ratio but also long-term stability are very important. Moreover, vacuum compatibility is also necessary in order to be used throughout ultra high vacuum.

2. Structure of Stack

Fig.2 shows 3-D image of stack. Fundamentally this is 3 stage stack. The breadboards of aluminium are loaded on three legs with three stages stack. SUS block with 64-67kg weight is used as the mass of stack. Surface processing by ECB(Electro-Chemical Buffing) is performed to this mass block in order to prevent

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Gravitational Wave Detection
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Surface due to ECB has roughness lower than 0.4 micron. It looks like mirror. ECB and EL process are compared, examine to the breadboard of aluminium, too. EL is oil free process using not oil but ethyl-alcohol.

Bellows are characteristic of this stack (Fig.3). From vacuum compatibility, fluorocarbon is generally used as rubber material. However the out-gassing is not satisfied specification of ultra high vacuum even fluorocarbon. Additionally contamination of optical devices by fluorocarbon is reported. Complete vacuum compatibility is achieved in this system by covering rubber using bellows. In this case, there is not a restriction to the material of rubbers. The bellows have 32 fins, it is more softly sufficiently designed than inner rubber to expansion and contraction and side lag. Since rubber is sealed tightly by bellows in air, bellows behave as air spring. Although the spring coefficient of air springs is smaller than that of rubbers, balance point of springs will deviate from initial one in vacuum. Since we will be able to adjust the height of the breadboard from outside the vacuum chambers using DBB(Double Balanced Bellows) mechanism, the deviation will be compensated instead of vacuum braking. Table 1 shows specification of parts of stack. Elastic modulus \(G\) of rubbers are variable for each stage. As result contraction of rubbers by weight of the stack are almost same, about 2 mm, when the stack system is built.

### 3. Experiment Using Prototype of Stack

We had developed stack system using a prototype corresponds to one leg of total stack system. Fluorel\(^a\) with elastic modulus of 1.9 MPa is used as rubber for every stages on this prototype. The prototype was loaded on vibrator and the transfer functions were measured by acceleration sensor. Fig.4 shows the measured transfer functions with and without bellows. Shaken and measured direction are horizontal(X-X). The transfer functions in these condition are almost same. Influence on vibration isolation property of this bellows was not admitted at a band below 300 Hz. Vertical to vertical mode(Z-Z), and coupling mode such as vertical to horizontal(Z-X or Z-Y) were also measured using similar method(Fig.5). Coupling

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\(^a\)A kind of fluorocarbon by 3M corporation.
such as vertical to horizontal (Z-X or Z-Y) is not small, but totally isolation ratio of more than $10^3 (60\text{dB})$ is achieved at 300 Hz, sensitive frequency of TAMA300.

Next important point is drift motion of stack. There is expansion and contraction of rubbers and air springs due to temperature diversity. Fig.6 is sample data of stack for 20m prototype at NAO. Viton is used as rubber material. Tilt of the breadboard on the stack was measured using optical lever. Exactly it is correlated to room temperature. Maximum tilt change of 50 $\mu\text{rad}$/day was observed. Take it into consideration that stack of the 20m prototype is located on the SUS legs with length of 70 cm. It is predict that this leg affects tilt of the stack as well as rubber. Also for TAMA stack prototype, motion of vertical direction were investigated.

When rubber length $x$ is increased by $\Delta x$ due to pressure change $P \rightarrow P'$ on one bellows unit,

$$PS + kx = P'S + k(x + \Delta x),$$

where

$$P = nRT \frac{S}{SL},$$

$$P' = nRT \frac{T + \Delta T}{S(L + \Delta x)}.$$  

$T$ is temperature, $S$ is square measure of the air spring, $L$ is length of the air spring. Using $P = 1 \text{ atm} = 1 \times 10^5 \text{ Pa}$, $S = 9 \times 10^{-4} \text{ m}^2$, $L = 8 \times 10^{-2} \text{ m}$, $k = 6 \times 10^4 \text{ Pa} \cdot \text{m}$, and $T = 300 \text{ K},$

$$\frac{\Delta x}{\Delta T} \approx \frac{L \cdot nR}{nRT - L^2k} \left(\frac{\Delta x}{L} \ll 1\right)$$

$$= \frac{PS}{PS - Lk} \frac{L}{T}$$

$$\approx -\frac{PS}{kT} \quad (PS \ll Lk)$$

$$= -5 \times 10^{-6} \text{ m/K}.$$  

On 3 stage, 15 $\mu\text{m}/\text{K}$ is predicted from this calculation.

At the 3 points relative positions of top of SUS block are monitored using shadow sensors. Fig.7 shows drift motion of stack prototype. I’m sure that the motion of 3 points (A, B, C) are correlated to room temperature. In vacuum response is slow to temperature change. Not only rubber but also air inside bellows expand and contract due to temperature in the opposite direction. As a result $3 \sim 10 \mu\text{m}/\text{K}$ is predicted from calculation and measurement (Table2). In this connection, temperature of the center room of TAMA changes about 1 K/day. This is thought, it is possible to correct so that alignment of the interferometer can be prevented from this effect using adjustment system of mirror suspension unit.

\footnote{National Astronomical Observatory}
Table 2: Summary of drift motion of stacks. Not only rubber but also air inside bellows expand and contract due to temperature in the opposite direction.

<table>
<thead>
<tr>
<th>Material</th>
<th>20m</th>
<th>300m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length(cm)</td>
<td>70</td>
<td>6</td>
</tr>
<tr>
<td>Calculation(μm/K)</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Measurement(μm/K)</td>
<td>20**</td>
<td>4 (Air)</td>
</tr>
</tbody>
</table>

*No accurate data
**Estimation from tilt measurement

4. Combined Isolation System

As mentioned above, vibration isolation ratio of 150 dB at 300 Hz is necessary. From data using prototypes or test machines, we estimated combined transfer function of vibration isolation system. These data is respectively confirmed by experiments. X-pendulum has only one dimensional freedom with fundamental frequency of 0.1 Hz. The intermediate mass of Double Pendulum is damped by eddy-current. Active Isolation is performed by acceleration sensors and piezoelectric actuators. Fig.8 shows combined transfer function of X to X and Z to Z. These transfer function data were offered by developers of each isolation system. In any case, aimed isolation of 150 dB is cleared around 300 Hz.

5. Present Status

Already two stack systems for the mode cleaning cavity have been inserted. As preliminary result, it is recognized out-gassing decreases in comparison with nude fluorocarbon(Fluorel) because of using bellows. We are going to confirm specifications, for example isolation effects and drift motion, on these stack systems.

Acknowledgements

We are most grateful to our colleagues in NAO for their encouragement. We also wish to express our thanks to VIST300 group for contribution to this result.

References


cVibration Isolation, Suspension, Thermal: one of working group of TAMA
Figure 1: Total vibration isolation system. Double pendulum, X-pendulum, active isolation, and stack are introduced.

Figure 2: 3-D image of stack. This is fundamentally 3 stage stack.
Figure 3: Longitudinal cross section of stack. Al breadboard is put on SUS block and rubber stack. Rubbers are covered by bellows.

Figure 4: Measured transfer functions with and without bellows. Two conditions, in vacuum and in air, are shown. These transfer functions are almost same.
Figure 5: Variation of transfer functions. X and Y mean horizontal direction. Z means vertical direction. X to Z, Z to X, and Z to Y mean cross coupling.

Figure 6: Data of stack for 20m prototype at NAO. Tilt of breadboard on the stack correlated to room temperature. Maximum tilt change of 50 $\mu$rad/day was observed.
Figure 7: Drift motion of stack prototype. The motion of 3 points (A, B, C) are correlated to room temperature. In vacuum response is slow to temperature change.

Figure 8: Combined transfer function of total isolation system. X to X and Z to Z are shown. In any case, aimed isolation of 150 dB is cleared around 300 Hz.
A 2D X-PENDULUM VIBRATION ISOLATION SYSTEM

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ABSTRACT
A laser interferometric gravity wave detector requires very effective filtering of seismic noise, and this is especially difficult at very low frequencies. The X-pendulum, a technique for constructing a passive low-frequency mechanical oscillator, was used to make a compact vibration isolation table suitable for use in the TAMA detector. The table has a very long period (5-10 s or more) in both horizontal directions and is relatively stiff in all remaining degrees of freedom, so that it makes a good base for further stages of vibration isolation. A normal mode analysis of the elastic and pendulum modes of the system gave fair agreement with the observed mode spectrum and will be useful as a design tool.

1. Introduction
A typical laser interferometric gravity wave detector is limited in sensitivity at low frequencies principally by seismic noise. A major difficulty in providing low frequency isolation is that of creating a good low-frequency mechanical oscillator. Below about 1 Hz this is quite difficult, particularly in the modest space budget of a medium scale detector such as TAMA300. A simple pendulum is an effective horizontal isolator with many other desirable properties such as low drift and unconditional stability, but a 10 s period would require a 25 m pendulum.

The X-pendulum is a mechanism which preserves many of the advantages of a pendulum while greatly extending the period. The main mass is fixed rigidly below a movable plate which is in turn is suspended below a fixed horizontal support table using three or four wires crossed over at a shallow angle. The wires outline an ‘X’ when viewed from the side, hence the name of the technique. As the net center of mass approaches a certain critical point from below, the period tends to infinity. The fourth-order term in the potential is positive, so the response of the system is always bounded, even for large swings or if the center of mass is mistakenly moved above the critical point.

A practical one dimensional vibration isolation system uses two light-weight
X-mechanisms in parallel to support a load table as in Figure 1. Each X-plate is connected to the load table by an inverted pyramid of wires. Being wide at the top and under tension, each pyramid is rigid over most of its length. However the wires are able to bend at the bottom vertex and form a hinge there. This design saves weight and decouples the load from the rotation of the X-plates.

The one dimensional table is moderately rigid in all but the working degree of freedom, and so makes a good platform for further stages of isolation. Therefore an obvious approach to a two-dimensional design is to cascade two one-dimensional tables at right angles, using the load table of the first as the support table of the second. However a better approach described here allows the intermediate table to be eliminated.

2. 2D X-Pendulum Mechanism

The observation that allows a better two dimensional design is that a set of two or more X-pendulums in parallel works just as well upside-down, with the X-plates at the bottom near the load plate. The requirement for more than one X-mechanism comes from the need for a horizontal reference. A normal X-pendulum can rely on the rigid support table, but the inverted mechanisms are dependent on the load table, which is not guaranteed to remain horizontal along the working direction unless it is supported at at least two positions. The dynamics in the two orientations are very slightly different but this can easily be compensated for.

An inverted X-pendulum has no particular advantages used alone, but can be combined very effectively with a normal X-pendulum via the arrangement of four intermediate wires shown in Figure 2. We call such a set of two X-mechanisms a corner unit, and our design uses one at each of the four corners of the load table.
We take the working direction for the upper, normal X-mechanisms to be the x direction, and that of the lower, inverted ones to be y. The principle of operation can then easily be seen by considering the views from along the two axes. The y axis view closely resembles the standard 1D system in Figure 1, whereas from along the x-axis the system presents the same profile but inverted.

![Diagram of X-pendulum design](image)

**Figure 2:** Schematic of one of four corner units in the 2D X-pendulum design.

The number of corner units is not critical. Two is possible but would have poor stability. Three is entirely satisfactory, and arguably optimal, but we decided on four to conform better to the right-angled symmetry of the interferometer.

A convenient approximation to the dynamics of the system at low frequency can be obtained by supposing that the lower X-mechanism is rigidly attached to the load table for the purposes of analysing the upper X-mechanisms and vice versa. Then the x axis motion reduces to the one dimensional problem analysed in Barton et al. The y axis motion can be analysed by the same method. There are two minor differences to be allowed for: (i) the X-plates, via their mass, contribute negative restoring force in the normal position but positive restoring force in the inverted position, and (ii) the mass of the lower X plates is part of the load for the upper mechanisms. The resonant frequencies are then

\[
\omega_N = \frac{K_N + g [M_N(D_N - H_N) - m_N(H_N - C_N)]}{I_N + M_N(D_N + R_N)^2 + m_N(C_N + R_N)^2}
\]

(1)

\[
\omega_I = \frac{K_I + g [M_I(D_I - H_I) + m_I(D_I - C_I)]}{I_I + M_I(D_I + R_I)^2 + m_I(D_I - C_I)^2}
\]

(2)

Here, \(m_{N/I}\) and \(I_{N/I}\) are the combined mass and moment of inertia of the X-plates, and \(M_N = M + m_I\) and \(M_I = M\) are the effective loads (\(M\) is the mass of the
load table alone). \( R_{N/1} \), \( C_{N/1} \), \( H_{N/1} \), and \( D_{N/1} \) are the vertical distances from the level where the X-wires attach to the X-plates to, respectively, the point where the X-wires cross, the centre of mass of the X-plate, the critical position and the hinge position. \( K_{N/1} \) is the net angular spring constant due to the elasticity of the X-wires. The critical position \( H_{N/1} \) is related to the X gap size \( 2R_{N/1} \) and the gap width \( L_{N/1} \) by

\[
H_{N/1} = \frac{L_{N/1}^2}{8R_{N/1}} - \frac{R_{N/1}}{2}.
\]

3. Application

We built two prototype vibration isolation tables on the above principle. The prototypes differed only in the design of the corner units, which were detachable modules. The support and load tables for both were identical and were designed to match the smaller size of tank used in the TAMA vacuum system. The main aim was to find out if such a complicated system could be successfully set up and tuned to a stable long period. We made no particular effort to optimise the high-frequency performance.

The support and load tables for both prototypes were 500 mm square and 50 mm thick, and made of aluminium. To leave the corners free for the X-mechanisms, the four stainless steel legs, of length 800 mm and diameter 40 mm were screwed into projections at the centre of the edges. The load table had matching recesses so as not to foul on the legs. The total range of movement allowed by the recesses was about 10 mm in any direction from the centre position.

Each corner unit consisted of two X-plates and two ‘reference’ plates, so-called because by being bolted to the support and load plates they supplied the horizontal reference for the X-mechanisms. In both prototypes, the top and bottom halves were identical except for orientation.

3.1. First Prototype

The first prototype mechanism used four X-wires, as in our previous designs. The adjustment mechanism was distributed between the X-plates and the plates which attached to the support and load tables. Each X-wire started at a fixed clamp on the X-plate, passed through a clamp on the outer plate, turned 180° around a roller bearing and terminated at a single adjustment screw mounted inside the dish-shaped outer plate. Tightening the wires reduced \( R_{N/1} \) and increased \( H_{N/1} \). The height of the intermediate wire attachment points on the X-plates could also be adjusted by screw mechanisms.

The top half of each corner unit was individually tuned to the correct period and centering using a test mass one quarter the weight of the load. However a single inverted mechanism is not stable alone. The obvious strategy is to turn the lower mechanisms right-side-up for tuning. However because of the slight dependence of period on orientation it turns out that the mechanism would need to be adjusted into the unstable regime in the tuning position to have a long but finite period in the working position. Thus we also added spacers between the X-plates and the
intermediate wire adjustment mechanisms so as to temporarily increase the effective height $D$. The optimum spacer size was calculated numerically using Mathematica. The first version of the calculation assumed unstretchable X-wires and gave a spacer size which was too large, so that the bottom mechanisms were unstable when the system was assembled. However when the effects of stretching were allowed for, both the top and bottom periods were close to the target value.

While the first prototype showed that the concept was sound, it had a number of problems which severely limited the period that could be obtained. First, the single screw adjustment mechanism was too coarse. Second, the clamps on the outer plate could not be tightened until after tuning, which caused a large random perturbation in the period. Third, the clamps, which had been made as small as possible to fit in the narrow gap, proved to be too weak. The resulting slippage during motion led to a very poor $Q$, which deteriorated as the inverse square of period. Thus it was not possible to do better than a period of around 5 s.

3.2. Second Prototype

In the second prototype we addressed the weaknesses identified above. To allow room for sturdier clamps, we interleaved the upper and lower clamps horizontally and mounted them well clear of the plate surfaces vertically using rigid posts for the X-plate clamps and movable levers for the reference plate clamps. This left plenty of headroom for the clamp screws. The clamps consisted of two pieces of tool steel mounted in a cavity in the post/lever. The jaw pieces were wedge shaped to make the angle of the wire in the clamp the same as that in the ‘X’.

The X-wires were installed using a jig to hold the reference plate and X-plate at the correct separation and and screw tensioners to tension the wires uniformly. Once the clamps were tightened, the jig and tensioners were removed and all subsequent adjustment was done with the levers. These were actuated by double screw adjustment mechanisms (using threads of 0.75 and 0.8 mm pitch).

The new mechanism and adjustment procedure worked very well. Provided the support table is mounted on a firm base, periods of 10 s are easy and 17 s has been achieved with a little more care. The $Q$ for a 5 s period is around 150, which is more than an order of magnitude better than the first prototype.

4. Optimization

Neither of the prototypes were optimised for good high frequency performance. Two effects cause the transfer function to depart from that of a simple harmonic oscillator. First, even for a system made of unstretchable wires, the finite mass and moment of inertia of the X-plate can cause the transfer function to saturate at a non-zero value at high frequency. Second, since real X-wires have finite rigidity, there are a large number of elastic modes. These can be seen in the transfer function of Figure 3. Note that the relatively small shaker table used for this measurement was not very stable against tilt, so the period was limited to 5-6 s.

The first effect can be largely eliminated by choosing the moment of inertia of the X-plate according to the optimization criterion given in. The formula depends
Figure 3: Transfer function (x→x) of the second prototype tuned to around a 5 s period. PZT accelerometers were used, so the data below 2-3 Hz is unreliable.
only on the geometry of the system and the mass distribution in the X-plate, but not on gravity or the mass of the load, and so is valid for the inverted case as well.

\[ I_{N/I(\text{opt})} = m_{N/I}(D_{N/I} - C_{N/I})(C_{N/I} + R_{N/I}). \] (4)

To explore the elastic mode spectrum we made a normal mode analysis using Mathematica. We neglected the internal modes of the parts, the violin modes of the wires and the vertical modes of the support plate (z, pitch and roll), but included all other rigid body modes, for a total of \( 1 \times 3 + 9 \times 6 = 57 \). As might be hoped, the lowest frequency modes predicted by the model were the characteristic x and y pendulum modes. The elastic modes ranged from the torsional mode of the load at 2.7 Hz to tipping modes of the X-plates at around 560 Hz. All the predicted modes up to 60 Hz corresponded closely to observed modes in character and frequency - the remaining modes were above 150 Hz and much harder to identify confidently.

The most troublesome modes are those involving x or y motion of the load plate, especially the pitch and roll modes of the plate itself (5.6 Hz), the x and y modes of the support plate (8 Hz), and certain transverse rocking (17-20 Hz) and vertical (57 Hz) modes of the X-plates.

Fortunately the coupling into and/or out of most of these modes can be reduced close to zero through attention to the relative position of the various centres of mass and wire attachment points. If the X-wire attachment points are placed at the same height as the centre of mass of the load, input x motion will not couple to tilt. If the suspension point of the final stage (i.e., the TAMA double pendulum) is placed at the centre of mass, any tilt vibrations that are excited will not couple to output x motion. Since the vertical modes of the X-plates act via load plate tilt, they are suppressed as well. In the same way, the tipping modes of the X-plates can be decoupled at both input and output by placing both the X-wire and intermediate wire clamp points at the same height as the centre of mass. The support table modes can be pushed to much higher frequencies (where the double pendulum is effective) by increasing the stiffness of the table. The only elastic modes below 100 Hz which cannot be suppressed are certain modes with top and bottom X-plates moving horizontally in phase.

5. Conclusion

Our 2D X-pendulum system behaves as predicted and can easily give a stable period of 10 s in both horizontal directions. The high-frequency performance is currently poor, but straightforward optimizations should improve this substantially and largely remove all but one resonance peak from the transfer function below 100 Hz. A design with these optimizations is being drawn up with a view to incorporation in TAMA300.

Acknowledgements

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References

LISA - A GRAVITATIONAL WAVE OBSERVATORY IN HELIOCENTRIC ORBIT

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ABSTRACT

A spaceborne detector will enable the observation of low-frequency gravitational waves in a frequency range from $10^{-4}$ to $10^{-1}$ Hz which is totally inaccessible to ground based experiments. This frequency range is unique in that it is expected to contain signals from massive black holes, galactive binary stars, as well as the most violent events in the Universe.

LISA will attain this low-frequency sensitivity by employing laser interferometric distance measurements over a very long base-line of $5 \times 10^6$ km. Three of these baselines form an equilateral triangle with spacecraft at each vertex. The cluster of spacecraft is in an earth-like orbit around the sun trailing the earth by 20°.

The spacecraft contain infrared light-emitting Nd-YAG lasers and freely floating test masses made from a special platinum-gold alloy with vanishing magnetic susceptibility. The spacecraft are being kept centered on their test masses using drag-free technology and field-emission electric propulsion, thus letting the test masses follow purely inertial orbits.

LISA has been chosen by the European Space Agency ESA as one of the Cornerstone missions in its future science program Horizons 2000.

1. Overview

Conceptually, the idea of implementing an interferometer in space is straightforward, but the practical realisation requires an intricate blend of optical technology, spacecraft engineering and control. For a start, the interferometer mirrors can not simply float freely in space — they must be contained inside spacecraft. Nonetheless, they can be arranged to be floating almost freely inside the spacecraft, protected from external disturbances by the spacecraft walls. As long as the spacecraft do not disturb the mirrors, then, ideally, only gravitational waves would perturb their relative motion. “Drag-free control” can be employed to ensure that the spacecraft always remain centred on the mirrors.

In principle, then, the Michelson interferometer could be realised using three spacecraft: one at the “corner” to house the light source, beam splitter, and detector, plus one at each “end” to house the remote mirrors. But there would be immediate practical problems with such a configuration. All three spacecraft would drift around, and the corner spacecraft would not be able to keep itself aligned with both of the end spacecraft at the same time. One way around this would be to have steerable optics inside the corner spacecraft so that alignment could be maintained with the two arms independently. To avoid this complexity, LISA uses six spacecraft, arranged in a triangular configuration with two at each vertex. With this setup, each of the corner spacecraft can dedicate itself to pointing at only one of the end spacecraft, thus eliminating the need to steer the main optics. The corner
spacecraft must, nevertheless, communicate with each other using steerable optics — but the separation distance is so much less that the steerable components can be much smaller, and hence more manoeuvrable.

Each “corner” pair of spacecraft, separated by 200 km, is located at the vertex of a large triangle whose sides measure $5 \times 10^6$ km in length. This arm length has been chosen to optimise the sensitivity of LISA at the frequencies of known and expected sources. A factor of 2 increase may be desirable. However, an arm length increase beyond that would begin to compromise the high-frequency sensitivity when the light in the arms experiences more than half of the gravitational wave period. An interferometer shorter than $5 \times 10^6$ km would begin to lose the interesting low-frequency massive blackhole sources. It would give less scientific information but would not be any easier to build or operate because the spacecraft and the interferometry would be essentially the same.

Nominally in such an arrangement of spacecraft, any two sides of the triangle (i.e. four spacecraft) can be used for the main interferometry, with the third arm giving supplementary information and redundancy. With the six spacecraft configuration, up to two can be lost without jeopardising the mission (as long as the two failures are not at the same corner), since the basic group of four in an approximate “L” shape is sufficient to perform the full interferometry.

Each spacecraft is actually in its own orbit around the Sun. The six individual orbits have their inclinations and eccentricities arranged such that, relative to each other, the spacecraft rotate on a circle ‘drawn through’ the vertices of the giant triangle which is tilted at 60° with respect to the ecliptic. With this special choice of orbits, the triangular geometry of the interferometer is largely maintained throughout the mission. The centre of the triangle is located on the ecliptic — 20°
behind the Earth — and follows the Earth on its orbit around the Sun. Ideally, the constellation should be as far from Earth as possible in order to minimise gravitational disturbances. The choice of $20^\circ$ is a practical compromise based on launch vehicle and telemetry capabilities.

The once-per-year orbital rotation of the LISA constellation around the Sun provides the instrument with angular resolution, i.e. the ability to pin-point the particular direction to a source. An interferometer is rather omnidirectional in its response to gravitational waves. In one sense this is advantageous — it means that more sources can be detected at any one time — but it has the disadvantage that the antenna cannot be “aimed” at a particular location in space. For a given source direction, the orbital motion of the interferometer Doppler-shifts the signal, and also affects the observed amplitude. By measuring these effects the angular position can thus be determined. This is analogous to the technique used by radio astronomers to determine pulsar locations.

It is expected that the strongest LISA sources (from very distant supermassive black holes) should be resolvable to better than an arcminute; and even the weaker sources (galactic binaries) should be positioned to within one degree throughout the entire galaxy.

A LISA spacecraft is shown in Fig. 2, and a cross-section of the payload in Fig. 3. Each spacecraft has its own 1 W laser (actually two, one for redundancy), its own two-mirror telescope for sending and receiving light, and an optical bench which is a mechanically-stable structure on which various sensitive optical components are mounted. The mirrors enclosed in each spacecraft are actually 40 mm gold-platinum
cubes (also referred to as the ‘proof masses’). Each one is located inside a titanium vacuum can at the centre of the respective optical bench. Quartz windows allow access for the laser light.

Within the corner pair of spacecraft, one laser is the ‘master’, and a fraction of its light (10 mW) is bounced off the back surface of its cube, and sent to the neighbouring corner spacecraft (via the small steerable optics), where it is used as a reference to ‘slave’ the local laser. In this way, the main (~ 1W) beams going out along each arm can be considered as having originated from a single laser. This is vital to the function of the interferometer.

The light sent out along an arm is received by the end spacecraft telescope, bounced off its cube, then amplified using its local laser, in such a way as to maintain the phase of the incoming light. The amplified light is then sent to the corner spacecraft. Amplification at the end spacecraft is required due to divergence of the beam over the very large distances. Even though each outgoing beam is extremely narrow — a few micro radians — it is about 20 km wide when it reaches the distant spacecraft. This diffraction effect, together with unavoidable optical losses, means that only a small fraction of the original output power (~ 10^{-10}) finally reaches the end diode. If this was simply reflected and sent all the way back, only about 200 photons per hour would reach the corner diode after the round-trip. The phase-signals they carry would be swamped by shot noise, the quantum-mechanical fluctuations in the arrival times of the photons. The amplification brings the number back up to over 10^8 photons per second — which makes the signal detection straightforward using standard photodiodes. The phase precision requirement for this measurement is seven orders of magnitude less demanding than is routinely achieved (at higher frequencies) in ground-based prototype interferometers.

The resulting round-trip journey from the corner to the end and back, defines one arm of the large interferometer. On its return to the corner spacecraft, the incoming light is bounced off the cube and then mixed with a fraction of the outgoing light on a sensitive photodetector, where interference is detected. The resulting brightness variations contain the phase-shift information for one arm of the interferometer. This signal is then compared (in software on the on-board computer) with the corresponding signals from the other two arms, and some preliminary data processing is done. The results are then transmitted to Earth by radio link.

Figure 3: Cross-section of the payload on each of the six identical LISA spacecraft.
The LISA spacecraft must be designed to minimise the total mass and required power. Preliminary results yield a mass, per spacecraft, of 300 kg, and an operational power requirement, per spacecraft, of 192 W.

2. Lasers

Lasers have extremely narrow beams that can survive long journeys through space. In addition, they are very stable in frequency (and phase) which is crucial to interferometry since phase “noise” appears just like gravitational waves. Furthermore, the infrared light has a frequency of $3 \times 10^{14} \text{ Hz}$ which renders it immune from refraction caused by the charged particles (plasma) which permeate interplanetary space.

The lasers for LISA must deliver sufficient power at high efficiency, as well as being compact, stable (in frequency and amplitude), and reliable. The plan is to use solid-state diode-pumped monolithic miniature Nd:YAG ring lasers which generate a continuous 1 W infra-red beam with a wavelength of 1.064 $\mu$m.

3. Drag-Free and Attitude Control

An essential task of the spacecraft is to protect the mirrors from any disturbances which could jostle them around and create phase-signals that appear as gravitational waves. For example, consider the momentum of the light from the Sun which amounts to an average pressure of about $5 \times 10^{-6} \text{ N/m}^2$. The internal dynamics of the Sun lead to small variations — less than one percent — in this photon pressure, which occur at the low frequencies within LISA’s range of interest. Although this variable photon pressure may seem rather small, if it were allowed to act on the cubical mirrors, the resulting motion would be $10^4$ times larger than the tiny motions due to gravitational waves that LISA is looking for.

By simply “wrapping a spacecraft around each one”, the cubes are isolated from the solar pressure — but this is not the complete picture. When the solar pressure blows on the surface of the spacecraft, it will move relative to the freely-floating cube. Left alone, this motion would build up to unacceptable levels — in the extreme case, the cube would eventually “hit the wall”. To stop this from happening, the relative motion can be measured very precisely by monitoring the change in electrical capacitance between the cube and electrodes mounted on the spacecraft. This measurement is then converted into a force-command which instructs thrusters mounted on the outer structure of the spacecraft, to fire against the solar pressure and keep the spacecraft centred on the cube.

This concept is, for historical reasons, known as “drag-free control”, since it was originally invented in the 1960’s to shield Earth-orbiting satellites from the aerodynamic drag due to the residual atmospheric gases. The method was first demonstrated on the TRIAD spacecraft, flown by the US Navy in 1972, where the drag-free controller designed at Stanford University in collaboration with the Johns Hopkins Applied Physics Laboratory, was effective in reducing the effects of atmospheric drag by a factor of $10^3$. Since then, the technique has undergone
continued development, most notably for use on NASA’s Gravity Probe B mission, which is the proposed space experiment to search for the relativistic precessions of gyroscopes orbiting the Earth.

The thrusters used on conventional spacecraft are far too powerful for LISA. The drag-free system only needs to develop a force of a few micro-newtons. Furthermore, the delivered force must be smoothly controllable so that the varying disturbance forces can be matched without introducing a further disturbance from the thrust system itself. Surprisingly, it is not a trivial task to build a thruster which generates such a small force and yet operates smoothly and does not consume too much power. By good fortune, ESA has been developing them for years, as an alternative to hydrazine rockets for station-keeping of communication satellites.

They are called FEEP, for Field Emission Electric Propulsion. They operate by accelerating ions in an electric field, and ejecting them to develop the thrust.

4. Ultrastable Structures

The small variations in the intensity of sunlight will cause fluctuations in the heat-load applied to the spacecraft. This could lead to thermal gradients across the optical bench, which would upset the stability of the laser cavity. To obtain the required thermal stability, most structural elements are made from carbon-epoxy which has a thermal expansion coefficient of $4 \times 10^{-7} / \text{K}$ and the optical bench is made from ULE, which has a temperature coefficient at least a factor 4 lower over the possible temperature range of the LISA payload. Furthermore, low emissivity coatings are used on most surfaces inside the spacecraft and a thermal shield surrounds the payload cylinder, in order to provide isolation from the temperature variations of the spacecraft skin that is exposed to the Sun. These shields are only effective against heat fluctuations faster than a few hours to half a day. The slower variations will get through, thus making the sensitivity of LISA deteriorate rapidly below roughly $10^{-4}$ Hz. The use of carbon-epoxy structures also minimises any thermally-induced mechanical distortions which could produce physical changes in the optical path-length, as well as local gravitational disturbances on the mirror-cubes.

5. Data Transmission

Each spacecraft will be equipped with two (one spare) X-band transponders with steerable 30 cm high-gain antennas for communication with the Earth. On average, the transmissions will require about eight hours per day, at a data rate of roughly 600 bits per second. The entire LISA data set, after a nominal two-year mission, will be stored on about a hundred CD-ROMs.
ASTROD and Gravitational Waves

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ABSTRACT

ASTROD (Astro Dynamical Space Test of Relativity using Optical Devices) is a multipurpose mission concept using drag-free spacecraft in solar orbits to combine high-precision measurement of relativistic effects, measurement of solar angular momentum via Lense-Thirring effect, better determination of the orbit parameters of major asteroids, improvement in the measurement of G, and the observation of low-frequency gravitational waves and solar oscillations in a single mission. After presenting the basic concept of this mission, we analyze its sensitivity in gravitational-wave detection and the corresponding technology requirement.

The theoretical sensitivity limit (shot noise limit) in the strain for gravitational-wave detection is inversely proportional to $P^{1/2}l$ with $P$ the received power and $l$ the distance. Since $P$ is inversely proportional to $l^2$ and $P^{1/2}l$ is constant, this sensitivity limit is independent of the distance. For 1-2 W emitting power, the limit is $10^{-21}/\sqrt{Hz}$. Due to longer arms, the whole time-integrated sensitivity limit curve for ASTROD would be shifted to lower frequency by a factor up to 60 compared to LISA if the following technological requirements can be met.

First, the requirement on the intensity of weak-light phase locking is up to 3 orders of magnitude more stringent than that of LISA, and requires a capability of locking to 100 fW incoming light. For 100 fW light of wavelength 1.064 μm, there are $5 \times 10^5$ photons/s. This intensity gives about 100 kHz of tunability in bandwidth, and there is enough power (photons) to implement appropriate phase-locking or offset phase-locking schemes. In the technological implementation, we require 1 Hz-linewidth laser pre-stabilization. This will also be good for interferometry. Second, heterodyne detection for Doppler shift (for 1064 nm laser) up to 12 GHz with a maximum change rate of 3 kHz/sec needs to be implemented. Third, drag-free performance in the low-frequency region needs to be guaranteed. For this, we propose to use absolute laser metrology with a molecular-line stabilized laser as a principal constituent of an absolute accelerometer.

With its technological requirements achieved, ASTROD would complement LISA in probing the early Universe and study strong-field black hole physics in the lower frequency range. We discuss briefly the detectability of primordial gravitational waves and the resolvability of confusion limit in this regard. After LISA and first flight of ASTROD, when the outlook becomes clearer, a dedicated gravitational-wave mission with 4 spacecraft in separate solar orbits may be proposed. With this dedicated mission, the detectability of primordial gravitational waves from slow-roll inflation looks possible.

1. Introduction

The importance of gravitational-wave detection is twofold: (i) as probes to fundamental physics and cosmology and (ii) as tools in astronomy and astrophysics. In the following discussions, we extend the conventional classification of gravitational-wave frequency bands into the following ranges:
(i) High-frequency band (1-10 kHz): This is the frequency band that ground gravitational-wave detectors are most sensitive to.

(ii) Low-frequency band (100 nHz - 1 Hz): This is the frequency band that space gravitational-wave experiments are most sensitive to.

(iii) Very-low-frequency band (300 pHz-100 nHz): This is the frequency band that the pulsar timing experiments are most sensitive to.

(iv) Extremely-low-frequency band (1 aHz - 10 fHz): This is the frequency band that the cosmic microwave anisotropy and polarization experiments are most sensitive to.

A summary of projected sensitivity of ground and space-based gravitational-wave detectors is depicted in Fig. 1 for burst sources of short duration. For periodic or quasi-periodic sources, simply divide by (duration of detection times frequency)\(^{1/2}\).

![Figure 1: Design sensitivity of ground and space-based gravitational-wave detectors to bursts at high and low frequencies. The sensitivity curves of GEO600, LIGO I, VIRGO, LIGO II, LISA together with various possible sources are taken from Fig. 4 of Reference 2. The TAMA sensitivity curve is taken from Reference 3.](image)

For the very-low-frequency band and for the extremely-low-frequency band, it is more convenient to express the sensitivity in terms of energy density per logarithmic frequency interval divided by the cosmic closure density \(\rho_c\) for a cosmic background of gravitational waves, i.e., \(\Omega_g(f)(=\langle f/\rho_c \rangle d\rho_g(f)/df\).
The current upper limits from pulsar timing observations on a gravitational wave background is about $\Omega_g \leq 10^{-7}$ in the frequency range 4-40 nHz.\(^4\)-\(^6\) More pulsar observations with extended periods of time will improve the limits by two orders of magnitude in the lifetime of present ground and space gravitational-wave-detector projects. The COBE microwave-background quadrupole anisotropy measurement\(^7\)-\(^8\) gives a limit $\Omega_g(1 \text{ aHz}) \sim 10^{-9}$ on the extremely-low-frequency gravitational-wave background.\(^9\)-\(^10\) Ground and balloon experiments probe smaller-angle anisotropies and, hence, higher-frequency background. Planck Surveyor\(^11\) and MAP\(^12\) space missions will probe anisotropies with $l$ up to 2000 and with higher sensitivity.

From Fig. 1, we see that ASTROD probes lower frequencies than LISA. In Section 2, we review the ASTROD mission concept. Section 3 and Section 4, discuss, respectively, technological requirements and our laboratory progress. Section 5 gives the gravitational-wave sensitivity of ASTROD. In Section 6, we discuss the detectability of the primordial gravitational waves. In Section 7, we briefly discuss Mini-ASTROD, a down-scaled mission with one spacecraft, and Super-ASTROD, an up-scaled mission with four spacecraft in 5 AU orbits. In Section 8, an outlook is presented.

2. ASTROD Mission Concept

Recently, we have proposed the ASTROD (Astrodynamical Space Test of Relativity using Optical Devices)\(^13\) mission concept of using drag-free spacecraft in solar orbits together with a constellation of Earth orbiting satellites (including spacecraft near the Lagrange points) which provides high-precision measurement of relativistic effects, better determination of the orbits of major asteroids, improvement in the measurement of $G$, measurement of solar angular momentum via Lense-Thirring effect and the observation of low-frequency gravitational waves and solar oscillations in a single mission. In this section, we review this mission concept.

The basic concept of our mission is to use laser as a measuring rod to map the solar geodesics and to study astrodynamics as accurately as possible. We call this emerging realm of study Laser Astrodynamics. With laser astrodynamics, the accuracy will be improved by 3-6 orders of magnitude. All the effects which have influences in this range of accuracy shall be explored and studied.

First-order relativistic effects include Shapiro time delay ($\gamma$), perihelion shift, Lense-Thirring effect and so on. The ultrahigh precision measurement of these effects, e.g., $\gamma$ to $10^{-6}$-$10^{-9}$, will enable us to probe the cosmological influence and galactic influence on the first-order parameters in various relativistic gravitational theories, and at the same time allow us to test gravitational theories and cosmological models.

Precision measurement of relativistic effects demands accompanying precision in the determination of the orbit elements (including masses) of solar system objects. This can be achieved by fitting the influences of these orbit elements to the observed orbits of the spacecraft. The accuracy of this procedure is being explored in the determination of the masses of the major asteroids from simulated spacecraft
orbits. In many cosmological models, the variation of the gravitational constant is a natural outcome. From the interferometric laser ranging of the spacecraft to Earth satellites, the residual secular accelerations of the drag-free spacecraft can be separated in the large from the effects due to $G$ in the determination of the Earth-Sun mean separation (1 AU). The time change of the Earth-Sun mean separation can give a measurement of the $G/G$ to $10^{-13}$/yr resolution. When the influence of the asteroids is better monitored, $10^{-14}$/yr can be reached. With this precision, we could start to monitor total mass loss/accretion in the solar system and probe the contribution from solar neutrinos and solar axions.

In general relativity, the solar angular momentum drags the local inertial frames around. This is the Lense-Thirring effect. Due to this effect, the difference of elapsed times for the light signals to travel clockwise around the Sun and counterclockwise around the Sun is proportional to the Sun’s angular momentum. With the development of round-trip optical interferometry, ASTROD would be able to measure this effect to 1% or better. There is about 10-20% uncertainty in solar angular momentum in the present model of the Sun. Therefore, following the measurement of the Lense-Thirring effect of Earth in the GP-B Mission, the measurement of solar Lense-Thirring effect would constitute a measurement of the solar angular momentum. This will provide direct measurement of an important parameter for modelling the Sun and provide additional constraint on the internal models which will be developed using the observations from the recently launched SOHO (the SOlar and Heliospheric Observatory).

The strain sensitivity of ASTROD spacecraft will reach $10^{-20}-10^{-21}/\sqrt{\text{Hz}}$ in the 50 $\mu$Hz - 5 mHz frequency range. The low l p-modes solar oscillations will be readily detectable through their changing gravity field if there is no background "noise" masks these signals. The g-mode sensitivity of ASTROD is about 2 orders of magnitude better than LISA and SOHO. ASTROD will have a good chance to detect and observe these g-mode oscillations. This will create a new eye to see inside the Sun. However, all will depend on whether the background noise can be resolved. Hils, Bender and Webbink estimated that there about $3\times10^6$ short-period white-dwarf binaries producing a large gravitational wave background in the relevant frequency band. The recent discovery of two white-dwarf binaries (periods: 3.47 h and 4 h) give input for a better estimate of the population. In a future work, we will analyze whether and how this background noise can be lifted for the gravitational detection of the solar g-mode oscillations.

For the realization of this basic mission concept, it is desirable to have a fleet of drag-free spacecraft in solar orbits together with an Earth reference system. The Earth reference system could be ground stations, Earth satellites and/or spacecraft near Earth-Sun Lagrange points. Each spacecraft in solar orbit communicates and ranges with the Earth reference system. When two of them are close to each other, they can communicate and range with each other. For the fleet of spacecraft in solar orbits, a simple two spacecraft implementation is to have each spacecraft in separate solar orbit carry a payload of a proof mass, two telescopes, two 1-2 W
lasers, a clock, and a drag-free system. In order to calculate the orbit, we assume an initial spacecraft location in geostationary orbit, one located at the midday longitude and one at midnight, and we assume they are launched from these locations to their interplanetary trajectories on 10 June, 2005. Each spacecraft orbit is propagated using a fourth-order Runge-Kutta method with the gravitational force term calculated using the JPL DE403 Ephemeris data. The computed results are shown in Fig. 2. On the top of Fig. 2, the orbits are drawn in the Sun-Earth Fixed Frame. On the bottom, the apparent angles of the two spacecraft from 850 to 1000 days after launch as viewed from the Earth are shown. During the 150 days in this period, two-way ranging between each spacecraft and Earth reference system, and between two spacecraft will be implemented.

Figure 2: (Top) The inner and outer orbits of two ASTROD spacecraft in the Sun-Earth fixed frame; and the triangles formed by the positions of the two spacecraft and Earth at 700, 800, and 900 days of mission time. (Bottom) The apparent angles of the two ASTROD spacecraft located respectively on the inner and outer orbits near two and half years after launch.

To measure the solar Lense-Thirring effect, we need the difference, $t_1 - t_2$, in the
two round trip propagations, ERS (Earth-Reference System) $\rightarrow$ S/C 1 (Spacecraft 1) $\rightarrow$ S/C 2 (Spacecraft 2) $\rightarrow$ ERS and ERS $\rightarrow$ S/C 2 $\rightarrow$ S/C 1 $\rightarrow$ ERS. The sensitivity of measurement of the Lense-Thirring effect is $10^{-5}$ or better by means of this interferometric measurement.

For gravitational-wave detection, we compare the following two paths through the whole mission except for the period 850-1000 days: (i) ERS $\rightarrow$ S/C 1 $\rightarrow$ ERS $\rightarrow$ S/C 2 $\rightarrow$ ERS; (ii) ERS $\rightarrow$ S/C 2 $\rightarrow$ ERS $\rightarrow$ S/C 1 $\rightarrow$ ERS. The time difference of the two paths varies up to 0.34 sec. A laser linewidth of 1 Hz would keep the interference coherent. The sensitivity $10^{-20}$ to $10^{-21}/\sqrt{\text{Hz}}$ would be comparable to that of LISA, but more sensitive to lower frequency due to long path. The round-trip interference for measurement of the Lense-Thirring effect also has some sensitivity on the gravitational wave.$^{21}$

3. Technological Requirements

To achieve the sensitivity as mentioned in the last section with about 1W laser power, the following technological requirements need to be met.

First, the requirement on the intensity of weak-light phase locking is up to 3 orders of magnitude more stringent than that of LISA. LISA requires phase-locking capability to 180 pW incoming light. For ASTROD, we require a capability of locking to 100 fW incoming light. For 100 fW light of wavelength 1.064 $\mu$m, there are $5 \times 10^5$ photons/s. This intensity gives about 100 kHz of tunability in bandwidth, and there are enough power (photons) to implement appropriate phase-locking or offset phase-locking schemes. Pre-stabilization of lasers are required. In the technological implementation, we require 1 Hz-linewidth pre-stabilization. This will be adequate for interferometric coherence for our detection topology with up to 0.34 sec armlength difference.

For the configuration in Fig. 2, the radial velocity of the spacecraft relative to Earth varies up to $10^{-4}$ $c$ and the fractional frequency shift $\Delta f/f$ due to Doppler effect varies up to $10^{-4}$. For the 1064 nm Nd:YAG laser light, the Doppler shift of frequency, $\Delta f$, varies up to 12 GHz with a maximum change rate of 3 kHz/sec. In the laboratory, we will use light with variable attenuation from a stabilized laser to simulate laser light from space. Schemes will be implemented to offset phase-lock another laser to the simulated laser light from space with the offset frequency synthesized according to a predetermined program to simulate Doppler tracking.

The third technological requirement is on the drag-free performance in the low-frequency region. For LISA, the drag-free performance error is required to be below $10^{-15}$ m/s$^2$ (rms) in the band $10^{-4}$ to $10^{-1}$ Hz. For ASTROD, since the distance is up to 60 times larger, the local spacecraft control requirement can be up to 60 times looser. However the frequency range required needs to be 60 times lower. This requirement can be met using an absolute laser metrology system to monitor and correct changes. For the absolute laser metrology system, a stabilized laser locked to a molecular line is needed. For this, we study the potential of using Nd:YAG laser stabilized to an Cs$^+_2$ line compared to the frequency-doubled Nd:YAG laser
4. Laboratory Progress

Since 1992, we have started to work on the laboratory development for fundamental physics space missions using optical devices. We worked on the weak light phase-locking, fibre-linked heterodyne interferometry, fibre delay line, picometre real-time notion control and laser stabilization. We demonstrated that with a suitable noise cancellation scheme, for two lasers with offset locking up to 2.5 GHz, the heterodyne linewidth after travelling through a 26.27 km fibre-linked interferometer is less than 1 mHz. For weak light phase-locking, we achieved 4.3 nW locking with a 3.4 mW local oscillator. Extrapolating to 340 mW local oscillator power and five times lower detector dark current, 86 pW weak light optical phase-locking can be achieved. To improve on this, laser prestabilization would be necessary and we are currently working on it. Our next goal is to achieve 10 pW weak light phase locking. We improved our side-polishing technique to polish more than eight fibres simultaneously and reached a tunable sensitivity as high as 85-90 dB in liquid-drop tests. Using these side-polished fibres, we have made tunable directional couplers and are now in the process of making continuously variable fibre delay lines. We use tunable directional couplers to achieve fixed delays and both PZTs and EOMs to achieve continuous delays. For laser metrology, we use mid-point cyclic averaging to reduce the nonlinearity error, and use a fitting method to cancel the drift, and have reached 1.5 pm linearity. Currently, with modulation and real-time cyclic averaging, we reach subnanometer real-time measuring precision and subnanometer real-time motion-control precision.

For the purpose of absolute stabilization of Nd:YAG lasers, we measure the spectroscopic positions of Cs$^+$ lines near 1064 nm and work on laser stabilization to some of these lines.

5. Gravitational-Wave Sensitivity

In this section, we discuss the ASTROD sensitivity for gravitational waves following our presentation in the LISA Symposium. Let the distance between the spacecraft (S/C 1) in the inner orbit and the Earth Reference System (ERS) be $l_1$, and that for the spacecraft (S/C 2) in the outer orbit and the ERS be $l_2$. To make a Michelson interferometer from these two arms would require a linewidth of 1 mHz. To minimize the armlength difference, we propose to compare the following two paths through the whole mission except for the period 850-1000 days: (I) ERS$\rightarrow$S/C 1$\rightarrow$ERS$\rightarrow$S/C 2$\rightarrow$ERS; (II) ERS$\rightarrow$S/C 2$\rightarrow$ERS$\rightarrow$S/C 1$\rightarrow$ERS. The time difference of the two paths varies only up to 0.34 sec. A laser with linewidth of 1 Hz would keep the interference coherent. The phase sensitivity of this topology to gravitational wave is calculated as follows. In the plane containing S/C 1, S/C 2 and ERS (for simplicity, we use L1 as ERS), draw a reference line L1$\rightarrow$A as in Fig. 3. Let the angle between ERS$\rightarrow$S/C 1 and ERS$\rightarrow$A be $\theta_1$, and that between ERS$\rightarrow$S/C 2 and ERS$\rightarrow$A be $\theta_2$. First consider a monochromatic gravitational wave.
wave with + polarization and strain amplitude $h_+$ coming in perpendicular to the orbital plane. The change of length scale in the L1→A direction at time $t$ is $h_+ \sin(2\pi f_G t)$. For laser light travelling through Path I and Path II to return at $t$ simultaneously, the optical path difference is

$$\Delta l = 4h_+(c/f_G)(\cos2\theta_1+\cos2\theta_2)[\cos2\pi f_G(\tau_1 - \tau_2)+\cos2\pi f_G(\tau_1 + \tau_2)]\cos2\pi f(t + \phi_0)$$

with $\tau_1 \equiv 2l_1/c$, and $\tau_2 \equiv 2l_2/c$. Hence phase difference of the laser light is

$$\Delta \phi = 4h_+(f/f_G)(\cos2\theta_1+\cos2\theta_2)[\cos2\pi f_G(\tau_1 - \tau_2)+\cos2\pi f_G(\tau_1 + \tau_2)]\cos2\pi f(t + \phi_0)$$

This is the laser phase sensitivity for our gravitational detection topology. For $\times$ polarization, simply rotate by 45°. For non-perpendicular incidence, insert a cosine factor.

The theoretical sensitivity limit (shot noise limit) in the strain for gravitational-wave detection is inversely proportional to $P^{1/2}l$ with $P$ the received power and $l$ the distance. Since $P$ is inversely proportional to $l^2$ and $P^{1/2}l$ is constant, this sensitivity limit is independent of the distance. For 1-2 W emitting power, the limit is $10^{-21}/\sqrt{\text{Hz}}$. As noted in the LISA study, making the arms longer shifts the whole time-integrated sensitivity curve to lower frequencies while leaving the bottom of the curve at the same level.24 With the same laser power, the ASTROD sensitivity would be shifted to lower frequency by a factor up to 60 (30 in average) if the technological requirements in the preceding section are met. This sensitivity curve is shown in Fig. 4 using the basic plot of LISA.

For pulsed ranging, 1 ps accuracy in the detector transit time variation is achievable.25 With $10^5$-$10^6$ Hz repetition rate and a distance of 2 AU, the sensitivity can reach $10^{-18}/\sqrt{\text{Hz}}$ when an ultrastable clock of comparable stability is available in the future. If one leg is on earth, due to influence of atmospheric turbulence, the sensitivity will be down-graded. In the T2L2 project study of satellite time transfer by laser light26,27, precision and accuracy are aimed at the order of 10 ps. For gravitational-wave detection, this implies a sensitivity of $10^{-16}$-$10^{-17}/\sqrt{\text{Hz}}$. 

Figure 3: Gravitational detection topology Path I: ERS → S/C 1 → ERS → S/C 2 → ERS. Path II: ERS → S/C 2 → ERS → S/C 2 → ERS.
6. Detectability of Primordial Gravitational Waves

From Fig. 4, we can see that the main gravitational wave sources for LISA and ASTROD are large black hole ($10^5$-$10^8 \, M_\odot$) coalescence/formation, nearest compact binaries, binaries at galactic center and compact white-dwarf binaries. This will enable us to study astrophysics (population, evolution and characteristics) of binaries and physics/astrophysics of black holes. As to the detection of the primordial gravitational waves, a comprehensive analysis and summary is presented by Battye and Shellard. Fig. 5 adapts their diagram to incorporate ASTROD sensitivity. ASTROD sensitivity curve is shifted to the left in the diagram to about 30 times compared to LISA sensitivity. The sensitivity to primordial gravitational energy in terms of $\Omega_g$ is about 2 orders of magnitude better. With this sensitivity, the global string scenario could be touched upon. With a dedicated Super-ASTROD mission in 5 AU orbits as will be discussed in the next section, the separation of 2 spacecraft is typically 10 AU and the sensitivity will reach the ”slow-roll inflation-upper bound”. However, this will depends on whether the gravitational-wave signals from short-period white-dwarf binaries can be separated out as discussed in Section 2.

7. Mini-ASTROD and Super-ASTROD

ASTROD is a relativity mission concept encompassing multipurposes. With its technological requirements achieved, it would give a gravitational-wave sensitivity similar to LISA, but shifted to lower frequencies. ASTROD would complement LISA in probing the early Universe and study strong-field black hole physics.
A down-scaled version, i.e., Mini-ASTROD with one spacecraft carrying a payload of a telescope, two lasers, and a clock will test the optical scheme and yet give important scientific results. These scientific results include a better measurement of $\gamma$ to 1 ppm, a better sensitivity (1-2 orders better) in using optical Doppler tracking method for detecting gravitational wave, and a potential of measuring the solar angular momentum via Lense-Thirring effect. It is important to do things in appropriate steps and we are now studying this precursor mission concept in more details.

With the advance of laser technology and the development of space interferometry, one can envisage 15 W (or more) compact laser power and 2-3 fold increase in pointing ability. With these developments, one can increase the distance from 2 AU for ASTROD to 10 AU (2×5 AU) and the spacecraft would be in orbits similar to Jupiter’s. Four spacecraft would be ideal for a dedicated gravitational-wave mission (super-ASTROD).

Figure 5: Battye-Shellard diagram with the sensitivity of ASTROD and Super-ASTROD added. This diagram shows the design sensitivity of ground and space-based gravitational-wave detectors to the potential cosmological sources of a stochastic gravitational radiation background.
8. Outlook

In this TAMA Workshop, we see that the building construction work for the TAMA300 interferometer is completed and the three buildings plus tunnels are open for visit. Vacuum pipes and vibration isolation systems are being installed. 10 W Nd:YAG laser is completed. Optics, control and data acquisition schemes are well worked out. If everything goes well as it has been, the design sensitivity (rms strain of $3 \times 10^{-21}$) should be reached by the target date of 1999. TAMA300, together with GEO600 (target date: 2000), VIRGO (target date: 2001), LIGO (target date: 2001 and 2002) will usher us into an era of large-interferometer gravitational wave detection. This will give us a good sensitivity for the high-frequency gravitational waves and allow us to probe supernova collapse and compact binary coalescence.

For low-frequency gravitational waves, LISA and ASTROD are two mission concepts under active studies. This will give us an excellent sensitivity to study fundamental physics in strong gravitational field and cosmology together with various astrophysical phenomena. LISA is considered as the third Cornerstone Mission in the ESA Horizons 2000 Plus Programme. ASTROD is looking for various flight opportunities. Mini-ASTROD could be considered as a pre-ASTROD mission with a good scientific return.

After the successes of LISA and ASTROD and the clear-up of the confusion limit, Super-ASTROD with orbit similar to Jupiter’s can be considered for the detection of the primordial gravitational waves.

Acknowledgements

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References

10. R. L. Davis, H. M. Hodges, G. F. Smoot, P. J. Steinhardt, and M. S. Turner, 
11. ESA D.Sci/RMB/SV/val/218, Planck Surveyor mission (M3), call for Letters of 
   Intent, 7 January, 1997.
12. Microwave Anisotropy Probe (MAP), a NASA mission to be launched in the 
   G. Spalding, and X. Xu, *Astrodynamical Space Test of Relativity Using Optical 
   Devices*, presented to 31st *COSPAR Scientific Assembly* (Birmingham, United 
   Kingdom, 14-21 July 1996).
   Possible Determination of Asteroid Masses through the ASTROD Space Mission," 
   presented to ACM96 COSPAR Colloquium 10: Asteroids, Comets and 
   Meteors (Versailles, France, July 8-12, 1996).
15. W.-T. Ni, "ASTROD Mission Concept and Measurement of the Temporal Vari- 
   ation of the Gravitation Constant", *Proceedings of the Pacific Conference on 
   Gravitation and Cosmology*, February 1-6, 1996, Seoul, Korea, in press (World 
   Scientific, Singapore, 1997).
   Order Relativistic Effects, Solar Angular Momentum and Low-Frequency Grav- 
   itational Waves", in *Proceedings of the Seventh Marcel Grossmann Meeting on 
   Mission Concept", paper in preparation; and references therein.
   tivity in Gravitational-Wave Detection”, presented to *First International LISA 
   Symposium*, 9-12 July, 1996, Rutherford Appleton Laboratory, Oxfordshire, UK.
   Laser using Cs2 Absorption”, p. 122 in *Conference Digest of Conference on Pre- 
   cision Electromagnetic Measurements*, 17-20 June, 1996 (Braunschweig, PTB, 
   1996).
24. P. Bender, I. Ciufolini, K. Danzmann, W. Folkner, J. Hough, D. Robertson, 
   a. Rüdiger, M. Sandford, R. Schilling, B. Schutz, R. Stebbins, T. Sumner, P. 
   Touboul, S. Vitale, H. Ward and W. Winkler, *LISA Pre-Phase A Report*, MPQ 
   208 (February, 1996).
25. E. Samain, "Le laser Lune millimétique et nouvelles méthodes de datation 
EFFICIENT GW TEMPLATE COMPUTATION FOR STEADY STATE BINARY STAR SOURCES WITH ANY ORBITAL ECCENTRICITY

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ABSTRACT

Closed-form spectral truncation error evaluation criteria and a fast computational tool for building waveform templates of gravitational radiation emitted from steady-state binaries with general elliptical orbits are introduced and applied to the simplest Peters-Mathews model.

1. Introduction

Gravitational radiation from steady state binary stars could be observed from planned spaceborne VLBI gravitational wave (henceforth GW) observatories, e.g., LISA\(^1\).

A substantial fraction of binary stars are expected\(^2,3\) to have non-negligible orbital eccentricities. A few representative examples from different star populations are listed in Table I below:

<table>
<thead>
<tr>
<th>Name</th>
<th>Period $[d]$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V1143 CYG</td>
<td>7.639</td>
<td>0.54</td>
</tr>
<tr>
<td>SX Cas</td>
<td>36.76</td>
<td>0.5</td>
</tr>
<tr>
<td>NY Cep</td>
<td>15.37</td>
<td>0.49</td>
</tr>
<tr>
<td>PSR1913+16</td>
<td>0.3229</td>
<td>0.617</td>
</tr>
<tr>
<td>PSR2303+46</td>
<td>12.3395</td>
<td>0.659</td>
</tr>
<tr>
<td>PSR2127+11</td>
<td>0.3353</td>
<td>0.681</td>
</tr>
</tbody>
</table>

- Table I -

Binary stars with elliptical orbits emit several GW harmonics of the orbital frequency $\Omega_{orb}$ with comparable intensities, and the maximum GW luminosity occurs at $\omega = N_{\text{max}}(e)\Omega_{orb}$. The function $N_{\text{max}}(e)$ and the ratio $G_{\text{max}}(e)$ between the total (sum over both polarizations) luminosity at $\omega = N_{\text{max}}\Omega_{orb}$ and the total luminosity of a circular–orbit binary (emitting only at $2\Omega_{orb}$) with the same companion.
masses and orbital period are shown in fig. 1 for the well known Peters-Mathews (henceforth PM) model\textsuperscript{4,5}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{- The functions $N_{\text{max}}(e)$, $G_{\text{max}}(e)$.}
\end{figure}

It is understood that using incorrect (e.g., circular orbit) templates implies a potentially large \textit{under-estimate} of the $L^2$ scalar product:

$$\rho = (h, \tilde{h}), \quad (1)$$

where $h$ and $\tilde{h}$ denote the sought signal and the template, respectively. This entails a \textit{sensible} degradation of the matched-filter detector performance, described by a template-mismatch factor also referred to as signal-to-noise-ratio degradation (henceforth $\text{SNRD}$):

$$\text{SNRD} = \frac{(\tilde{h}, h)}{\| \tilde{h} \| \| h \|}, \quad (2)$$

where $\| \cdot \|$ is the $L^2$ norm \textsuperscript{a}.

In this paper we set-up an efficient computational framework for computing templates of GWs from steady-state binaries with general elliptical orbits, based \textit{i)} on closed-form evaluation of the truncation error of the spectral (Fourier) metric representations and \textit{ii)} a fast approximation tool for the special (Bessel) functions involved. For simplicity, in this communication reference is made to the PM model \textsuperscript{b}.

The proposed approach is however quite general, and is readily adapted to higher order, post-newtonian models, as e.g. the one expounded in\textsuperscript{15}.

\textsuperscript{a}The possible relevance of an albeit \textit{residual} orbital eccentricity in matched filter detection of GW chirps from \textit{coalescing} binaries has been discussed in\textsuperscript{6} and\textsuperscript{7}.

\textsuperscript{b}For the PM model, the (known) keplerian integral of motion relating time to the orbital polar anomaly $\theta$ can be numerically inverted to compute GW templates as closed-form functions of $\theta$, for any orbital eccentricity\textsuperscript{11}.
2.1 Checking the Applicability of Peters Mathews Model

For most steady-state binaries (far before coalescence) the PM model turns out to be adequate. As a matter of fact, the dimensionless parameters:

\[ \xi_1 := \frac{\text{source gravitational radius}}{\text{aphastral separation}} = 2\chi^{-2/3}(1-e)^{-1}, \]

\[ \xi_2 := \frac{\text{aphastral velocity}}{\text{velocity of light}} = \chi^{-1/3}(1+e)^{1/2}, \]

\[ \xi_3 := \sup\left\{ \left|\frac{dT}{dt}\right|, \left|e^{-1}\frac{de}{dt}\right| T\right\} = \frac{152\pi}{15}(1-\Delta^2)\chi^{-5/3} \cdot \left(1 + \frac{121}{304}e^2\right)(1-e^2)^{-5/2}, \]

gauging, respectively, the weak field, slow motion and adiabatic assumptions behind the model\(^9\), are all much less than one. For instance, for PSR1534 + 12 one has \(\xi_1 = 3.48 \cdot 10^{-6}, \xi_2 = 1.32 \cdot 10^{-3}\) and \(\xi_3 = 7.65 \cdot 10^{-14}\).

2.2 The Peters-Mathews Far-Field Metric

The relevant PM far-field metric components\(^10\) can be computed from:

\[ h_{x\pm y} = \sum_{n=1}^{\infty} h_{x\pm y}^{(n)} \cos (n\Omega_{\text{orb}} t), \quad h_{xy} = \sum_{n=1}^{\infty} h_{xy}^{(n)} \sin (n\Omega_{\text{orb}} t), \]

where \(h_{x\pm y}\) is a shorthand for \(h_{xx} \pm h_{yy}\),

\[ h_{x+y}^{(n)} = -4h_0J_n(ne), \quad h_{x-y}^{(n)} = 2h_0n \{J_{n-2}(ne) - J_{n+2}(ne) + 2e [J_{n-1}(ne) - J_{n+1}(ne)] + (2/n)J_n(ne)\}, \]

\[ h_{xy}^{(n)} = h_0n(1-e^2)^{1/2} [J_{n-2}(ne) + J_{n+2}(ne) - 2J_n(ne)], \]

\[ h_0 = \frac{cT_0}{4\pi r} \chi_0^{5/3}, \]

and \(J_n(x)\) is the Bessel function of the first kind. For circular orbits one has simply:

\[ h_{x-y} = 2h_{xy}^{(n)} = 4h_0T^{-2/3}\delta_{n2}, \quad h_{x+y}^{(n)} = 0, \]

where \(\delta_{pq}\) is the Kronecker symbol.

\(^c\)The TT far-fields are computed from eqs (6) to (9) as follows

\[ h_x = \cos \vartheta \left[2h_{xy}\cos 2\varphi - (h_{x-y})\sin 2\varphi\right], \]

\[ h_\vartheta = \frac{1}{\sqrt{2}} \left\{ \frac{3 + \cos 2\vartheta}{4} [2h_{xy}\sin 2\varphi + (h_{x-y})\cos 2\varphi] - \frac{1 - \cos 2\vartheta}{4} (h_{x+y}) \right\}, \]

where \((\vartheta, \varphi)\) specify the direction of the observer (the orbit lies in the \(\vartheta = \pi/2\) plane).
3. Generalized Carlini-Meissel Formula

The main issue for efficient template construction is related to fast and accurate computation of terms like:

\[ J_{n\pm k}(ne), \]  

(11)
e.g., in equations (7) and (8). In general, whenever the argument is close to the order, Bessel functions are *inefficiently* computed both by series expansion\(^{12}\), and by (re-normalized, downward) recurrence\(^ {13}\). We found that a suitable generalization of the well-known Carlini-Meissel expansion\(^ {14}\) yields the following pretty convenient computational recipe\(^ {10}\):

\[ J_{n\pm k}(ne) \sim J_{n}^{(CM)}(ne) \Psi_{\pm k}(n, e), \]  

(12)

where:

\[
J_{n}^{(CM)}(ne) = \frac{(d/2)^n}{n!} \left( \frac{1 + \sqrt{1 - e^2}}{2} \right)^{-n} (1 - e^2)^{-1/4},
\]

\[
\cdot \exp \left\{ n \left[ \sqrt{1 - e^2} - 1 \right] + n^{-1} \left[ \frac{-3e^2 - 2}{24(1 - e^2)^{3/2}} + \frac{1}{12} \right] \right\},
\]

(13)

and:

\[
\Psi_{\pm k}(n, e) = \frac{n!}{(n \pm k)!} \left( \frac{ne}{1 + \sqrt{1 - e^2}} \right)^{\pm k} 
\cdot \exp \left\{ \frac{1}{n} \left[ \mp k \frac{e^2}{2} - \frac{k^2}{2} \left( 1 - \frac{1}{\sqrt{1 - e^2}} \right) \right] \right\}.
\]

(14)

4. Spectral Truncation

Spectral truncation of (6) can be conveniently gauged in terms of total harmonic distortion (henceforth *THD*)\(^d\):

\[
THD = \frac{\|h - \tilde{h}\|}{\|h\|},
\]

(15)

\(^d\)Using the triangular inequality:

\[
1 - \frac{\|h - \tilde{h}\|}{\|h\|} \leq \frac{\|\tilde{h}\|}{\|h\|} \leq 1 + \frac{\|h - \tilde{h}\|}{\|h\|} \Rightarrow \exists \alpha \in (-1, 1) : \frac{\|\tilde{h}\|}{\|h\|} = 1 + \alpha \, THD,
\]

and the obvious relationship:

\[
SNRD = \frac{(h, \tilde{h})}{\|h\||\tilde{h}\|} = \frac{1}{2} \left( 1 - THD^2 \right) \frac{\|h\|}{\|h\|} + \frac{\|\tilde{h}\|}{\|h\|}.
\]

*SNRD* can be related to *THD* as follows:

\[
SNRD \sim 1 - \frac{THD^2}{2} + \mathcal{O}(THD^3).
\]
which reads explicitly:

\[
THD = \left( \frac{\sum_{n=1}^{\infty} |h^{(n)}|^2 + \sum_{n=1}^{NT} \left| \tilde{h}^{(n)} \right|^2 - 2 \sum_{n=1}^{NT} h^{(n)} \tilde{h}(n)}{\sum_{n=1}^{\infty} |h^{(n)}|^2} \right)^{1/2}.
\]

Here \(NT\) is the truncation order of (6), and \(h^{(n)}, \tilde{h}^{(n)}\) are the Fourier coefficients of the sought signal and the template. All infinite sums in (16) belong to the class of Kapteyn series \(^{14}\) and can be evaluated in closed form after some (lengthy but) straightforward calculations. Using e.g. PM eq.s (6)-(9), one gets:

\[
||h_{x+y}||^2 = \sum_{n=1}^{\infty} |h_{x+y}^{(n)}|^2 = 8 \left( (1 - e^2)^{-1/2} - 1 \right),
\]

\[
||h_{x-y}||^2 = \sum_{n=1}^{\infty} |h_{x-y}^{(n)}|^2 = e^{-4} \left\{ 4(1 - e^2)^{-1/2}(8 - 12e^2 + 9e^4) - 8(e^2 - 2)^2 \right\},
\]

\[
||h_{xy}||^2 = \sum_{n=1}^{\infty} |h_{xy}^{(n)}|^2 = e^{-2(1 - e^2)^{-1/2}} \left\{ 12 + e^2 + 8e^{-2} \left[ (1 - e^2)^{3/2} - 1 \right] \right\}.
\]

From eq.s (16), (17) one can verify that if \(e \geq 0.3\) the circular orbit assumption yields a substantial underestimate of the scalar product (\(\tilde{h}, h\)), viz. exceedingly large \(THD\) and \(SNRD\) values\(^{10}\).

5. Results

Truncating the spectral expansions (6) according to Table II below:

<table>
<thead>
<tr>
<th>(e_0)</th>
<th>.1</th>
<th>.2</th>
<th>.3</th>
<th>.4</th>
<th>.5</th>
<th>.6</th>
<th>.7</th>
<th>.8</th>
<th>.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>(NT)</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>11</td>
<td>15</td>
<td>22</td>
<td>36</td>
<td>68</td>
<td>206</td>
</tr>
</tbody>
</table>

- Table II -

and using the generalized Carlini-Meissel approximations (13) and (14) gives \(THD < 10^{-2}\) (worst value among \(THD_{x+y}, THD_{xy}\)) \(e\). As an illustration, the \(h_x\) and \(h_+\) waveforms produced by \(PSR1913 + 16\) at \(\dot{\vartheta} = \varphi = \pi/4\) are displayed in fig.2 below.

6. Conclusions

Closed-form spectral truncation error evaluation criteria, based on Kapteyn series, and use of a generalized Carlini-Meissel expansion allow to setup an efficient framework for computing gravitational radiation waveform templates for steady state binary sources with general elliptical orbits, which can be extended to higher order PN models with relative ease.

\(e\)Upper \(THD\) bounds for \(h_+\) and \(h_x\) are readily shown to be of the same order as those pertinent to \(h_{x \pm y}\) and \(h_{xy}\)\(^{10}\).
Figure 2: - PSR1913 + 16 waveforms; \( h_0 = 2.057 \times 10^{-23} \)

References

1. K. Danzmann, this Conference.
The LIGO Phase Noise Interferometer, a suspended-mirror power-recycled Michelson interferometer located at M.I.T., is used to study technical limitations in optical phase sensing and to demonstrate techniques for achieving the photon shot noise limit in LIGO interferometers. Operating at a wavelength of $\lambda = 514.5$ nm, this interferometer has achieved a recycling gain of 450 and a carrier circulating power of 60 W incident on the beamsplitter. Under these conditions the interferometer’s optical phase sensitivity is limited by photon shot noise at $(2.9 \pm 0.2) \times 10^{-10} \text{ rad Hz}^{-1/2}$ at frequencies from 1.5 kHz to over 10 kHz. At lower frequencies the phase noise power spectrum is dominated by excess laser frequency noise and by thermally-excited mechanical resonances. The interferometer’s planned conversion for operation at $\lambda = 1064$ nm wavelength is also discussed.

1. Introduction

LIGO interferometers\(^1,2\) will rely on precise determination of the relative phase shift between optical fields stored in their orthogonal arm cavities. Specifications for the initial LIGO detector limit the equivalent spectral density of optical phase measurement errors to $10^{-10} \text{ rad Hz}^{-1/2}$ or less at measurement frequencies between 100 Hz and 10 kHz.\(^3\) This level corresponds to the limit due to Poisson counting statistics (“photon shot noise”) in the detected light at the expected circulating optical power. Division of an interference fringe at this level of phase precision has not been demonstrated to our knowledge.

The LIGO Phase Noise Interferometer is designed to identify and study possible technical limitations to the sensing of optical phase and to demonstrate the required phase sensitivity on a laboratory scale. As a result this machine is configured as a power-recycled, single-bounce Michelson interferometer without arm cavities; this minimizes the differential phase contamination attributable to external and ther-
mally driven forces on the mirrors. Despite this minimized sensitivity to motion, the target phase sensitivity still demands residual mirror motions below $10^{-17} \text{m Hz}^{-1/2}$ at fluctuation frequencies above 100 Hz. As a result, the interferometric components must be seismically isolated and suspended in vacuum like those of the LIGO interferometers.\textsuperscript{5}

2. Configuration of the Interferometer

The physical arrangement and operating schematic are depicted in Figure 1. The Michelson interferometer mirrors (TM) and beamsplitter (BS) are all located on a common table which is seismically isolated by multi-stage mechanical filters\textsuperscript{7} inside a 2.1-meter diameter vacuum chamber. The passive isolation system is itself supported by three active seismic isolation mounts\textsuperscript{a} placed below the chamber; compliant bellows are used to decouple the mount supports from the chamber walls. When activated, the active isolators reduce the seismic noise transmitted to the interferometer by approximately a factor of ten between 3 Hz and 30 Hz (Figure 2). Since much of the seismic activity originates from traffic and other human activity in the MIT laboratory’s city environment, the distribution of seismic amplitude is highly nongaussian with large impulsive excursions. A particular benefit is thus seen in the drastically reduced occurrence of impulsive overloads to the interferometer controls and readout (Figure 3).

The recycling mirror (RM) is concave with a 10 meter radius, and is mounted on a similar isolation system in a second chamber at a distance of 6 meters. The end mirrors, beamsplitter and recycling mirror are all suspended from their isolated platforms by single-loop suspensions with pendulation frequencies of approximately one Hz and rotational eigenfrequencies of approximately 0.5 Hz about vertical (“yaw”) and horizontal (“pitch”) axes. The angular orientation and position of each mirror is sensed and feedback-damped by integrated optical shadow sensors and electromagnetic force transducers placed at its periphery. These transducers also provide a means for externally controlling the axial position and angles of each optic, and for calibrating the interferometer.

The interferometer is illuminated by a prestabilized single-mode argon ion laser operating at 514.5 nanometer wavelength. The commercially-supplied laser is modified to isolate its cavity mirrors from coolant-induced vibration. It is further stabilized by feedback control using the radiofrequency phase-modulated reflection technique\textsuperscript{8} to a seismically isolated passive reference cavity suspended in a vacuum chamber. To correct the laser emission wavelength, one laser mirror (the output coupler) is mounted on a wide-range piezoelectric stack transducer to provide coarse tuning control at frequencies up to 1 kHz. The laser’s high reflector is attached to a smaller high-speed piezoelectric transducer backed by a damped acoustic delay line\textsuperscript{9} to effect fast correction. At still higher fluctuation frequencies, the optical phase of the emission is directly corrected by transmission through an electrooptic modulator.\textsuperscript{10} Electronic tailoring of the signals driving each of these transducers

\textsuperscript{a}Barry Controls, Inc. STACIS\textsuperscript{TM}
provides an integrated feedback characteristic, having a unity-gain bandwidth exceeding 1 MHz and dynamic reserve (at low frequencies) comparable to the laser’s free spectral range.

The laser output is passed through an acoustooptic modulator (AO), which performs in a feedback loop to stabilize the power detected on a reference photodetector. The beam is matched to a single-mode polarization-preserving optical fiber to suppress beam shape and direction fluctuations. Phase modulation sidebands are applied by an Pockels cell (PC) at frequency $f_m \approx 25$ MHz with a modulation depth $\Gamma_m \approx 0.5$. The beam is then expanded and modematched to the interferometer recycling cavity.

2.1. Control of Laser Frequency and Common-Mode Length

The light reflected from the recycling mirror is detected by tuned RF photodetector D1, and the photocurrent signal is demodulated at frequency $f_m$ and lowpass filtered. The resulting baseband signal is proportional to the difference between the common-mode length of the recycling cavity $\bar{l} = (l_1 + l_2)/2$ and the nearest
integral multiple of half the laser wavelength, where $l_1$ and $l_2$ are the optical paths between the recycling mirror and the respective arm mirrors. This signal is applied through two actuation paths; at low frequencies (up to 150 Hz) the signal is fed to the electromagnetic actuators to adjust the position of the recycling mirror. At higher frequencies, the signal is applied to a high-speed summing node in the laser prestabilization loop. The combined loop achieves a unity-gain bandwidth of approximately 20 kHz, in principle affording adequate gain to suppress residual laser frequency fluctuations to an insignificant level. However, since a portion of the feedback is effected by altering the interferometer length rather than correcting the laser frequency, the true suppression of frequency noise is only given by the ratio between the gains of these two paths. As we will show, this resulted in a limitation to phase sensing performance at low frequencies.

2.2. Sensing and Control of Differential Length and Alignment

The modulation frequency $f_m \approx 25$ MHz is chosen so that the optical carrier and its upper and lower sidebands resonate on adjacent longitudinal modes of the
compound cavity formed by the recycling mirror, the beamsplitter and the two arm mirrors. The two arm lengths are chosen to be macroscopically different by $\Delta l = 20 \text{ cm}$; thus, when adjusted to interferometrically cancel the carrier wave at the antisymmetric output (a “dark fringe”), the upper and lower sidebands are incompletely cancelled. This yields a net sideband field at the output which can beat against any residual carrier. Demodulation of the antisymmetric port photodetector D2’s photocurrent against the modulating waveform thus gives a measure proportional to the net phase difference between the two arms.$^{12-14}$

This differential phase error signal is filtered and processed into corrective force signals which are applied to the arm mirrors through their electromagnetic transducers. The control loop provides a unity-gain bandwidth of approximately 300 Hz.

A portion of the antisymmetric output beam is also analyzed by a tuned radiofrequency quadrant photodetector system (a “Wavefront Sensor”) to determine the transverse spatial gradient of the phase error.$^{15-20}$ Each spatial quadrant’s

Figure 3: Histogram of impulsive event displacement amplitudes over a 24 hour observation period, measured at the ground (solid bars) and at the bottom table of the passive seismic isolation stack (dashed bars with asterisks). The reduction in rate of impulses exceeding 20 microns in amplitude, from several hundred to only a few per day, greatly simplifies operation of the interferometer.
photocurrent is separately amplified and demodulated; the four phase signals are combined algebraically to provide differential pitch and yaw errors, which are fed back to one of the arm mirrors as corrective torques. The effect of this system on the interferometer’s circulating power is depicted in Figure 4, which shows a sample of the intracavity circulating power before and after engaging the wavefront sensor control loop.

Figure 4: Effect of engaging wavefront sensing control on differential alignment of the Michelson interferometer. A sample of the circulating optical power (residual transmission through one arm mirror) is plotted as a function of time. Initially the mirrors are locally damped by their suspension sensors with some static error and some fluctuation due to environmental noise. After engagement the waste power leaving the antisymmetric port is minimized, increasing the recycling gain. The circulating power approaches and stabilizes near the optimum.

2.3. Readout and Calibration

The demodulated antisymmetric-port photocurrent signal is monitored to measure the differential length deviation as a function of time. To calibrate the readout, a sinusoidal test force of known amplitude is applied to one of the suspended mirrors through its suspension controller. The differential-mode control loop gain is measured independently and applied as a normalization. The coefficient linking applied
calibration voltage to mirror displacement is established by comparison with the laser wavelength; with the control loops deactivated and the recycling mirror deliberately misaligned, one arm mirror is swept through a full order while monitoring the antisymmetric port output signal. The fractional accuracy in transfer of this calibration to the recorded output signal is estimated at approximately ±5%.

3. Experimental Results and Discussion

The noise power spectral density of the interferometer as of January 10, 1997 is shown in Figure 5. Here the machine was operated with an optical power of approximately 190 mW incident on the recycling mirror. With the wavefront sensing alignment control active, a recycling gain \( R \approx 450 \) was achieved with contrast defect \( 1 - C \approx 3 \times 10^{-4} \). The phase noise spectrum has three distinct spectral regions, each with a different limiting noise mechanism.

At frequencies from 1.5 kHz to above 10 kHz, the phase sensitivity of \( \tilde{\phi}(f) \approx (2.9 \pm 0.2) \times 10^{-10} \text{rad Hz}^{-1/2} \) is approximately equal to the calculated photon shot noise in the antisymmetric output photocurrent (dotted line in Figure 5). The observed spectrum also closely matches the measured response to incoherent light from an incandescent light bulb at comparable photocurrent (dot-dashed line), when this response is properly compensated for stationarity of the photocurrent.\(^{22}\) Between 1 kHz and 1.5 kHz is a series of resonant features, each of which is associated with the flexural resonance of one of the five force actuator magnet assemblies bonded to each mirror. Excitation amplitudes of these features are constant and consistent with the predicted effect of Brownian thermal noise, given the measured quality factors and calculated reaction inertias of the flexures. The design of these magnet attachments has since been modified to increase the lowest resonant frequency.

\( \Phi \)From 100 Hz to 1 kHz, statistical correlation between the differential phase signal and the common-mode error signal indicates that residual laser frequency noise is the dominant contributor. This is confirmed by the observation that the noise in this regime varies inversely with the ratio \( G(f)/R(f) \) (Figure 1), the effective frequency noise suppression gain in the common-mode loop. This effective suppression is less than the open-loop gain, since feedback to the recycling mirror cooperates in suppressing the loop’s error signal but does so only by disturbing the interferometer length to accommodate frequency excursions. Our intrinsically asymmetric Michelson interferometer directly detects fluctuations in absolute frequency, contributing a phase shift signal \( \delta \phi_\nu \approx 4\pi \delta \nu \Delta l/c \) for frequency error \( \delta \nu \) (in our case, approximately \( 8 \times 10^{-9} \) radian per Hz of frequency). While the feedback gain to the recycling mirror \( R(f) \) was strongly attenuated at frequencies above 150 Hz, the finite filter attenuation permitted by control stability considerations resulted in significant contamination up to 1 kHz, and could not be further reduced in this experiment due to dynamic range limitation in the additive offset actuator path. This shortcoming will be addressed in the future, by incorporating a third common-mode feedback path having wider dynamic range which directly corrects the prestabilized laser reference frequency. Use of a more stable laser, like the solid-state Nd:YAG
laser to be installed in the next phase, should also prove helpful.

Two further observations are worth noting. First, optical feedback between the interferometer and the laser system was seen to limit the sensitivity at frequencies below 100 Hz, and often contaminated the measurement at higher frequencies during periods of high seismic activity. The upconversion of seismic noise due to multi-wavelength relative motions of scattering surfaces is well understood, and will be addressed by placing the prestabilized laser table itself on a third set of STACIS™ active isolators. Secondly, the high contrast and recycling gain described here were difficult to achieve repeatably due to thermal lensing in the beamsplitter. This effect caused a mismatch between the transmitted and reflected beam parameters to arise within about 1 second of acquiring resonance, thus degrading the contrast defect (by as much as a factor of three). The effect appeared to worsen over time, but performance was partially restored by translating the beam to a new location on the beamsplitter. As a result we suspect locally enhanced optical absorption may have been induced by photochemical or thermal action on surface contaminants.

Figure 5: Equivalent phase noise spectral density, with calculated shot noise (dotted) for the measured parameters at the time of the measurement. Pure shot noise generated by equivalent photocurrent from an incandescent light bulb, with stationarity corrections applied, is also shown (dot-dashed line).
4. Conclusions and Future Work

With the Phase Noise Interferometer we have demonstrated what is to our knowledge the highest optical phase measuring sensitivity ever achieved, a sensitivity comparable to that required for LIGO and other large scale interferometric gravitational-wave detectors. However, we suspect many technical noise mechanisms and practical difficulties encountered are peculiar to the laser and wavelength used. Since LIGO and others will use solid-state lasers at $\lambda = 1.06 \mu m$ it is important to repeat (and further improve) the measurement at the design wavelength.

As a result the interferometer has been shut down and will be rebuilt for operation using a solid-state infrared laser. Initially we will use a Lightwave Electronics, Inc. model 126, which can deliver over 500 mW to the recycling mirror. Interferometer internal optics and the laser prestabilization system have been replaced, and the controls are being reworked to accommodate the new laser and to incorporate the feedback topology enhancements discussed above. As a result we are optimistic that the Phase Noise Interferometer will achieve still better performance working in the infrared.

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References

1. The construction, operation and supporting research and development of a laser interferometer gravitational-wave observatory. Proposal to the National Science Foundation, California Institute of Technology (1989).

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CURRENT STATUS OF 20M PROTOTYPE

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ABSTRACT

We started to operate a recombined 20 m Fabry-Perot prototype with a mode cleaner whose mirrors were independently suspended. The light source was an LD pumped Nd:YAG laser with a wavelength of 1.064 μm and an output power of 500 mW, and its frequency was stabilized by using the mode cleaner cavity as a reference. Consequently, the contrast of the prototype improved up to 99.2% owing to the mode cleaner. The prototype could be operated with pre-modulation method using the 40 MHz phase modulation before the mode cleaner which was designed to have about 40 MHz Free Spectral Range by setting its length to 3.75 m to permit this modulation sidebands to pass it into the interferometer. The finesse of the arm cavities were 516 and the common mode rejection ratio was about 40 dB. The displacement noise was $4 \times 10^{-17} \text{m/} \sqrt{\text{Hz}}$ at 1kHz and it was examined to be limited mainly by shot noise.

1. Introduction

The recombined Fabry-Perot prototype with 20 m arm-length, at National Astronomical Observatory of Japan, has been operated since 1996. The characteristics of this prototype are the use of an Nd:YAG laser as a light source and the operation with the mode cleaner using independently suspended mirrors. The aim of this prototype is to test all the optical design which is necessary for the full-scale gravitational wave detectors.

Recently a major advance was made; the operation with a mode cleaner which can transmit the modulation sidebands. It was very stable and the method to transmit the sidebands is proved to be promising. And by replacing the mirrors, the finesse of two arm cavities were equalized and it results the improvement of CMRR from 20 dB to 40 dB.
Figure 1: Schematic diagram of the recombined Fabry-Perot prototype with 20 m arm-length. The transmitted light from the mode cleaner is directed to the main interferometer. A small portion of reflected light from each arm cavity is extracted (by anti-reflection surfaces of the beam splitter) to keep the cavities on resonance. The servo gain in the observation band is designed to be below unity in order to avoid any compensation of the effects of gravitational waves by the servo loop. The remainder of the reflected beams are optically recombined at the beam splitter.

2. Configuration of the Prototype

The configuration of our prototype with 20 m arm-length is schematically shown in Fig. 1. This system is separated into two parts. One is a light source with a mode cleaner and the other is a recombined interferometer.

The light source is an LD pumped Nd:YAG laser with a wavelength of 1.064 μm and an output power of 511 mW (LIGHTWAVE Electronics; Model 122-1064-500-F), and its frequency is stabilized by Pound-Drever method using the mode cleaner cavity as a reference. The modulation frequency is 18MHz.\(^2\)

The mode cleaner (MC) is designed 3.75 m in length to permit the passing of the 40 MHz frequency modulation sidebands for the operation of the recombined interferometer. In other words, a Free Spectral Range (FSR) of MC cavity is set to 40 MHz. It is necessary to keep the sideband frequency with accuracy 2 Hz around the FSR because the frequency difference results a excess intensity noise caused by FM-AM conversion of the cavity. So we must use a stable oscillator such a synthesizer. The required frequency stability is $10^{-8}$.

The mirrors of MC are all 20 mm-in-diameter superpolished substrates with
coatings manufactured by Japan Aviation Electronics (JAE). These mirrors are attached to a silica bar by optical contact and suspended independently by magnet-damped double pendulums. The finesse of MC cavity is about 2100. The power throughput of MC is 91%, which indicates each mirror loss is below 100 ppm. Due to losses in the input optics, the incident laser power to MC is diminished from 511 mW at the laser down to 320 mW. The resulting power transmitted from MC is about 270 mW. MC remarkably improved the contrast of prototype from 95% (without MC) up to 99.2%.

Figure 2 shows the MC servo system. At low frequencies, the servo controls the cavity length to follow the free-running laser, and at high frequencies, it stabilizes the laser frequency. Such a servoloop at low frequencies was necessary because of the instability of the cavity length, which was due to the pendulum motion excited by seismic noise. We chose the crossover frequency between these loops to be about 30 Hz, considering the cavity stability. The transfer function of the servo is shown in Fig. 3. The frequency stabilization gain is 80 dB at 1 kHz. The operation of MC is extremely stable and we observed no failures of the locking during the continuous operation for several hours.

The recombined interferometer consists of two arm cavities and a beam splitter. The arm cavity mirrors are made of monolithic fused silica which has a diameter
of 5 cm and a length of 6 cm. The curvature radius of the end mirror is 30 m and the front mirror is flat. These substrates are superpolished and coated by JAE. Each of them is suspended by a magnet-damped double pendulum whose resonant frequency is about 2 Hz (See Fig. 4). The Q-value of the pendulum is about 5 or so. The alignment of the mirrors can be controlled by motor drives and PZTs.

Each arm cavity of finesse 516 is kept on resonance by using a light reflected from the anti-reflection(AR) surface of the beam splitter. For this purpose, the beam splitter (two silica plates 100 mm-in-diameter are attached by optical contact) has symmetric AR surfaces and those reflectivities are set to 0.7% because at least 1 mW is necessary for a good signal-to-noise ratio in the cavity locking servo. In our configuration, the Unity Gain Frequency(UGF) of the cavity locking servo must be below a signal frequency because the servo suppresses any mirror motions including gravitational wave signals. So it was designed to be 300 Hz.

The reflected lights from the cavities are recombined at the beam splitter. The interferometer is held to a dark fringe by pre-modulation method. The frequency of the phase modulation is 40 MHz and the modulation index \( m_{\text{eff}} = m \sin (\omega \Delta \ell / c) \) is 0.12 with a Michelson path difference \( (\Delta \ell) \) of 17.5 cm. To keep a dark fringe, the beam splitter position is controlled. The UGF of the servo loop is about 50 Hz.

All the system is housed in aluminum vacuum system which is shown in Fig. 5. When the chamber was empty, it could be pumped to a vacuum of \( \approx 10^{-8} \) torr. Two turbo molecular pumps, which are placed in the middle of the 20 m tubes, keep
Figure 4: Suspension system for the arm cavity mirrors, which is consisted of the aluminum upper mass and the lower mass of the monolithic fused-silica mirror. Eddy-current damping is applied to the upper mass to suppress the large pendulum motion. To isolate the vertical vibration, the rectangular blade springs are inserted at the suspension points of the upper mass.

the vacuum level below $10^{-6}$ torr. The mirrors, the beam splitter and the optics are set on the aluminum table which is isolated from the ground vibration by one layer stack.

3. Experimental Results and Discussions

At first we measured the excess noise caused by the MC system.\(^3\) We compared the intensity noise spectra observed after EOM and after MC with that observed just after the laser. The result is shown in Fig. 6. In the intensity noise spectra measured after EOM, we observed up-converted intensity noise produced by EOM. After MC, we observed a bigger excess noise because frequency noise of the laser is converted into intensity noise by FM-AM conversion in the MC cavity. The excess noise is very sensitive to the difference between the FSR of MC cavity and the modulation frequency. So we adjusted the modulation frequency within $\pm 2$Hz around the FSR of MC. Consequently, the excess noise was sufficiently suppressed and its effect could be reduced below the shot noise in the displacement noise spectrum of 20m interferometer due to the recombination.

After these preparations, we measured Common Mode Rejection Ratio (CMRR) and obtained a value of 40 dB. It was estimated to be limited by the difference of the storage time of the arm cavities. The contaminations which occurred during the mirror installation degraded the finesse of the cavity. It was also found that CMRR was very sensitive to the alignment of arm cavity, so we could not keep the best condition for a long time.

Finally, the noise spectrum of the interferometer was obtained (See Fig. 2). The
displacement noise was $4 \times 10^{-17} \text{m/} \sqrt{\text{Hz}}$ at 1 kHz. The calculated value considering the effective modulation index is smaller than this value by an order of magnitude. But the photodiode current was much larger than the expected one because the dark fringe was not so dark. The typical value of the contrast during the operation was 99% and the residual light power at the dark port was more than 1 mW. According to the incandescent lamp test for the anti-symmetric port photodetector, the displacement noise was proved to be limited mainly by shot noise.

4. Summary and Future

We succeeded the operation of the recombined interferometer with the suspended mode cleaner. The 40 MHz modulation sidebands for pre-modulation are passed through the mode cleaner. For this purpose, the length of mode cleaner was designed to be 3.75 m. The resulting displacement noise was $4 \times 10^{-17} \text{m/} \sqrt{\text{Hz}}$ at 1 kHz and it was almost limited by shot noise. At lower frequencies, the noise was limited by the frequency noise of the laser. So we must improve the frequency stabilization.

In near future, we will reconstitute the servo system for the recombination of the interferometer. After several checks (cavity reflectivities, misalignment effects,
Figure 6: Observed intensity noise of the modulation sidebands obtained by mixing the output of a photodetector with the 40 MHz reference. The lowest curve represents the shot noise. The middle curve shows the intensity fluctuation produced by the mismatch of the light angle. Top curve is the intensity noise of the MC transmitted beam.

etc.), a recycling mirror will be installed. At the first stage of the power recycling, we will examine the matrix-diagonalization method.\textsuperscript{5}

**Acknowledgements**

We thank Norikatsu Mio and Seiji Kawamura for their helpful discussions and cooperations in construction of this prototype.

**References**

3. S. Telada, et al. This issue.
Figure 7: Observed noise spectrum of the 20 m prototype. The noise level of $4 \times 10^{-17} \text{m/Hz}^{1/2}$ at 1 kHz was almost limited by shot noise. At lower frequencies, the noise was limited by the frequency noise of the laser. For calibration of the measured spectrum, a sinusoidal signal of known amplitude was applied to one front mirror through its coil driver.
The Laser Interferometer Gravitational-Wave Observatory (LIGO) is being developed to detect gravitational waves emitted from Astrophysical sources. The facility will consist of two widely separated laboratories housing highly sensitive long baseline interferometers using suspended test masses. The construction of LIGO began in 1995 with scheduled completion in 1999. The status and goals are discussed.

1. Introduction

Evidence that gravitational waves indeed exist in nature has been provided by the beautiful experiment of Hulse and Taylor, who measured the orbital period of a binary neutron star system, over a 15 year interval with great precision. They observed, with 1% accuracy, a decreased in the ~8 hour period of the orbit by about 10 seconds. This measurement is completely consistent with expectations from general relativity due to the emission of gravitational radiation.

The present situation experimentally is reminiscent of what occurred after the emission of neutrinos were proposed as the explanation for observations of apparent missing energy (and angular momentum) from some nuclear beta decay reactions. That lead to a concerted effort to ‘directly observe’ neutrinos through their interactions, which was finally accomplished 20 years later by Reines and Cowan. Since the time of the first direct observations, a rich field of neutrino physics has developed both to study the neutrino itself (this continues with the search for neutrino mass and oscillations), and the use of the neutrino as a sensitive probe of fundamental particle physics (quark structure of nucleon, neutral currents, etc.).

In analogy to neutrinos, for gravitational waves, following indirect observation, sensitive new instruments are now being developed for in the future, spherical detectors, to directly detect these waves. In this talk, I concentrate of the interferometer approach, and particularly those on the earth’s surface (there are long range proposals for such devices in space) and specifically for the U.S. Project (LIGO). More detailed technical talks are presented elsewhere in this workshop.

2. LIGO

LIGO, the Laser Interferometer Gravitational-Wave Observatory, is a joint MIT/Caltech project funded by the National Science Foundation. It will consist of two widely separated sites (3000 km), each having two 4 km arms in an L-shape and under high vacuum. These vacuum systems will house sensitive suspended mass interferometers which will be used for coincidence detection of gravitational waves. The experimental goal is to measure changes in distance of as little as $10^{-18}$ m in the separation of the masses in the arms over a frequency interval of 10-1000 Hz. The expected signal from a binary neutron star system, like the one observed by
Hulse and Taylor, is a so-called ‘chirp’ signal (Figure 1) that increases frequency and amplitude as it crosses our frequency bands during the last seconds of the inspiral and coalescence.

The ‘benchmark’ design goal of LIGO is to have sufficient sensitivity capability to observe such systems. The best estimates for rates, based on extrapolations of the statistics of neutron stars in our galaxy yield ~ 200 Mpc (650 million light years) as the distance LIGO must be sensitive to, in order to observe three neutron inspirals/year. More optimistic estimates yield a distance of 23 Mpc, and ultraconservative estimates yield 1000 Mpc. With this guidance and experimental practicalities, the strategy for LIGO is to build an initial device that will approach the interesting region and be straightforwardly improvable to the best guess estimate. Furthermore, the overall LIGO facility design is such that future more sensitive interferometers can reach the conservative bound without being limited by the facility (e.g. the vacuum, the seismic isolation, etc.).

The LIGO configuration is shown in Figure 2. A high power laser (initially 10W), which has been highly stabilized, is used as the light source. The laser is a Nd:YAG type with wavelength $\lambda = 1064$ nm. The laser beam is injected into the two arms of the interferometer by splitting the beam, and then the beam is servo-locked to the length of the interferometer arms. The detectors are set on a dark fringe and using the high photostatistics, small changes in distance are recorded by very finely ‘splitting’ the fringe.

The reflectivity’s of the mirrors are selected and positions controlled to build up the beam in the resonant cavities and to optimize sensitivity. The parameters
Figure 2: The optical system for LIGO.

<table>
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<th>OPTICAL CHARACTERISTICS</th>
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Table 1: LIGO interferometer optical parameters.
used in the initial and a sample enhanced interferometer are given in Table 1.

3. The Sensitivity of Ligo

A comparison of LIGO sensitivities with predictions of various astrophysical sources is given in Figure 3. The shape of the sensitivity curve is determined by three basic noise sources; seismic noise at low frequencies (10-50 Hz); thermal noise at intermediate frequencies (50-100 Hz); and shot noise at the highest frequencies (100 - 1000 Hz). Both the initial LIGO and an advanced detector (where all three noise floors have been significantly improved) are shown. In addition to binary neutron star systems, possible signal levels from binary blackhole systems and from supernovae collapse are shown.

Figure 3: LIGO sensitivity compared with expected gravitational wave signals.
4. Ligo Status

LIGO is presently a full scale construction project. This is an exciting time, when after years of preparation, dirt is being moved, concrete poured, the enormous vacuum system construction is underway and the first interferometer elements are in final design.

The two sites for LIGO are in Hanford, Washington and Livingston, Louisiana as shown in Figure 4. Each site consists of a large corner station, which houses the lasers, the input-output optics and vacuum equipment. This station ‘feeds’ the long 4 km beam pipes with beams that reflect off test masses at the far end. The vacuum chambers are 4 ft in diameter allowing installation of multiple interferometers in the same vacuum. In fact, the Hanford facility will initially house two interferometers (2 km and 4 km) to help reduce noise through a ‘local’ coincidence.

Both sites have been cleared and graded, and the concrete slab on which the beam tube is erected, and the first area of the beam tube and enclosure (Figure 5) are almost complete in Washington.

The beam tubes are made from 3 mm thin stainless steel, especially selected and carefully cleaned to reduce outgassing properties. They are rolled in a special spiral mill designed for this purpose and welded. The tube has stiffening rings to maintain the stability and bellows every 130 ft to allow for expansion and contraction.
Internal to the beam tube, baffles are inserted to reduce scattered light off the walls getting into the system. Vibrations of the walls cause modulation of this light which can create phase noise background. To reduce this problem, special baffles treated to be absorptive at laser wavelengths are installed. They are serrated at the edge to minimize diffractive effects.

Figure 5: Beam tube and enclosure for LIGO.

The vacuum requirements are stringent in order to insure that gas scattering will never a factor to the noise floor, even in advanced detectors. The pressure required to meet this goal is that we obtain \(< 10^{-9}\) torr for all residual species. That has been achieved in a 130 ft prototype beam pipe assembly that in all essential ways is identical to the actual LIGO beam tube. We believe the production tube is of higher quality due to rigorous quality control and about 3 km have been completed with no known leaks.

All the conventional facilities for LIGO are well underway and will be completed in 1999. The initial interferometer design is being finalized this year and some long lead items are already under construction. By the year 2000 we expect to be commissioning the initial interferometers, testing them and soon after we will do an initial search for gravitation waves. Our goal is to reach our initial design sensitivity indicated in Figure 3 by 2002 at which point we hope to detect gravitational waves.

In any case, the detector sensitivity will be improved systematically by incremental enhancements in the following years, allowing detection if the rates are low.
The indicated improvements expected in a 5-10 year program are also shown in Figure 3. If gravitational waves have already been observed, the improvements will afford the opportunity to increase the rates to where LIGO will be able to initiate a new field of research - gravitational wave astrophysics.

References

VIRGO STATUS REPORT, NOVEMBER 1996

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ABSTRACT

We describe the present situation of the Virgo project, as concerns its goals, its organization, its planning, and the advancement of the R&D.
1. Introduction

Virgo is the French-Italian project for the construction of a 3 km interferometer for the detection of gravitational waves. Feasibility studies started independently in Italy and in France at the beginning of the 1980’s and progressively led to an informal collaboration which resulted in 1989 in the production of a proposal for the construction of a joint large interferometer,\(^1\) which was reviewed independently by CNRS in France and INFN in Italy. Both institutions promoted a national enlargement of the collaboration. At the request of a small binational committee, created in 1991, the collaboration produced, in 1992, the ”Final Conceptual Design” document,\(^2\) which was approved and is still considered as the reference for the cost and manpower estimated needs of the project. An agreement was signed in June 1994 by the Directors of CNRS and INFN, which defines the responsibilities, the sharing of tasks and the financial rules for the construction phase of Virgo. The site of Cascina (close to the Pisa laboratory) was selected in 1994. The legal problems concerning the expropriation process lasted until August 1996, but INFN took the risk of anticipating the final steps and allowed on-site construction to start in April. The first buildings will be ready in Summer 1997.

2. Description

The objectives of Virgo, are:

- to realize or participate in the first direct detections of the gravitational waves emitted by astrophysical sources,
- to test the theoretical predictions concerning the properties of the waves,
- and finally to generate the new field of gravitational astronomy.

(Note that both last goals (and probably all three) require that other interferometers of comparable sensitivity will be built in the world and will function in coincidence with Virgo). For these purposes, the infrastructures, which represent the largest part of the investment, are foreseen for a minimal lifetime of 20 years, and are slightly overdesigned compared to the size and the expected performance of the first interferometer, in order to allow for the implementation of an improved interferometer and for the future simultaneous operation of two or three interferometers sharing the same vacuum system.

The initial interferometer is a recycled Michelson interferometer, containing a 3 km Fabry-Perot cavity of finesse \(F = 50\) in each arm. This geometry is considered to be a good compromise between the requirements of sensitivity, bandwidth, and simplicity. The end mirrors of the Fabry-Perot cavities will have a radius of curvature of 3450 m, all other surfaces being plane. The beam waist is 2 cm in the vertex area, and 6 cm near the extremities. The shot noise limited sensitivity goal asks for storing a power of about 1 kW. The compromise we have presently chosen is to use a 10 to 25W Nd-YAG laser, with a power recycling factor of about 100 to 50, and
to aim for a high efficiency of the whole optical system, including input and output optics and photodectors. The seismic isolation is provided for each interferometer mirror by a "superattenuator" (see below), which is designed so that the seismic noise will be negligible at all frequencies higher than 3-4 Hz.

The Virgo facilities consist mainly in a large vacuum vessel containing the interferometer, and a few buildings. The vacuum vessel is L shaped, with two, 3 km long, orthogonal arms (the tubes). The tubes have an internal diameter of 1.2m. They house a number of baffles (about 100 per arm) designed to trap the scattered light, which leave a free aperture of 1m in diameter. The tubes are connected to vertical tanks (the towers), which house the superattenuators suspending each optical element of the interferometer. The tubes are protected by light concrete/metal structures (the tunnels) at ground level. At all the extremities small buildings house the towers and associated equipment and maintain proper environmental conditions (temperature, cleanliness, vibrations,...). The largest one (about 1000m$^2$) is the central building which contains seven towers. In the central area, three additional smaller buildings house the computing system (control-command building), some technical facilities (technical building), and the extremity of the mode-cleaner (a 144 m long optical filtering cavity which is used in transmission, to filter the laser beam).

3. Performance Goals and Main Requirements

Figure 1 shows the spectral sensitivity curves of VIRGO, from 1 Hz to 10 kHz. The full (upper) curve represents the expected equivalent noise level for the initial interferometer. From low to high frequencies, the sensitivity will be limited by seismic noise, by the thermal noise of the last isolation stage, by the thermal noise of the mirrors, and finally by the quantum noise of the optical detection system. The numerical data injected in this model have been measured on existing materials and components which will be used in VIRGO: they correspond to the present state of the art. A negligible fraction of the spectrum will be spoiled by very narrow "noise peaks" corresponding to mechanical resonance frequencies in the wires of the last isolation stage, excited by thermal noise. They are not shown here, because their narrowness cannot be properly represented, due to the limited resolution of the picture. Initially, the very low frequency part of the spectrum (below 10 Hz) could be partly spoiled by noise from the interferometer locking system, the requests on its dynamic range being very high. The expected sensitivity is such that, according to average astrophysicists estimates, the initial Virgo should be able to detect a few events per year. The dotted (lower) curve shows the ultimate sensitivity level which could be achieved in Cascina. This curve does not constitute a realistic sensitivity goal, but rather shows that the facilities being built will not become rapidly obsolete soon: approaching this level of sensitivity would allow the observation of a large fraction of the Universe, with the detection of many events per hour. From what we understand today, the ultimate limitations are: seismic noise below 3 Hz, fluctuating gravity gradients (atmospheric and ground motions) up to 10 Hz, the "standard
quantum limit ” up to 2 kHz, and residual pressure fluctuations in the vacuum system, at higher frequencies. In practice, there are already indications that it could be possible in the near future to enhance the initial sensitivity by about an order of magnitude at all frequencies above 5 Hz. But, reaching the ultimate sensitivity would require very large, and yet inconceivable, improvements in the interferometer’s thermal noise and shot-noise.

4. Organization

Virgo is a collaboration between eleven laboratories of CNRS and INFN, involving about 80 physicists and 70 engineers and technicians. The Virgo Council, with three representatives from each institution, meets officially twice a year: its role is to organize and supervise the construction phase, to prepare and to finalize the yearly budget, to appoint the project Direction team and to prepare the operation phase. Its chairmanship alternates every 2 years between France and Italy. The STAC (a Scientific and Technical Advisory Committee of 5 external experts) meets twice a year with the project Direction and reports to the Council on selected topics. In June 1996, the Council appointed A.Brillet as Project Leader for the next three years, with A.Giazotto as Deputy Project Leader, and D.Enard as Technical Manager. Since then, the organization of the project has been clarified with the elaboration of a ”Management Plan”. The Virgo Executive Committee (VEC)
is a consulting and advising body where all the laboratories are represented. The members of the VEC are the Direction and the group leader from each laboratory of the collaboration, or his representative, plus a few experts. The Virgo Technical Management Team (VTMT) meets once per month to help the Technical Manager in surveying the progress of the construction of each system. A Scientific Committee will be set up in order to advise the Direction as concerns the long-term scientific issues. A yearly review of the project has been recently set up. It will start in January 1996 with a detailed description of each subsystem, presented by its responsible.

5. Construction Planning

On request from the funding agencies to extend the construction period over 5 years, it was decided in 1994 that the most efficient way to build Virgo would be the following: In a first phase, we are going to complete all the buildings and the vacuum system of the central area, and install a short recycled Michelson interferometer, called the “test interferometer”. Except for the mirrors, this instrument will use the final components of the Virgo interferometer: superattenuators, laser system, input and detection optics, alignment system, local controls and monitoring, electronics, networks, global control and data management system. The operation of the test interferometer should start at the end of 1998. The construction of the tunnels, the vacuum tube, and the end buildings will start in 1998 and go on until the beginning of 2001, in parallel with the debugging of the test interferometer, which corresponds actually to the commissioning of most of the subsystems of Virgo. Consequently, one can expect that Virgo will start taking useful data only a very short while after the end of the construction. This will be an advantage, and the construction planning should be very efficient, but only at the condition that the operation phase can start progressively, with an appropriate staff on the site, a few years before the end of the construction. This is an unusual condition which still has to be fully integrated in the project management. In parallel with these operations, also takes place in Lyon the construction of a specific laboratory for the realization of the final large optical components of the Virgo interferometer. It started in November 1996, and the first large coatings should be realized in 1999.

6. R&D

6.1. Construction R&D

After a phase of feasibility studies (1983-1990), the R&D activities relative to the construction of Virgo are now finishing. Let’s recall some of the most remarkable achievements:

Suspensions

Since the beginning of the project, we have decided to make a large effort in order to obtain a good sensitivity at low frequencies: the initial goal was to make the seismic noise negligible for all frequencies above 10 Hz, and we developed in
Pisa the "superattenuators" (SA). The SA is a chain of cascaded pendulums, in which each intermediate mass (or filter) is a heavy steel structure which provides vertical seismic isolation due to a combination of vertical springs: the next mass is suspended to a spring consisting in an ensemble of blades and having a resonance frequency of about 800 mHz. Their stiffness is reduced by the addition of a magnetic system (antispring), which adds a tunable negative spring constant: this can reduce the resonance frequency of each stage below 100 mHz. A large effort was done during the last years in order to test, to improve, and to simplify the SA as well as to understand its potential problems, in particular temperature sensitivity and creep. Each SA will be suspended from an "inverted pendulum": this is a 3-legged structure, about 8 m high, with a high flexibility at the bottom. The inverted pendulums can be passive, with only some active damping, or they can use sensitive accelerometers to enhance the very low frequency isolation, and to deduce the RMS motion of the suspension point of the SA far below 1\(\mu\)m. This should be extremely useful because it would relax the constraints on the dynamic range of the control system, and should minimize the nonlinear effects which can contaminate the low frequency part of the spectrum. The "last stage" of a suspension consists in a marionette, from which are suspended the mirror (or the beamsplitter) and a "reference mass", which is used to protect the mirror and also to provide forces for the fine positioning of the mirror. The critical problem here is the suspension of the mirror from the marionette, which should have a low thermal and excess noise. Today, we have very good hopes to reach the goals of the whole design, which are:

- to provide enough isolation for the seismic noise to be negligible at all frequencies above 3-4 Hz
- to understand and control excess noise in the SA in such a way that the only concern should be the thermal noise (and creep noise) of the last stage
- to ensure vacuum compatibility and safety of operation
- to reduce thermal noise at the level of the initial sensitivity curve

A number of studies are still going on to optimize the design, but the main parts of the superattenuators are now being built.

Lasers

The light source is a high power diode-pumped Nd-YAG laser, injection-locked to a stable low power laser. These choices were made more than ten years ago, and they start proving to be very efficient:

- the low power laser was frequency locked to a very stable high finesse Fabry-Perot cavity, and its stability was measured with a second cavity, used as a frequency discriminant. The results are satisfying: this stabilized laser shows the best short term stability (or spectral purity) ever achieved for any kind of oscillator, and, above all, fully complies with the stability requirements for

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Virgo. The fast servo system detects and recovers in a few seconds after a perturbation. During a one week long test in normal laboratory environment, it never unlocked, except when intentional perturbations were applied.

- the high power laser presently delivers 10.2 Watts single frequency, after spatial filtering. The beam is very stable, and the injection-locking technique functions. The lock acquisition is performed automatically in less than 3 seconds. A long term test was recently started. No unlocking has happened after a few days.

- a fast power stabilization, which consists in a variable current sink in parallel with the high power pumping diodes, has been built and tested on a 5W prototype laser. It reduces the power fluctuations down to shot-noise level (for a power of 10 mW) in the frequency domain below 10 kHz.

A 10 W stable laser with all the final input optics and local controls will be installed in Cascina in 1998 for the test interferometer. The final 25 W laser will be built meanwhile and should be installed during year 2000.

Vacuum System

A large effort was made in Pisa and in Orsay in order to design a low cost and low maintenance vacuum tube. The most remarkable achievement is that we understood the physics of the well known hydrogen outgassing of clean stainless steel and that we can reduce it to a negligible level: after a controlled air bake (at about 700K) of each tube module, which diffuses hydrogen outside of the steel, its outgassing rate is reduced to about $10^{-15}$ mbar l s$^{-1}$ cm$^{-2}$, about three orders of magnitude lower than the standard value which is used by ultra high vacuum specialists. This is specially important in Virgo, due to the large area of the tube (about 20,000 m$^2$). Then the permanent pumping of the 6 km of tube will be realized with 10 Ti getter pumps, which have no moving part and will require no other maintenance than the annual evaporation of a fresh Ti layer. Both groups also studied together the problem of the baffles which are necessary to prevent scattered light from recombining with the main beam after reflections inside the tube. A prototype tower was delivered in Annecy in 1994. It has been very useful in order to set up the procedures for pumping and controlling the pumping system of the towers, mainly for performing the introduction of the mirrors without contaminating them.

Automatic Alignment

The procedure for automatic alignment of a recycled interferometer has been studied in Frascati, on paper first, and more recently on a rigid table-top interferometer. The procedure, and its precision, will fulfill the needs of Virgo and of the test interferometer. Similar results have just been achieved by LIGO at MIT. The rough alignment system is being designed in Rome. Satisfying preliminary results have been obtained in a collaboration between Orsay and Annecy: the readout of the position of suspended masses can be performed from outside the vacuum system with a repeatability of about 1$\mu$m and 1 mrad, using a computerized CCD camera.
Thermal Noise

Experiments have been set up in Orsay and in Perugia to measure the mechanical Q factor of mirror substrates\textsuperscript{14} and of pendulums.\textsuperscript{15} Q’s of about $10^6$ have been obtained in both cases, which correspond to the initial goals. Moreover, the understanding we have acquired, and some additional tests performed in collaboration with GEO on silica fiber pendulums\textsuperscript{16} and with ACIGA on sapphire mirror substrates,\textsuperscript{17} let us hope for large improvements in the future.

Optics

The R&D on optical metrology in Paris and on the realization of low loss mirror coatings in Lyon has been very successful. The progress of optical technology is strongly dependent on the progress of optical metrology, and we have had to create appropriate measurement tools in order to test and to improve the optical components of Virgo: - a new optical absorption measurement technique allowing to measure substrates absorption\textsuperscript{18} down to the level of $10^{-7}$ cm\textsuperscript{-1} and coatings absorption\textsuperscript{19} down to $10^{-8}$ was the starting point for the German company Hereaus to develop a new quality of very low loss, very high homogeneity silica, which will be used for the Virgo mirrors, and helped the development in Lyon of excellent coatings. - a specific test bench has been set up to check the birefringence, the homogeneity, and the absence of local defects (such as bubbles) in the substrate of a Virgo beamsplitter: all results are satisfying, and much better than what could be specified by the manufacturer and by the polisher, since they don’t have such measuring facilities. - the recent availability, in Paris and from commercial companies, of very sensitive surface inspection instruments, with mapping facilities, helps improving the polish roughness down to sub-Angstrom levels, and to reduce the number of local defects (scratch and dig). - a specific absolute wavefront sensor is also developed in order to check the wavefront deformation by coated mirrors to better than $\lambda/100$ peak to peak, over the useful surface of the Virgo mirrors (about 200cm$^2$): our goal is to map the wavefront distortions once the multilayer coating is nearly finished, and to make wavefront corrections by adjusting accordingly the last layer thickness. This new technique, that we called ” corrective coating technique ” is now operative for medium size mirrors (up to 10 cm diameter). The only limitation to its performance seems to be the quality of the metrology. All the mirrors specifications are already met for 5 cm diameter mirrors.

Photodetectors

The Japanese company Hamamatsu has developed, in collaboration with the Annecy group, some very efficient 3mm$^2$ InGaAs photodiodes: their quantum efficiency is close to 90% and can stand permanently a laser power of more than 50 mA, with a linear response. For safety and redundancy reasons, each important beam is detected by one or more arrays of 4 photodiodes.
Simulation

A large effort has been made in Annecy for more than 4 years in order to develop a dynamic simulation program of the whole interferometer. The goal of SIESTA is to simulate, in the time domain, all the relevant subsystems of Virgo (superattenuators, mirrors, detectors, laser, demodulators, ADC’s, electronic filters, etc...) with various levels of description. For instance, one can run a quasi static detailed optical model of the interferometer in order to study the sensitivity to small misalignments, or a real time model, using plane waves, for the study of the locking system. Its most important uses are probably the study of the automatic locking and alignment systems, and the generation of simulated data for the preparation to data analysis.

Length and Alignment Control

A common effort is made to understand and design the general locking strategy. In order to extract all the information needed to calculate the longitudinal and transversal position errors of the mirrors and beams, it was chosen to set up InGaAs and quadrant detectors on each beam which is reflected or transmitted by the interferometer. The incoming beam into the interferometer is phase modulated. In phase and out of phase demodulation of each detector output provides a redundant information which allows in principle to control each degree of freedom of the interferometer. Another important choice was to use a completely digital feedback system, following initial recommendations and studies of the Napoli group. Each signal from the InGaAs photodetectors, and from the alignment quadrant photodiodes will be received, after an ADC, by the ”global control” unit, which ”diagonalizes” the matrix relating these signals to the transducers which are used in the local control units (laser control, suspensions controls). The redundancy of the signals allows for a $\chi^2$ optimization, which will be performed on-line by the global control unit developed at LAL- Orsay. The choice of appropriate ADC’s, DSP’s, and CPU’s is now converging.

Data Management

Data acquisition, formatting, storage, and distribution are being elaborated in a collaboration between Annecy and Napoli. It has been decided to use a common data format with the LIGO interferometers, and to propose to extend this definition and standardization effort to other interferometer groups. The general directions concerning monitoring and data quality evaluation are being discussed and presently being drafted in Napoli. The preparation of the data (validation, and removal of the instrument transfer function) and the data analysis (on-line and off-line) are being organized. To some extent, these activities involve all the collaboration. Simulated data provided by SIESTA are going to help defining the computing needs and selecting the appropriate filters to be implemented for the recognition of each kind of signal.
6.2. Long Term R&D

Though the R&D which was necessary for the construction of the initial interferometer is essentially finished, it is clear that the way for improving further the sensitivity is: more R&D. The obvious fields for R&D are thermal noise (better materials, bonding techniques, cooling, direct measurement of thermal noise, ...) and optical shot noise (laser power, overall efficiency, thermal effects, new optical schemes and components, in-situ mirror cleaning, squeezed light, ...), and also better control techniques, efficient data analysis and storage systems, low noise and faster digital filters, ... It will be difficult to find enough resources to start these activities during the next few years, when most of the physicists of Virgo should be busy with the realization, the installation, and the debugging of their subsystems, in Cascina. The solutions are to expand the research community outside of the present Virgo collaboration, and to keep a minimum of activity in the Virgo groups, which could restart later at full speed. All this has first to be organized and funded. This is one of the concerns of the project Direction team.

References
1. Virgo: proposal for the construction of a large interferometric detector of gravitational waves, CNRS-INFN, May 1989
4. Pisa group Seismic isolation: the use of blade springs, Virgo Note PJT94-008, 1994
5. R.DeSalvo private communications, and notes in preparation Gaddi et al., Pre-isolator stage for Virgo, Virgo Note NTS96-34, 1996
8. P.Marin internal report AM950413, 1995
10. R.Morand et al. VIRNOT-1390-074 and 075, 1996
11. Frascati group Alignment procedure for the Virgo interferometer: experimental results from the Frascati prototype, Virgo Note, 1996
13. Pham Tu, private communication
14. C.N.Man and M.Barsuglia An optical method for measuring the resonance characteristics of a silica block, Virgo Note PJT92-027, 1992
16. J.Hough, this issue

172
17. D.Blair, this issue
19. C.N.Man et al. *Optical absorption measurements in monocrystalline sapphire at 1.06mm*, *Virgo Note NTS96-20*, 1996
20. ESPCI group *Multilayers coatings characterization*, *Virgo Note PJT94-019*, 1994
21. F.Marion, this issue

The list of the available Virgo Notes can be consulted on: http://www.lal.in2p3.fr/virgo/virgo.html
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GEO 600: CURRENT STATUS AND SOME ASPECTS OF THE DESIGN

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ABSTRACT

GEO 600, the German/British gravitational wave detector with arms of 600 m length, is now under construction near Hannover in Germany. It will use a four bounce Michelson interferometer arrangement with power and signal recycling between fused silica test masses suspended by fused silica fibres in double pendulum arrangements. Seismic isolation will be provided by encapsulated stacks of stainless steel and graphite loaded RTV.

1. Introduction

The design for the German/British GEO 600 detector is based on the experience at Glasgow and Garching with prototype detectors of arm length 10 m and 30 m, and is essentially a descoped version of the original GEO proposal for a detector of 3 km arm length.

GEO 600 is an interferometer system with 600 m arms designed to be low cost but high performance. Its planned sensitivity is close to that of the LIGO and VIRGO detectors in their initial configuration, in a frequency range from a few hundred Hz to approximately one kHz. In more detail, the design specification calls for a noise level, expressed in gravitational wave amplitude, of better than $2 \times 10^{-22}/\sqrt{\text{Hz}}$ in a frequency range of 50 Hz to 500 Hz, with a sensitivity of $10^{-22}/\sqrt{\text{Hz}}$ between 100 Hz and 300 Hz.

Construction should proceed on a timescale similar to that of the larger detectors.
so that the instrument will be able to participate in coincidence measurements with LIGO, VIRGO and TAMA 300. The need to achieve sensitivities close to that of the longer detectors means that relatively high performance from the interferometry and from the suspension systems for the test masses has to be achieved and thus the GEO 600 system will serve as a useful testbed for advanced techniques for future interferometers.

2. The Interferometer Outline

As shown in Figure 1 the GEO 600 system uses a four bounce Michelson interferometer to sense the relative displacements of the test masses, which, excluding the beamsplitter, will be of 14 kg mass. The beam splitter will be approximately 9 kg in mass. Power recycling is included in the design with a proposed power recycling factor of approximately 2000. Signal recycling is also included and it is proposed that this be selectable in bandwidth and tunable in centre frequency. The parameters chosen for signal recycling at any time will depend on the particular application.
of the detector at that time, i.e. whether it is being used to search for broad band pulsed sources or narrow band signals such as might be expected from pulsars. Research on power recycling is being carried out on the 30 m prototype detector at Garching\textsuperscript{8} and work on signal recycling and its control is being addressed on bench top systems in Hannover\textsuperscript{9,10} and on the 30 m prototype detector at Garching.

In Figure 1 the technique of external modulation is shown as being used for signal recovery at the output of the interferometer; there are, however, other methods which may be used based on the ideas of Schnupp (i.e. frontal) Modulation\textsuperscript{11} and this remains an open point in the design at present. Research in this area is currently being carried out mainly at MPQ in Garching.

Two triangular modecleaner cavities, each of optical length 8 m, are placed in front of the interferometer. The finesse of each cavity is chosen to be 1900 and thus suppression of beam geometry fluctuations by up to 6 orders of magnitude should be achievable. Research on modecleaners is being carried out at Glasgow\textsuperscript{12}.

The proposed laser system is an injection locked diode pumped Nd:YAG system with 10 W single frequency output power and this is currently under development at the Laser-Zentrum, Hannover. Specifications on the stability of the laser light incident on the power recycling mirror are as follows:

- Frequency jitter - $\Delta f \leq 10^{-3}\text{Hz}/\sqrt{\text{Hz}}$. To achieve this the laser must be prestabilised to a short reference cavity. Further stabilisation to the required level of $\leq 10^{-5}\text{Hz}/\sqrt{\text{Hz}}$ at the main beamsplitter will be obtained by frequency locking to the cavity formed by the power recycling mirror and the rest of the interferometer.

- Fractional beam geometry fluctuations - $10^{-11}/\sqrt{\text{Hz}}$. This level will be achieved by the use of the two modecleaners described above provided that the geometry fluctuations from the laser do not exceed $10^{-5}/\sqrt{\text{Hz}}$ which is a reasonable experimental value.\textsuperscript{13}

- Power fluctuations - $\Delta P/P \leq 10^{-8}/\sqrt{\text{Hz}}$. To achieve such a level will require a significant degree of active power stabilisation of the light from the laser.

These levels are set mainly by the size of the expected contributions to the detector noise level from the different laser fluctuations, given likely imbalances and asymmetries in the interferometer.

3. Vacuum

The vacuum system for GEO600 will consist of 11 vacuum tanks to house the test masses and optics, the tanks at the ends of the arms being joined by 60 cm diameter convoluted vacuum pipes of wall thickness 0.9 mm. The system will be pumped at the centre station and the ends by turbomolecular pumps of 4000 l/sec total pumping rate, the target vacuum being $5 \times 10^{-8}$ mbar for Hydrogen and $5 \times 10^{-9}$ mbar for heavier gases. It is intended that the system be hydrocarbon free and thus no viton gaskets or organic insulation on wires will be used.
4. Seismic Isolation and Suspension

Of the many noise sources which may degrade the sensitivity of ground based laser interferometric gravitational wave detectors, thermal noise associated with the mirror masses and the last stage of their suspensions is likely to be the most significant at least at the lower end of the operating frequency range of the detector. The operating range of the detector lies between the resonances of the test masses and their pendulum suspensions and thus it is the thermal noise in the tails of the resonant modes which is important. In order to keep the off-resonance thermal noise as low as possible the mechanical loss factors of the material of the test masses and of the last stage of the pendulum suspensions need to be kept low and if we assume that mechanical damping is structural in nature (loss independent of frequency) this situation is achieved if the mechanical quality factors ($Q$ factors) of the masses and pendulum resonances are arranged to be as high as possible. To be more specific the design sensitivity of the GEO 600 detector ($h$ smaller than $2 \times 10^{-22}/\sqrt{\text{Hz}}$ between 50 Hz and 500 Hz) is based on the premise that at the lower end of this frequency range the dominant noise source results from losses in the internal modes of the fused silica test masses, experiments having shown that $Q$ factors of $5 \times 10^6$ can be achieved. This assumes that the thermally induced motion at 50 Hz, and above, from the pendulum motion of each test mass is significantly lower than the contribution from the internal modes. If a target pendulum motion of approximately $2 \times 10^{-20} \text{m}/\sqrt{\text{Hz}}$ at 50 Hz - a tenth in power terms of the internal mode contribution - is chosen, it can be shown that a loss tangent associated with the pendulum mode of each suspension of less than $4 \times 10^{-8}$ at 50 Hz is required. If the damping of the pendulum is frequency independent this implies that the $Q$ factor of the pendulum mode must be greater than $2.5 \times 10^7$. $Q$ factors of this level and higher have been measured for pendulums using ultra-pure fused silica fibres welded to fused silica test masses and to the points of support and for pendulums using fibres of commercial grade fused quartz either clamped very carefully or welded to the test masses and the support points. It is thus proposed that the final stage of the pendulum suspensions in GEO 600 should use fibres of commercial grade fused quartz. Different methods of jointing the fibres to the test masses and to the suspension points, which will be the intermediate masses of double pendulum systems, are currently under investigation at Glasgow and at Hannover.

In order to achieve the proposed sensitivity of the detector - an amplitude of $2 \times 10^{-22}/\sqrt{\text{Hz}}$ above 50 Hz - a significant degree of seismic isolation has to be provided for each test mass in the horizontal and vertical directions. Assuming a typical seismic noise level of $10^{-7}/f^2 \text{m}/\sqrt{\text{Hz}}$ in three dimensions, an isolation factor at 50 Hz of approximately $6 \times 10^9$ is required in the principal horizontal dimension. If a coupling factor of 0.1% of vertical motion into horizontal motion is adopted, an isolation factor of approximately $6 \times 10^8$ in the vertical is adequate. This level of seismic isolation can be achieved using a double pendulum system suspended from a top plate mounted by means of cantilever springs from three stacks, each consisting of two layers of stainless steel and graphite loaded RTV rubber. The
stacks will be encapsulated in stainless steel bellows internally damped by means of layers of silicone grease loaded with graphite powder and the stacks will be pumped out independently from the main system. In order to provide isolation in the rotational degree of freedom flex pivots are placed at the top of each stack below the mounting point of the cantilever springs. A further vertical spring will be incorporated in each of the two steel suspension wires for the intermediate mass of each double pendulum to allow a higher degree of isolation than would otherwise be achievable in the vertical direction. Consideration is currently being given to providing three dimensional piezo motors below each of the stacks to allow reduction of the effect of microseismic noise by feedback from geophones placed at the tops of the piezoelectric motors. This should allow reduction in the required dynamic range of the servo systems used for locking the arm lengths of the interferometer. Reaction masses on a second double pendulum will be provided to allow mounting of control transducers for the main pendulum. Coil/magnet transducers will be used for the intermediate mass and the use of an electrostatic transducer for longitudinal control of the main test mass for some pendulums is proposed. A schematic diagram of the suspension system is presented in Figure 2, where, for clarity, only the reaction mass for the intermediate pendulum mass is shown. In the diagram the test mass is suspended by 4 fused quartz fibres; consideration is also being given to using 2 fibres in the plane of the pendulum swing.

5. Location and Building of GEO 600

The detector is being built near Ruthe, close to Hannover and the arms bear essentially NNW and ENE. As shown in Figure 3 the buildings to house the vacuum system are completed. The vacuum pipes are slung from a rail above a trench in the ground. The rail and trench are covered by a corrugated cover for protection from the elements. The vacuum pipes are being delivered to the site in sections 4 m long and are being welded together in one of the end station houses before being pushed out along the suspending rail. At the time of writing the tube in one arm is fully installed and is 60% installed in the other arm. The vacuum tanks and remainder of the vacuum system should be in place by summer 1997.

6. Conclusion

Construction of the GEO 600 detector is advancing well. Research into the suspension design and into the interferometry is continuing and it is planned that the detector will be available for initial operation in 1999/2000.

7. Acknowledgments

We wish to thank the State of Niedersachsen, the Volkswagen Foundation, MPG, BMBF, the Albert Einstein Institut, PPARC, the University of Glasgow, the University of Hannover and the University of Cardiff for supporting this work.
Figure 2: Schematic view of suspension system for main optics in GEO600.
Figure 3: Aerial view of the site for the GEO 600 detector near Ruthe, Hannover.
References


TAMA PROJECT

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ABSTRACT

In 1995 we started a project involving a 300-m arm-length laser interferometric gravitational wave detector (TAMA300). We have already constructed the buildings and tunnels to hold vacuum chambers and ducts. An outline of the design of the detector is presented here.

1. TAMA Project

TAMA is a project to construct and operate a 300-m arm-length laser interferometer (TAMA300) to detect gravitational waves. There are two aims of this project: one is to establish techniques necessary for the future km-class laser interferometer; the other is to operate it to detect possible gravitational waves from nearby galaxies. The LIGO, VIRGO and GEO projects have already started construction around the world. In the near future TAMA300 will play an important role in an international network of gravitational wave detection.

Figure 1: Conceptual drawing of the TAMA300 laser interferometer for gravitational wave detection.

Figure 1 shows a conceptual drawing of the TAMA300. The fundamental scheme is a Michelson-type laser interferometer with suspended mirrors; the arms are replaced with Fabry-Perot (FP) cavities. Two reflected light beams from the FP cavities are recombined at the position of the photo-detector (PD). A change in the fringe signal obtained from the output of the PD will inform us of any incoming...
gravitational waves. A power-recycling scheme has been implemented to enhance the laser power in the interferometer.

The fundamental parameters of the detector are listed in Table 1. The projected sensitivity for the gravitational waves is $3 \times 10^{-21}$ in terms of the root-mean-square metric perturbation ($h_{\text{rms}}$) at 300Hz with a bandwidth of 300Hz. The detector comprises two orthogonally oriented FP cavities with 300-m length. Each FP cavity has a finesse of 520, and is illuminated by an injection-locked Nd:YAG laser. The laser has an output power of 10W and its wavelength is 1064nm. We expect to obtain a power-recycling gain of about 10, and the effective input laser power of the interferometer will be around 30W. The diameter of the vacuum duct is 40cm. To avoid any phase fluctuation of light due to residual gas in the tube, a vacuum is to be maintained inside of the enclosure at a level of $10^{-6}$Pa.

<table>
<thead>
<tr>
<th>Sensitivity</th>
<th>$h_{\text{rms}} = 3 \times 10^{-21} @ 300\text{Hz}$ (Bandwidth 300Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Fabry-Perot-Michelson (FPM)</td>
</tr>
<tr>
<td>Baseline</td>
<td>300m</td>
</tr>
<tr>
<td>Finesse of the FP cavities</td>
<td>520</td>
</tr>
<tr>
<td>Light source</td>
<td>Injection locked Nd:YAG laser</td>
</tr>
<tr>
<td></td>
<td>Output power: 10W, Wavelength: 1064nm</td>
</tr>
<tr>
<td>Power recycling</td>
<td>Gain: 10</td>
</tr>
<tr>
<td></td>
<td>(Effective input power of the interferometer: 30W)</td>
</tr>
<tr>
<td>Vacuum duct</td>
<td>Diameter: 40cm, Vacuum: $10^{-6}$ Pa</td>
</tr>
</tbody>
</table>

Table 1: Fundamental parameters of the TAMA300 laser interferometer.

![Expected sensitivity spectrum of the detector calculated from several noise sources.](image)
Next figure (Figure 2) shows the expected sensitivity spectrum of the detector, calculated from several noise sources. At lower frequencies the seismic noise and the thermal noise are dominant, while at higher frequencies the sensitivity is limited by the shot noise of the laser power. The bottom of the sensitivity is \( \sim 3 \times 10^{-21} \) at around 300Hz.

The TAMA project is being carried out by people from National Astronomical Observatory (NAO), Institute for Cosmic Ray Research (ICRR), National Laboratory for High Energy Physics (KEK), Institute of Space and Astronautical Science (ISAS), University of Tokyo, University of Electro-Communications and Kyoto University.

2. Design of the TAMA300 Detector

The development and construction of the detector is divided into several parts, and people involved have been organized to work on each part. As described in the following, a design study is being carried out to achieve high sensitivity and high reliability of the detector.

2.1. Optical Design

![Figure 3: Optical design of the interferometer.](image)

The optical design of the entire detector is shown in Figure 3. The output of the pre-stabilized laser is frequency-modulated at 15.25MHz with an EOM (Electro-Optical-Modulator). Then, the beam is injected into the mode cleaner thorough the lenses for mode matching and mirrors for steering. The mode cleaner is a 10-m length ring cavity. After passing through lenses the light beam is injected into the
beamsplitter through a recycling mirror. A pick-off plate is inserted in front of each FP cavity to extract information necessary to control the alignment of the mirrors.

A mode-cleaner cavity is implemented between the laser and the interferometer to decrease any unnecessary higher mode light and to eliminate beam jitter. We have already tested the performance of the mode cleaner with suspended mirrors by developing a 1-m linear FP cavity. In the TAMA300 detector we use a ring cavity consisting of two inline flat mirrors and one end mirror with a curvature of 15m. The distance between the flat and curved mirrors is set to be 9.74m, so that the FSR of the cavity is equal to the modulation frequency (15.25MHz) of the light. With this setting the modulation can go through the mode-cleaner cavity without attenuation. Each mirror is suspended as a double pendulum. The finesse of the cavity is designed to be 1,800.

2.2. Control Design

In order to operate the interferometer properly, we have to control the positions and orientations of the mirrors as well as the laser frequency.

Fringe control is necessary to keep the photo detector output dark. Also, the cavities must be resonant with the incoming laser light. These conditions are accomplished by controlling the mirror positions along the laser-beam direction. The change in the length of 300-m FP cavity is estimated to be a maximum of 2mm, which is caused by long-term affects, such as tidal forces or temperature change. These slow changes are compensated for by vacuum-compatible picomotors (New Focus, Inc.), while fast change is suppressed by coil and magnets attached to the surface of the mirror. Four degrees of freedom of the mirror position must be controlled: the common and differential changes of the FP cavities \((L_+, L_-)\) and those of Michelson arms \((\ell_+, \ell_-)\). Recombination of the Fabry-Perot-Michelson interferometer was demonstrated by using 3-m FP cavities with suspended mirrors.

The mirror orientation is automatically controlled relative to the direction of the light beam. In the wavefront-sensing method a quadrant photo detector is used to separate the error signals for controlling the pitch and yaw motions of the mirrors. A feedback signal is applied to the picomotors and PZTs at the suspension point of the mirrors.

We aim to obtain a power recycling gain of 10, which will provide more than 30W as an effective input laser power of the interferometer. The recycling mirror curvature is 9km in radius: the measuring method for such a very large curvature is under consideration. The problems of signal separation for control and lock acquisition are being studied.

2.3. Laser and Optics

A 10W Nd:YAG laser was developed by a group at the research center of SONY corporation. As shown in Figure 4, the slave laser pumped by two 10-W laser diodes (SDL p-6) is injection-locked to the master laser of a 700-mW output NPRO (Lightwave Electronics, MISER). The system has already been delivered to us, and an output of 10W was confirmed in the laboratory.
Also, we are studying the high-power, large-aperture photo-detector, EOM, and isolator.

2.4. Mirror

The substrate of the mirrors used in the interferometer is fused silica (Sprasil-P10) made by Shin-Etsu Quartz Products Co., Ltd. The specifications of the mirrors are summarized in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>diameter (mm)</th>
<th>thickness (mm)</th>
<th>curvature (m)</th>
<th>reflectivity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300mFP front</td>
<td>100</td>
<td>60</td>
<td>flat</td>
<td>98.80</td>
</tr>
<tr>
<td>300mFP end</td>
<td>100</td>
<td>60</td>
<td>450±25</td>
<td>≥ 99.99</td>
</tr>
<tr>
<td>300mFP BS</td>
<td>150</td>
<td>40</td>
<td>flat</td>
<td>50±0.1</td>
</tr>
<tr>
<td>10m ring MC near</td>
<td>100</td>
<td>30</td>
<td>flat</td>
<td>99.82 ± 0.05</td>
</tr>
<tr>
<td>10m ring MC far</td>
<td>100</td>
<td>60</td>
<td>15 ± 1</td>
<td>≥ 99.99</td>
</tr>
<tr>
<td>recycling mirror</td>
<td>100</td>
<td>60</td>
<td>9,000</td>
<td>96.67</td>
</tr>
</tbody>
</table>

Table 2: Specifications of the TAMA mirrors.

First, the substrate is super-polished, and then coated with an ion-beam sputtering machine by Japan Aviation Electronics Industry Ltd. (JAE). They will deliver the mirrors by the end of 1997. A small sample mirror made by JAE showed a very small loss as 6ppm.8

2.5. Mirror Suspension

The mirrors are suspended with a double-pendulum system, as shown in Figur 5. The mirror position (X,Y,Z) and orientation (pitch, yaw) are controlled by picomotors and metal-bellows sealed PZTs at the top of suspension point. The motion
of the pendulum is damped by the eddy-current produced by permanent magnets around the upper mass. This passive damping scheme makes the system very simple and gives high reliability. The effectiveness of the double pendulum has been verified in the 3-m FP interferometer at the University of Tokyo and 20-m FP interferometer at NAO. The affect of the leaked magnetic field of the damping magnets on the mirror has been checked, confirming that it is negligible. The vibration-isolation performance is being tested using a vibration table.

![Diagram of double-pendulum suspension system](image)

Figure 5: Double-pendulum suspension system for the mirrors. The motion of the pendulum is damped by the eddy-current produced by permanent magnets around the upper mass.

### 2.6. Vibration Isolation

The mirror-suspension system is suspended by an X-pendulum mounted on the isolation stack. A stack comprises three alternating layers of stainless steel and rubber. The rubbers are housed in a metal-sealed bellows to avoid contamination by outgassing. The X-pendulum is a special device for obtaining good vibration isolation at low frequencies due to its long period. We have developed a two-dimensional X-pendulum for TAMA interferometer.

### 2.7. Data and Monitor

Data acquisition is divided into two categories: one is slow-rate (sampling rate ≤100Hz) data, such as a vacuum or temperature monitor; the other is high-rate
(sampling rate $\sim 20$kHz) data, including the main signal from the interferometer. It will amount to $\sim 600$kByte/sec and will be stored on 8-mm magnetic tapes. To control the peripheral device, two software programs are being tested: EPICS (Experimental Physics and Industrial Control System) and HP VEE system (Visual Engineering Environment, Hewlett-Packard Co.). Optical fibers are installed along 300-m vacuum ducts. Analog and video signals are transmitted a distance of 300m through the fibers with signal converters at both ends.

2.8. Vacuum System

The vacuum system of the TAMA300 is shown in Figure 6. The mirrors and beamsplitter are housed in eight vacuum chambers with 1 or 1.2m in diameter. Each vacuum duct has a diameter of 40cm, and is made from SUS304. Inside of the duct is a surface treated with electro-chemical buffing (ECB) to achieve a $10^{-6}$Pa vacuum level without baking. It is evacuated with turbo-molecular and ion-sputtering pumps. As of now, one arm (300m) duct and two vacuum chambers have been finished and are ready to be evacuated.

![Vacuum system of the TAMA300. It comprises eight vacuum chambers and two 300-m length vacuum ducts. They are evacuated by turbo molecular and ion sputtering pumps.](image)

2.9. Infrastructure

The TAMA detector is being built at the campus of National Astronomical Observatory (NAO) in Mitaka. Mitaka is 20km from the center of Tokyo. TAMA is the name of the area including Mitaka city. In 1995, excavation work started to make 300-m tunnels and space for buildings. Tunnels 1.5m in width and 2m in height are made of concrete boxes just beneath the ground. The center corner building is 16m in width and 12m in length. It took about one year to complete all of the tunnels and buildings.
3. Discussions and Conclusion

The schedule of TAMA300 is shown in Figure 7. We started to construct the detector in 1995. As of now, the infrastructure, including all of the buildings and about half of the vacuum system, have been completed. We will begin installing the optical system inside vacuum enclosures in 1997. In 1998 we will start operating the interferometer, and after one-year of improvements we will hopefully achieve the desired sensitivity in 1999.

TAMA300 is not the final goal of our project. Our goal is to establish a new field of astronomy, gravitational wave astronomy. Since a laser interferometer has a
directivity pattern, in order to cover the entire sky we need at least one more km-class laser interferometer somewhere in the world (see Figure 8). We have already started a study of the next large-scale laser interferometer in Japan.

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References


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The **VIRGO Collaboration** have been designed for detection GW signals down few Hertz. At such a low frequency the sensitivity is limited by seismic and thermal noise. The device for seismic noise attenuation, the Superattenuator, is presented together with some ideas for building a test device able to measure thermal noise with sensitivity of $10^{-18} \div 10^{-20} \frac{m}{\sqrt{Hz}}$.

presented by A. Giazotto

1. Introduction

Gravitational waves (GW) are predicted by the General Relativity theory.¹ Their direct observation will open a new picture of the Universe due to the comple-
mentary information that they carry with respect to electromagnetic and neutrino observations. Until now there is only an indirect evidence of the GW existence due to the study of J. Taylor et al. The radio emission of the binary system called PSR 1913+16 has been followed for tens of years. The discovered phenomenon was a change in the revolution period of the emitting pulsar. All the unknown parameters of the orbit of the pulsar have been determined from the observations, and with the hypothesis of a GW emission also the change in the distance between the pulsar and its invisible companion has been determined. Using this hypothesis the theoretical and experimental curves agree within less then 1%. This was the first, even if indirect, Gravitational wave emission evidence. The aim of the VIRGO collaboration is to detect directly the GW and to perform astronomical observations. VIRGO is designed for a broad band detection, from few Hz up to 10 kHz, the current improvements in the low frequency range should give the best possibilities for detecting GW emission by pulsars and coalescing binary stars.

2. Gravitational Waves Sources

In the weak field approximation the GW perturb the space-time metric. The GW have two states of polarization, named $h_+$ and $h_\times$ with quadrupolar patterns. The dimensionless perturbation $h$ to the flat space-time metric causes a change $\Delta L$ of the distance $L$ between two free falling masses such that

$$\Delta L = \frac{hL}{2}$$

in the hypothesis that the wavelength $\lambda$ of GW is much larger than the mass separation $L$. Eq. 1 is used by the ground based detectors to measure the GW signals. GW are emitted by very massive body in strong gravity condition, laboratory GW are, at the moment, not conceivable and only astronomical sources are considered promising.

The Supernovae collapse is a very terrifying event in which a lot of energy is emitted. Also a short gravitational radiation burst is predicted. Even if the models of the Supernova event suffer of the uncertainty of some important parameters like the collapse velocity, some estimation of the radiation strength can be made. For galactic sources the predicted value for the strength of GW is $h \simeq 3 \cdot 10^{-18}$ with a rate of events equal to 0.05 per year. For Supernova events coming from the VIRGO cluster (about 10 Mpc) a strength of $h \simeq 3 \cdot 10^{-21}$ is expected with a rate of 5 per year. The frequency range of these events is around 1 kHz.

The coalescing binaries, similar to the Taylor system, are another source of GW. In this case it is necessary a long integration time and the bandwidth of the detector should go from few Hz up to 2 kHz. The expected rate of this event inside a sphere of 100 Mpc radius is 5 per year, the strength is well described by the law:

$$h = 10^{-23} \left( \frac{100 Mpc}{R} \right) \left( \frac{M}{M_0} \right)^{2/3} \left( \frac{\mu}{M_0} \right) \left( \frac{\nu}{100 Hz} \right)^{2/3}$$

(2)
where $R$ is the distance from hearth, $M, \mu, M_0$ are the total and reduced mass of the star system and the solar mass; $\nu$ is the frequency of the emitted radiation. From Eq. 2 is evident that the binary coalescing events have very high intensity.

Asymmetries in the mass distribution of neutron stars are believed to produce GW emission. If $\varepsilon$ is the value of the ellipticity then the strength of the GW emitted should be:

$$h = 10^{-23} \varepsilon \left( \frac{\nu}{10Hz} \right)^2 \left( \frac{10kpc}{R} \right)$$

(3)

All the contribution coming from each star cannot be summed coherently due to the unknown parameters of the stars, namely the rotating frequency, the initial phase, the direction in sky etc. For this reason a non-linear analysis, which takes into account of the Doppler shift due to Earth motion, has been proposed to perform the detection of the signal coming from the whole ensemble.\textsuperscript{5}

The last expected GW source is the Relic radiation. This is the background radiation produced soon after the Big Bang event. The information embedded inside this radiation are those that they were $10^{-44}$ sec after the Big Bang. The detection of this stochastic background will give a picture of the real status of the Universe at the Planck Time.

3. **VIRGO Detector**

VIRGO is a laser interferometer with 3 km long arms contained in long tunnels. The optical path is increased with the use of Fabry–Perot cavities and the effective optical path will be about 100 km. Using digital feedback loops, the interferometer will be kept on the dark fringe. This operation will permit the power recycling of the laser light improving the sensitivity of the VIRGO antenna (see Fig. 1).

The shot noise and the scattered light noise determine a change in optical phase without a real displacement of the mirrors, while the seismic noise and the thermal one really change their position. Stringent requirements on the fluctuation of the laser frequency\textsuperscript{5} is needed to reduce the inducted phase noise. Strong requirements has been put on the mirror quality to increase the finesse of the long arms Fabry–Perot cavities. Very strong requirements\textsuperscript{7} on the vacuum level in the pipes and towers are needed to reduce the noise induced by the change in the refractive index and the pollution on the optical components surfaces. Moreover the use of special baffles along the vacuum pipe will reduce the noise contribution due to the scattered light to a negligible level.\textsuperscript{7} Even if these phase noises are important because they affect the entire operating bandwidth of the interferometer, they are expected to become negligible in the low frequency region, below 500Hz, where thermal and seismic noise are both higher than shot noise. For this reasons in the following sections VIRGO seismic isolation system will be described in some details, and an R&D project for the study of thermal noise will be presented.
4. VIRGO Seismic Isolation System.

The seismic noise at the ground level is well described in the frequency region of interest by the following formula:

\[ \tilde{x}(\nu) \simeq \frac{10^{-6}}{\nu^2} \frac{m}{\sqrt{Hz}} \]  

(4)

where \( \tilde{x}(\nu) \) is the square root of the power spectral density of the horizontal ground vibration\(^a\). The expected displacement of the mirrors, due to a GW signal at 10 Hz, is about \( 10^{-18}m \), at this frequency the displacement induced by the seismic horizontal noise alone is about \( 10^{-8}m \), so it is clear that the VIRGO seismic noise attenuation required to detect the GW signal has to be \( 10^{-10} \). Even if from the theoretical point of view a vertical displacement of the mirror does not affect the laser beam, unavoidable mechanical couplings make necessary also a noise reduction of this component. An attenuation of horizontal and vertical component of seismic noise will be obtained with the use of complex both active and passive isolation system called Superattenuator (SA, Fig. 2).

The working principle of the passive part of the SA is simply related to the mechanical filter action of a pendulum. If an horizontal noise affects the suspension point of a simple pendulum then, for frequency greater than its resonant frequency \( f_{0,i} \), the displacement transmitted to the suspended mass is attenuated by the factor \( f_{0,i}^2/f_2^2 \). With this effect in mind a cascade of pendula can be used to reduce passively the seismic noise. With an N-fold pendulum cascade with \( f_{0,i} \) resonant frequencies, a total reduction of \( (\Pi f_{0,i}^2)/f_2^{2N} \) can be obtained for frequency larger than the chain resonant modes.\(^b\) In order to reduce vertical seismic noise the same principle can be used if it is possible to build up a cascade of vertical oscillators; this can be obtained by replacing each stage of the SA with a special mechanical filter exhibiting the elasticity in the vertical direction.\(^c\) The crucial point is to lower the chain resonance frequency as much as possible to achieve the required attenuation down few hertz.

4.1. VIRGO Mechanical Filters.

The general characteristics of this system will be described together with the solution adopted for the creep problem that arise with this solution. The interested reader can refer to\(^d\) for detailed discussion about this topic. The vertical oscillators are obtained clamping the suspension point of each intermediate stage to cantilever metal blades. Each blade is pre-bent to a rest position in such a way that, with load, it returns to the horizontal position. The very high stiffness value needed for the payloads suspension will be reached, but this fact is contrasting with the requirement of resonant frequency as low as possible. It is easy to understand that two magnets face in repulsive configuration can generate an elastic force that

\(^a\)The vertical component is of the same order of magnitude
\(^b\)The attenuation factor for VIRGO SA becomes, from about 30Hz on, \( 10^{-2} \) due to higher filter’s mechanical resonances.
pertain to a *negative* stiffness value, with a suitable choice of magnetic fields a very high value of this stiffness can be obtained. So the blades and the magnetic antisprings can be used together to obtain a vertical cascade of oscillator with resonant frequencies less than about 2.5 Hz\textsuperscript{11} allowing to start the GW detection from about 4Hz.

Two problems arise from this solution: the dependency of the vertical resonant frequency with the temperature due to the change of magnetic stiffness, and the creep phenomenon of the pre–bent blades. A 1°C temperature change will produce a vertical resonance shift of about 15mHz, making the SA mechanical response dependent on temperature. This fact could create troubles in the electromechanical control of the chain, if temperature changes with time. The creep phenomenon, namely a micro change in the metal structure, produces a sort of mechanical ”shot noise” on the mirror, limiting the antenna sensitivity. With the common steel C70 a displacement of few tens of μm/day in the vertical position of the far end of a blade has been observed. A solution to the creep problem can be achieved using different material for the blades. VIRGO will use MARAGIN steel, a material which allows to build blades with a stability of less than 1μm/day under the nominal stress. This material will reduce the noise induced by the creep phenomenon to a very low level.

4.2. The Preisolator Stage and the Active Damping

The preisolator stage has been introduced to improve the SA performance reducing as much as possible its main resonant frequencies. Other benefits has been produced by this solution and they will be discussed here. The preisolator stage consists in an inverted pendulum (IP) in which soft flexural joints give the mechanical restoring forces (see Fig. 2). The main resonant frequency is well described by:

\[ f_{ip} = \frac{1}{2\pi} \sqrt{\frac{K}{M} - \frac{g}{L}} \] (5)

where \( K \) is the joints stiffness while \( M, L \) and \( g \) are, respectively, the total mass, the legs length and the gravity acceleration. It is worth noting that this formula assumes that the legs mass is negligible with respect to the top table one. VIRGO final prestabilization stage will have 6 meter long alluminium legs with a steel top circular table satisfying this assumption. A more complete treatment of the IP can be found elsewhere.\textsuperscript{12} From Eq. 5 is evident that the resonant frequency can be tuned with suitable choice of the total mass; in this way a main frequency of 30mHz can be easily achieved for the translational degree of freedom. The very soft stiffness of the IP will give another important improvement to the SA performance: a softer movement of the suspension point can be easily and noiselessly obtained with magnet–coil pairs actuatuors instead of stepping motors. Even if the IP attenuates the horizontal component of the seismic noise, it is harmfull against a tilt of the ground. Tilt seismic noise, in fact, will produce a motion of the suspension point that cannot be easily controlled neither mechanically nor electronically due to the amplification produced by the legs length and gravity effect.\textsuperscript{12}
To simplify the control of the tilt noise a mechanical element has been added: the rigid ring at the bottom (see Fig. 2). The IP can move in the 3 directions \((x, y, z)\) and it can rotate around each principal axis \((\theta_x, \theta_y, \theta_z)\). In the ideal case two important facts can be noted: i) if an acceleration sensor is put on the IP then all the DOF will contribute to the measurement; ii) the tilt of the ground will produce a torque on the IP structure. By putting two orthogonal angular accelerometers on the rigid ring, it is possible to obtain (see Fig. 3) error signals that are directly proportional to the tilt of the ground; this feedback signals will go on the PZT actuators mounted below the ring, making the necessary restoring torque to the system. In this way, in principle, all the upper sensors, that are used to control the other DOF, will be not affected by this loop. The control of the SA has been reduced to the solution of two independent control loops, one at the bottom the other at the top of the prestabilization stage. Summarizing the prestabilization stage has given to the seismic isolation system the following advantages:

- improve the passive operation mode by lowering the main resonant frequency of the SA;
- obtain the required dynamic of the suspension point by using electromagnetic forces instead of stepping motors;
- simplify the active control of the tilt seismic noise.

5. The Thermal Noise R&D Project

The thermal noise is related to the dissipation in the system through the Fluctuation–Dissipation theorem, which states that stochastic forces arise with spectral density: \( \hat{F}^2(\omega) = 4k_B T R(\omega) \) where \( R(\omega) \) is the real part of the mechanical impedance, \( k_B \) and \( T \) are, respectively, the Boltzmann constant and the temperature. Due to the vacuum operation the dissipation is coming from the internal friction in the suspension materials. The dominant contribution\(^{13} \) to the dissipation in the VIRGO SA is given by the last filter stage, called marionetta. To improve the VIRGO sensitivity below 500Hz one special suspension could be built in order to create a facility for the study of thermal noise and materials.\(^{14} \) The read out system for this experiment will be a flat–flat cavity with very high finessse and very small length. The system is sketched in Fig. 4. A VIRGO Superattenuator with a special last stage will be built, to this special marionetta two mirrors are mounted to build a flat–flat Fabry–Perot cavity. This mechanical apparatus is used together with the control active system discussed in the above sections. A laser will be suspended to an input bench with two stage pendulum. All the system will work under vacuum. The technique used for the laser stabilization will be the same as in the VIRGO experiment. It is interesting to evaluate the effect of the noise induced by the stabilization cavity on this single–arm interferometer. Let us suppose to lock the laser to a stabilization Fabry–Perot cavity with finessse \( F_1 \), length \( L \). Let \( \nu_0 \) the laser
frequency, then for this system $\Delta \nu$ is given by:

$$\Delta \nu = \nu_0 \left[ -\frac{\delta L}{L} + \frac{1}{\omega_0 \tau} \sqrt{\frac{h \nu_0}{W_1}} \right]$$  \hfill (6)

where $\delta L$ is the variation of $L$, $\tau = \frac{2F_1 c}{\omega_0}$ is the cavity storage time and $W_1$ is the power on the photodiode. If $d$ and $F_2$ are, respectively, the length of the measurement cavity and its finesse, then the minimum $\Delta \phi$ measured by the single-arm interferometer to the first order in $\delta \nu$ and $\delta d$ is:

$$\Delta \phi = 2 \pi (\nu_0 + \Delta \nu) \cdot \left( \frac{d + \delta d}{c} \right) \approx 2 \frac{\pi}{c} \left( \nu_0 \cdot d + \Delta \nu \cdot d + \delta d \cdot \nu_0 \right)$$  \hfill (7)

The quantity of interest is $\delta d$ that measures the change of the measurement cavity length; from the above relationships, introducing also the shot noise of the measurement's cavity we finally obtain the following limit for the sensitivity of the experiment:

$$\delta d \geq \left[ \left( \frac{d}{L} \right)^2 + \left( \frac{\delta L}{\nu_0} \right)^2 + \left( \frac{\lambda}{F_1} \right)^2 \frac{h \nu_0}{W_1} \right] + \left( \frac{\lambda}{F_2} \right)^2 \frac{h \nu_0}{W_2} \right]^{\frac{1}{2}}$$  \hfill (8)

where $W_2$ is the power in the measurement cavity. Looking at Eq. 8, it is clear that making the ratio $d/L$ very less than 1 a large reduction of the noise induced by the stabilization cavity can be achieved. In this way, the experiment sensitivity is essentially limited by the shot noise of the measurement's cavity and so, in principle, $\delta d \approx 10^{-18} \div 10^{-20} \frac{m}{\sqrt{Hz}}$ can be measured.

6. Conclusion

A big challenge of gravitational wave interferometers is the low frequency detection of GW signals. Below 500Hz the main contribution to the noise, after the elimination of seismic noise, is the thermal one. The VIRGO isolation system composed by the SA and a control strategy for the active damping of the seismic noise down to few Hertz has been presented. An R&D project for the improvement of VIRGO sensitivity by reducing thermal noise is sketched and, perhaps, can be realized.

References

7. VIRGO: Final Design (1997)
9. J. Y. Vinet and V. Brisson, VIRGO internal note PJT00-011
11. M. Beccaria et al. *Extending the VIRGO gravitational wave detection band down to a few hertz: metal blade springs and magnetic antisprings*”, March 1997 submitted paper to NIM, also published as VIRGO internal note VIR-NTS-1390-068.
14. M. Beccaria et al. “*The measurement of the displacement noise of mirrors suspended as in the VIRGO antenna to improve the low frequency antenna performance*” submitted paper March 1997 to Physical Review, also published as VIRGO internal note VIR-NOT-PIS-1390-061, 1996
Figure 1: The VIRGO optical scheme

Figure 2: The VIRGO Superattenuator. The Prestabilization Stage, the intermediate filter stages and the rigid ring are also visible. See text for detailed discussion.
Figure 3: Control strategy: schematic position of sensor and actuators on the prestabilization stage

Figure 4: Experimental Setup for the thermal noise measurement
ABSTRACT

We report on the first run of the ultracryogenic gravitational wave detector AURIGA. Up to now, the experimental goal has been to measure the noise of the antenna, and to perform diagnostic tests on the cryogenics, the transduction chain, the calibration procedures, the data acquisition and analysis system, and the mechanical suspensions. The first results show that the antenna noise is very close to thermal at liquid helium temperatures. The operating temperature of the bar reached 0.14 K. We present also preliminary results of a room temperature experiment which demonstrate the capability of detecting the arrival time of an impulsive signal with resolution well below 1 ms. In this respect, we briefly discuss the advantage of widening the effective frequency bandwidth of the detector by means of optimized transduction schemes.

1. Introduction

The AURIGA collaboration\(^1\) started its experimental activity in 1990 at the INFN National Laboratories of Legnaro, Italy. By June 1995 we were able to begin the first cryogenic run of the detector, which lasted about one year and has been dedicated to measure the noise of the antenna and to perform diagnostic tests on the cryogenics, the transduction chain, the calibration procedures, the data acquisition and analysis system, and the mechanical suspensions. The antenna is a 2.3 t Al5056 bar, equipped with a capacitive resonant transducer coupled to an internal SQUID amplifier and cooled by a \(^3\)He – \(^4\)He dilution refrigerator. AURIGA and the other INFN ultracryogenic detector NAUTILUS,\(^2\) in their present configuration have similar expected sensitivities to broad-band gravitational wave bursts, i.e. a minimum detectable amplitude \(h_{\text{min}} \sim 3 \times 10^{-19}\) with an effective bandwidth of...
about 1 Hz. In 1997 we plan to operate AURIGA at this level of sensitivity and to begin a coordinate coincidence program with NAUTILUS and the other operating cryogenic detectors, ALLEGRO, EXPLORER and NIOBE. In order to maximize the probability of detecting in coincidence, the orientations of these detectors are parallel. The plan of the paper is as follows: the Section 2 deals with the main specific features and performance of the AURIGA detector; in Section 3 we present the antenna noise measurements during its first cryogenic run, and the results of a room temperature experiment to measure the arrival time of a burst excitation on the antenna; in Section 4 we discuss some perspectives for future improvements.

2. The AURIGA Detector

2.1. The Cryostat

The general designs of the liquid Helium cryostat, the mechanical suspensions and the resonant displacement transducer are derived from those of NAUTILUS. However, the development of the present configuration of AURIGA involved the implementation of specific features, which gave satisfactory results in the diagnostic tests. As for the cryostat, the most relevant ones are the set-up of i) the internal mechanical suspensions and ii) the $^{3}$He – $^{4}$He dilution refrigerator.

i) To maximize the vibration isolation, the elastic rods of each stage of the cryogenic suspensions have been equally tensioned, the room temperature stacks of rubber disks have been loaded by suitable lead masses, and the dilution refrigerator include mechanically soft pipelines and thermal links. The resulting mechanical attenuation of the internal suspensions is $\sim 245 \text{ dB}$ at the resonances of the bar and transducer, and is not affected by the presence of the dilution refrigerator. This figure should provide sufficient vibration isolation for the expected sensitivity of the detector under normal vibrational noise. In fact, the antenna was usually not excited during activities of cryogenic maintenance in the $1 - 10 \text{ K}$ energy range. Moreover, the vibrational noise related to human activities can be regulated, since the detector is housed in a dedicated building and rests on a massive concrete platform separated from the other foundations of the building.

ii) To improve the cryogenic availability, the dilution refrigerator is equipped with twin condenser lines and additional cold traps. However, the overall availability of the detector was satisfactory only during the first six months of operation at liquid Helium temperatures. Subsequently, the cryogenic working point has been lost several times due to plugging of the liquid Helium line which refills the 1 $K$ pot of the dilution refrigerator from the main liquid Helium tank. Unfortunately, when the cryostat is operating, this line is not accessible and it can be only partially heated to evaporate condensed impurities. These difficulties caused the dilution refrigerator to perform properly only for 10 days, when the bar was operated at 0.14 $K$. Afterwards, we warmed up the detector at room temperature to purify the cryostat. For the next run of AURIGA, we expect to overcome the plugging problems by using purer liquid Helium, now available from a liquefier installed at
INFN Legnaro, and by providing a second transfer line, which will be removable and will operate in parallel to the first.

2.2. Calibration and Signal Amplification

The specific features of the AURIGA signal line include a cryogenic switch that can connect the transducer either directly to an external test port or to the internal d.c.SQUID amplification chain. Up to now all measurements have been made through the test port with room temperature electronics both for calibrating the detector and for monitoring the antenna noise. The absolute calibration procedure is carried out by measuring the electric impedance of the mechanical resonances as seen from the test port. Then, the direct noise temperature of the bar-transducer system can be estimated by comparing the Nyquist prediction of the thermal noise of these impedances with the measured antenna noise (see Sect. 3).

The internal d.c.SQUID amplification chain includes an impedance matching transformer whose primary inductance is \( L \approx 5 \) H. It has been developed\(^6\) to obtain very low losses device; in fact, the resonator consisting of the primary inductance and the transducer capacitance has a quality factor of the same order of those typical for the mechanical resonators, \( Q \approx 10^6 - 10^7 \). In principle this feature allows to tune the \( LC \) resonator to the antenna resonances without worsening their quality factors, an opportunity which will be useful for future improvements in bandwidth and sensitivity of AURIGA.\(^8\) In the present configuration of the detector, the \( LC \) oscillator is not tuned to the mechanical resonances in order to keep the system as simple as possible: even an optimal tuning would affect only marginally the expected performances, due to the comparatively low energy resolution of the commercial d.c.SQUID used, \( \sim 5 \times 10^3 \) h.

2.3. Data Acquisition and Analysis

The fast data acquisition system of the AURIGA detector is described elsewhere.\(^7\) The sampling of the antenna signal is performed by a 5 KHz, 18 effective bits A/D converter which is synchronized to UTC within 1\( \mu s \) by means of a GPS clock. The raw data (\( \sim 3 \) GB/day) are then archived on cassettes and analyzed on-line by a workstation. The analysis procedure is fully numerical and its main stage consists of an adaptive optimal Wiener filter matched to a \( \delta \) signal at the antenna input. New features of events above threshold can be detected, including the arrival time of impulses,\(^7\) the optimal reconstruction of the signal shape within the bandwidth of the detector,\(^10\) and a consistency test\(^9\) which allows to distinguish between direct mechanical excitations of the antenna and other kinds of spurious excitations of electrical or mechanical origin. This is accomplished by means of a goodness of the fit test in respect to the measured characteristics of a calibrating excitation of the antenna. This test can effectively reject some classes of spuria events in a single detector. Moreover, since the acquired data maintain full spectral information up to 2.5 KHz,
the behaviour of the detector outside its bandwidth can be monitored, a fact that is useful for diagnostic purposes.

3. Experimental Results

3.1. Cryogenic Run

During the first five months of the run, the bar was operated between 6 and 10 $K$, cooled by He exchange gas in the vacuum chamber inside the liquid Helium shield at a pressure $\sim 10^{-5}$ mbar. In subsequent months, the antenna has been cooled to $\simeq 2$ $K$ by pumping out the exchange gas and by operating the 1 $K$ pot stage of the dilution refrigerator. Due to the plugging problems, the cryogenic working point has not been sufficiently stable to allow a complete experimentation. Afterwards, we could operate the dilution refrigerator for about two weeks and the bar temperature was stable at 0.14 $K$. The average liquid helium consumption of the cryostat was about 90 liters/day. The typical bias field of the capacitive transducer was $\sim 0.3 \div 0.6$ MV/m and the electric charge showed a negligible leakage over 2 months of operation. In figure 1 we report the quality factors of the antenna-transducer resonances as a functions of the bar temperature; it is worth to notice that at 0.14 $K$ the Q-values are 4 and 6 millions respectively for the 913 Hz and 931 Hz modes.

The noise of the antenna has been measured by a room temperature FET ampli-

![Figure 1: mechanical quality factors of mode − , squares, and mode +, circles, as a function of temperature.](image)

fier connected to the transducer through the test port. The FET noise temperature
is $T_n \approx (0.10 \pm 0.05) \, K$ and its noise resistance is $R_n \approx (2.4 \pm 1.2) \, M\Omega$. The typical contribution of the amplifier to the antenna noise temperature in our experimental conditions has been $\sim 0.5 \, K$ from the wideband and $\sim 0.1 \, K$ from the narrowband noises respectively. Over a 31 day span while the antenna was kept at a thermodynamic temperature of $(6.5 \pm 0.5) \, K$, we could take data for 12.8 days. The acquisition was not continuous mostly because of the calibration measurements. The acquired data give average antenna noise temperatures $T_+ \sim 9.8 \, K$ and $T_- \sim 8.1 \, K$ for the + and - modes respectively, with a root mean square deviation of the data $\sim \pm 3 \, K$. Therefore, we can conclude that the antenna noise was quite close to a thermal behaviour. Moreover, the histograms made with the counts of the energy levels follow the expected Boltzmann distribution, apart from a few events per day. Also the energy innovation, as calculated by a Zero Order Prediction (ZOP) filter of the data, showed a Boltzmann distribution with effective temperatures $\sim 3K$, as can be seen in Figure 2.

During the operation of the 1 $K$ pot of the dilution refrigerator, the noise temperature of the antenna approached the thermodynamic temperature for 1.5 day, giving $\sim 3K$, and the effective temperatures given by the ZOP filter were $\sim 0.9 \pm 0.2K$ and $\sim 0.7 \pm 0.2K$ for the + and - modes respectively. During the last months of the run, we observed an excess of noise of the antenna of about $10^{100} \, K$, which we have not been able to diagnose because of the cryogenic instabilities. It is worth to notice that the normal cryogenic maintenance, including the operation of the dilution refrigerator, did not affect the noise at this level.

3.2. Measure of the Arrival Time

We tested the timing capability of the data acquisition and analysis system developed for AURIGA in a room temperature experiment. A room temperature antenna read by a capacitive transducer and a low noise FET amplifier can easily have a effective bandwidth up to $\sim 10 \, Hz$. This condition is similar to that expected for future configurations of ultracryogenic detectors. Such a bandwidth allows to estimate the arrival time of an impulsive excitation with a reasonable high Signal to Noise Ratio ($SNR$). In fact, for narrowband detectors the measurement of the arrival time $t_a$ can be separated in two part: $t_a = t_\phi + kT_0$, where $t_\phi$ is the phase part, i.e. a fraction of the semi-period $T_0$ of the detector, and $k$ is an integer. As for the estimate of the phase part, the uncertainty is given by $\sigma_{t_\phi} \approx T_0/(\pi SNR) = 174\mu s/\, SNR$. As for the uncertainty on $k$, in our experimental configuration with a bandwidth of $4 \, Hz$ for each mode, we found that it is inversely proportional to $SNR$ and is $\leq 1$ for $SNR \geq 20$. Above this threshold the only residual uncertainty on $t_a$ is that on the phase part, allowing a timing error $\leq 9 \, \mu s$. The present limiting accuracy of the system due to synchronization with UTC is $\sim 1 \, \mu s$.

The expected performances on timing will improve significantly if the bandwidth gets larger than the frequency separation between the antenna-transducer modes. In this case a $SNR \geq 8$ will be enough to neglect the uncertainty on $k$. 

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Figure 2: histograms of the energy distribution measured for mode + at a bar temperature of $\simeq 6.5K$ during 3 days, upper, and of $\simeq 2K$ during 1.5 day, lower. Open circles stand for the direct noise of antenna; black triangles for the energy innovation as calculated by the ZOP filter.
4. Conclusions

Future improvements of the AURIGA sensitivity rely mainly on the implementation of transduction and amplification schemes with a better energy resolution. This is at present quite far from the standard quantum limit, since AURIGA is equipped with a commercial d.c.SQUID amplifier that showed an energy resolution of about $5 \times 10^3 \ h$ in separate tests. This performance makes somewhat marginal the effort of cooling the bar to $\sim 0.1 \ K$, since operation of the bar at $\sim 1 \ K$ should still ensure a similar sensitivity. With a $\sim 100 \ h$ SQUID and without other changes of the present configuration, the g.w. pulse sensitivity of AURIGA would approach $h_{\text{min}} \sim 3 \times 10^{-20}$. Moreover, an effective bandwidth of $\sim 50 \ Hz$ could be reached by optimizing the transducer mass and by tuning the $LC$ resonator made of the impedance matching stage. Further improvements towards the quantum limit would need both a decrease of SQUID energy resolution and a proportional increase of the ratio between the quality factors of resonators and their thermodynamic temperature, $Q/T$, from the presently achieved values of $\sim 10^8$.

A bandwidth greater than $10 \ Hz$ is crucial to estimate efficiently the arrival time of impulsive signals, as we have demonstrated in room temperature experiments.

The timing measurement with submillisecond resolution allows to estimate the signal time delays among detectors located at separate sites. With at least three detectors, the direction of propagation of the incoming gravitational wavefront can be determined. In this coincidence analysis one can get a strong evidence of a gw burst if the delays are consistent with a wavefront travelling at the speed of light.

In our opinion, the resonant detectors will be competitive for gravitational wave astronomy if their sensitivity will approach the quantum limit and their bandwidth will allow high resolution measurements of pulse arrival time. At that level, the role of resonant detectors is complementary to that of interferometers under construction, since the firsts can test specific properties of the Riemann tensor, such as tracelessness, and therefore allow a more confident detection of gravitational wave signals such as bursts, stochastic background and periodic sources at frequencies $\sim 10^8 \ Hz$.

References

A B. A. E. - SQUID TRANSDUCER FOR THE RESONANT
GRAVITATIONAL WAVE ANTENNA

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ABSTRACT

The crucial problem of the R&D program for the resonant antennas is the design
and construction of new transducers with higher efficiency and bandwidth.
Here we present the main characteristics of the Back Action Evading (BAE) trans-
ducer which we are developing in Rome. The system is based on the use of a
parametric capacitive transducer configured as a capacitive bridge coupled to a dc
Squid. We discuss the optimisation of the BAE configuration and we derive the
present limits.

1. Introduction

The aim of the gravitational wave experiments is to detect events that are at the
limit of the present technology. For example in the case of a detection of a burst of
duration $\tau_g$ performed at angular frequency $\omega_o$ with a resonant gravitational wave
antenna of mass $M_{eq}$, we have to detect an energy innovation in the detector which

$$E_s = (1/4) M_{eq} \omega_o^4 h_{TT}^2 \tau_g^2$$  \hspace{1cm} (1)

where $h_{TT}$ is the amplitude of the metric tensor perturbation computed in the TT
gauge. The minimum detectable signal is usually expressed in Kelvin by introducing
the effective temperature of the detector which corresponds to a Signal to Noise ratio
(SNR) equal to one.

$$E_s \geq k T_{eff}$$  \hspace{1cm} (2)

where $k$ is the Botzmann constant. For an event characterised by an amplitude
$h_{TT} \sim 10^{-21}$ and $\tau_g \sim 1 \text{ ms}$ (an optimistic scenario of SuperNova collapse in the
Virgo cluster), the typical values of the equivalent mass and angular frequency of
the detector $M_{eq} \approx 1000 \text{ kg}$, $\omega_o = 2\pi 1000 \text{ rad/s}$, we get an effective temperature

$$T_{eff} \approx 5 \cdot 10^{-6}.$$  \hspace{1cm} (3)

$T_{eff}$ is related to the antenna fluctuations. In the case of a system dominated
by the thermal noise we have

$$T_{eff} = (T \omega_o / Q) \Delta t$$  \hspace{1cm} (4)

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where $\Delta t$ is the averaging time of the measurement, $T$ the physical temperature of
the system and $Q$ the overall quality factor of the detector which is directly related
to the dissipation of the system.

On this device the intrinsic limit of the detector accuracy is due to the quantum
nature of the detector and it is given by the following formula

$$\hbar \omega_0 = kT_{eff}$$

were $\hbar$ is the reduced Planck constant. It follows that the quantum limit is ap-
proached when

$$(T/Q)\Delta t = \frac{\hbar}{k} \simeq \frac{7}{12} \cdot 10^{-12} \quad K \quad s$$

Values as $T \simeq 10^{-2}$ K ( see for example the Italian antennas NAUTILUS and
AURIGA), $Q \simeq 10^8$ (the intrinsic Q of the Australian detector NIOBE) and $\Delta t \simeq$
$10^{-2}$ s are realistic targets of the present R&D research of the resonant detectors.
It results that

$$(T/Q)\Delta t \simeq 1 \cdot 10^{-12} \quad K \quad s$$

which is a value in the same range of the quantum limit. This implies that the
detector is treated as a quantum device and the strategy of measurements has to
be revised. For a free mass in equilibrium with a thermal bath, as in the case of
the free falling mirrors of the interferometer, similar consideration can be done. The
quantum limit is given by

$$(T/\tau)\Delta t^2 = \frac{\hbar}{k} \simeq 7 \cdot 10^{-12} \quad K \quad s$$

where $\tau$ is the relaxation time of the particle in the bath. Let us notice here that as
for the resonant antennas, the R&D of the interferometric detection of gravitational
waves is centred on the development of systems with very low elastic losses and large
bandwidth (short $\Delta t$). The quantum limit is more distant than for the resonant
detector case, but it is again the implicit target of the detector sensitivity ad it is
tightly related to the reduction of the dissipation mechanisms.

Thus, the intrinsic limit of a gravitational wave antenna based on standard de-
tection techniques is directly related to the quantum nature of the device. Although
this quantum barrier is not yet reached by present detectors, it has been investigated
the possibility to define alternative strategies of measurements that circumvent the
limit based on the Heisenberg principle (see the reference\(^1\) for a complete review ).
These techniques consist in coupling one component of the complex amplitude of the
oscillator (system) with the homologous one of an other harmonic oscillator, which
is the actual measurement apparatus. In quantum regime, it can be shown that the
uncertainty introduced into the system, when one component of the apparatus is
monitored, is driven onto the component of the system that is not measured. In the
past we have studied experimentally the classic analogy of such a system: a Back
Action Evasion transducer coupled to a classic harmonic oscillator(Cinquegrana et
al.\(^2\)). In such conditions the BAE technique is expected to prevent the measurement

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from the Back-Action of the measuring apparatus. This theoretical predictions was verified by our test performed on a BAE system at the liquid helium temperature (4.2 K).\textsuperscript{3} We performed also measurements on the resonant gravitational wave antenna ALTAIR equipped with a BAE transducer and a FET amplifier. The results of the experimental test are in agreement with our model predictions.\textsuperscript{4} On these bases we started the design and construction of a second generation device which we present here. In the following we present this experimental apparatus which should permit quantum level sensitivities in monitoring the motion of a macroscopic harmonic oscillator.

2. The BAE-SQUID Model

The theoretical model of the BAE transducer was extensively treated in the case of a single mechanical oscillator.\textsuperscript{2} Later the treatment was extended to the case of resonant transducer + resonant antenna configuration.\textsuperscript{5} In the following we limit our consideration to the case of a single harmonic oscillator. This corresponds in practice to compute the device sensitivity of one normal mode of a gravitational antenna equipped with a resonant mechanical transducer.

The basic Back-Action-Evading scheme consists of two interacting harmonic oscillators, the mechanical oscillator with resonance frequency $\omega_m$ and the read-out apparatus with resonance frequency $\omega_e$. The electro mechanical coupling between the two oscillators is due to the time varying electric filed $E(t)$ which fix the relations among the four quadrature components of the mechanical displacement $x(t)$ and the electrical output $q(t)$, which is the charge variation in the capacitance due to the mechanical motion at $\omega_m$. A BAE measurement can be obtained if the field is

$$E(t)_{BAE} = E_0 \cos \omega_m t \cos \omega_e t$$

We consider the mechanical dissipation due to the damping of the mechanical harmonic oscillator and the Langevin stochastic force that describes the Brownian motion. We also consider the electrical losses of the read-out system.

The amplification system and the biasing field contribute with stochastic forcing terms on the electric oscillator. We include these terms in the motion equations and we have:

$$\ddot{x} + \frac{1}{\tau_m} \dot{x} + \omega_m^2 x + \frac{E(t)}{m} q = \frac{F_n(t) + F_s(t)}{m}$$

$$\ddot{q} + \frac{1}{\tau_e} \dot{q} + \omega_e^2 q + \frac{E(t)}{L} x = \frac{R_n(t)}{L}$$

where $\tau_m$ and $\tau_e$ are the mechanical and electrical decay times respectively. $F_n(t)$ is the Langevin stochastic force and $R_n(t)$ contains the back-action noise of the amplifier.

In our experimental apparatus the mechanical oscillator is the central electrode of a differential transducer. This configuration is crucial to reduce the effect of
the force, due to the biasing field, on the mechanical resonator. The noise coming from the biasing field sources which acts on the electric circuit, is reduced inserting the transducer in a balanced capacitive bridge. We express with $\eta_z$ the balance parameter. In the limit of a perfect balance ($\eta_z = 0$), the contribution of the biasing field generators and the related noise to the voltage output of the system is cancelled. The electric oscillator is given by the resonance network of the capacitive bridge and the central arm coil with inductance $L$. The transducer output signal $I(t)$ which flows through the inductance $L$, converted at the electric frequency $\omega_e$, is sent to the input of a dc SQUID. The SQUID is treated as a linear current amplifier because the output signals due to gravitational waves are very small to the respect of its dynamic range.

The sensitivity of a transduction system is the energy variation related with the minimal detectable displacement. It can be calculated in term of a signal-to-noise-ratio and then usually expressed using the effective temperature parameter $T_{eff}$ which is the energy variation measured when the signal-to-noise ratio is equal to the unity.

One of the most important parameters which affects the $T_{eff}$ is the noise coming from the amplifier which is usually expressed in terms of temperature $T_n = (S_{I_n}S_{V_n})^{1/2}/k_B$.

Unlike the non-back-action evading systems where the noise of the amplifier represents a lower limit on their sensitivity, in a BAE system this kind of noise is related to the effective temperature by the relation:

$$T_{eff} = T_n \frac{\omega_m}{\omega_e} \frac{1}{r},$$

where the parameter $r$ can be greater than unity and mostly depends on the balance $\eta_z$ and the biasing sources noise.

Here we report the explicit formula of the parameter $r$ and of the detector bandwidth $\Delta \nu_{opt}$:

$$r = \frac{1}{16} \frac{E_0^2}{\pi \omega_m \omega_e m L^2}$$

and

$$\Delta \nu_{opt} = \frac{I_G (H_b + H_c)}{2\pi}$$

$I_G$ is a complicated function of the various parameters of the system and we express it by using some auxiliary variables as $H_0$, $H_a$, $H_b$ and $H_c$. We have:

$$I_G = \frac{\pi}{\sqrt{(H_b + H_c)(H_a + 2\sqrt{H_0(H_b + H_c)})}}$$

with

$$H_0 = (S_{I_n}/S_{V_n})^{1/2} = Z_n^{-1}$$

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and
\[ H_a = \left( \frac{1}{4\tau_e^2} + \frac{1}{4\tau_m^2} \right) Z_n^{-1} + \left( \frac{Z_n}{2} + \frac{T}{T_n} \frac{1}{\omega_e^2 \tau_e C} \right) + \eta_e^2 \frac{E_o^2 d^2}{4kT_n} (S_a + S_\Phi) \right) \frac{1}{L^2} \] (17)

Here \( C \) is the transducer capacitance, \( d \) its gap. The other \( H \) parameters depend on \( S_a \) and \( S_\Phi \) which are the power spectral densities of the amplitude and phase noise of the bias sources. It results that this noise contribution is weighted by the balance factor \( \eta_e \).

\[ H_b = \frac{1}{16\tau_e^2 \tau_m^2} Z_n^{-1} + \left( \frac{Z_n}{2} + \frac{T}{T_n} \frac{1}{\omega_e^2 \tau_e C} \right) + \eta_e^2 \frac{E_o^2 d^2}{4kT_n} (S_a + S_\Phi) \right) \frac{1}{16\tau_m^2 L^2} \] (18)

\[ H_c = \frac{E_o^2}{16kT_n L^2} \left( \frac{kT}{m\omega_m^2 \tau_m} + \frac{(\eta_e \beta d \omega_e)^2}{64} \left[ S_a + S_\Phi / (4\omega_m^2 \tau_e^2) \right] \right) \] (19)

Provided that we want to operate close to the quantum limit, we express the sensitivity in terms of effective change of number of phonons at \( \omega_m \):

\[ \Delta N_{eff} = \frac{K_B T_{eff}}{\hbar \omega_m} \] (20)

In order to improve the sensitivity of the BAE transduction system we try to optimise its effective temperature with respect to several parameters such as the biasing field sources noise, the balance and the amplifier noise temperature. We have found that using a room temperature amplifier, the system could not work near the quantum limit even if the sensitivity achievable is far better than for a non-back-action evading system. For this reason we proposed to couple a dc SQUID to the BAE transducer to monitor the current in the inductive central arm of bridge.

3. The Experimental Apparatus

3.1. The Transducer

In the real transducer the mechanical oscillator consists of a resonator made of Al 5056 which has the shape of a double umbrella. It is a disc, with the outer diameter of 17.0 cm and thickness of 0.65 cm, fixed at the inner radius (0.75 cm) and vibrating in its first flexural mode at \( \sim 930 \) Hz with an equivalent mass of 0.341 kg. We recall here that we have constructed and tested several resonators having this geometry. At liquid helium temperature we obtained values of the mechanical merit factor up to \( Q_m \approx 8 \times 10^6 \). The limiting dissipation mechanism is generally due to the clamping system. In this case the resonator is clamped both up and down to two identical lids.

The transducer is configured as a push pull device in order to compensate the force due to the bias sources acting on the resonator. The residual effect depends on the geometrical asymmetry between the upper and lower active capacitance’s of the transducer. For this reason we check the dimensions and the planarity of the various pieces in the assembling phase with particular care. The metrology control is performed with an accuracy of 1 \( \mu m \) by means of a POLI DIAMOND measuring...
center which is available at the INFN Section of Roma 1. Applying this method we are able to obtain vacuum gap of the transducer capacitance down to \( d = 13 \mu m \). In order to keep the balance of the transducer also at low temperatures, the transducer is designed to be symmetric to respect of the plane of vibration (see Figure 1). In fact, the vibrating body is the central plate of the push-pull capacitor made by two other electrodes which are rings of equal size rigidly locked to the upper and the lower lid of the transducer. The electric insulation is given by PTFE washers. The upper and lower capacitances of the transducer are set up as two branches of the capacitive bridge. The bridge configuration is completed by two identical capacitors made by means of a third electrode, locked to both the transducer lids and set around the vibrating plate of the transducer. The residual unbalance of the capacitive part of the impedance is compensated by using a vacuum variable capacitance, driven by an electrical motor and a high resolution variable capacitance. Using such a system we got in the previous tests an electrical merit factor \( Q_e \approx 6 \times 10^3 \) and a balance \( \eta_z \sim 1 \cdot 10^{-6} \).2 The resistive part of the impedance of the bridge is decreased using super conducting wires for all connections and the pick-up coil. Values of the balance factor of this order allows to neglect the noise term due to the bias sources.

3.2. The SQUID

Sensitivities close to the quantum limit can be obtained by means of a low noise amplifier SQUID which is in use, for instance, on the EXPLORER gravitational wave detector.7 Unfortunately the electronic control unit of the SQUID of EXPLORER is conceived to work at lower frequencies than those we need for the BAE transducer. For this reason in our present prototype we are using a Quantum Design dc SQUID which amplifies signals in the frequency range of 200 kHz (around the electric resonance of BAE system). The system uses a conventional electronics with a bias square modulation at 550 kHz. Thus, the SQUID works in the usual feedback loop configuration which has the frequency cut-off in the range of 20 kHz. Although the device is noise mismatched in the 200 kHz range, its behaviour is still linear for weak signals in this frequency zone. We have performed a complete characterisation of the frequency response of the device in the range of 200 kHz and then we have measured its voltage and the current noise. This has been done by measuring the output noise of the SQUID in the open circuit configuration and by loading the SQUID input by a super conducting coil of 4 \( \mu H \). Then, we modelled the SQUID as a current amplifier with an input impedance purely inductive of 2 \( \mu H \). On the base of this assumption, we have derived the spectra reported in the following figure 2.

4. Discussion and Conclusion

We notice that the value of the current noise is low enough to limit the wide band noise of the BAE transducer and to keep the bandwidth of the device in the range of 100 Hz. Moreover, the voltage noise of this SQUID is well above the values that are expected for the SQUID of EXPLORER at 1 kHz. In principle the
Figure 1: Scheme of the new push-pull resonant transducer.
higher value of this noise source does not affect the performances of the transducer. In fact, it contributes mainly to the back action noise of the amplifier and this effect is beaten by the back action evasion configuration. On the base of this result we computed the expected sensitivity of our BAE transducer equipped with this commercial SQUID. If we assume realistic values for the parameters of the BAE-SQUID system: $d = 35 \, \mu m$, $E_O \sim 1 \cdot 10^7$, $Q_e \simeq 2000$, $S_a = 4 \cdot 10^{-21} \, Hz^{-1}$, $S_\Phi = 4 \cdot 10^{-19} \, Hz^{-1}$ and $\eta_e \simeq 1 \cdot 10^{-6}$, we get using the formulae (12), (14) and (20) a bandwidth of $\Delta \nu_{opt} \sim 100 \, Hz$, $T_{eff} \simeq 7 \cdot 10^{-6} \, K$ which corresponds to $\Delta N_{eff} \simeq 160$.

We stress that in this configuration both sensitivity and bandwidth are greatly improved by a factor $10^2$ with respect to the prototypes of the first generation.\(^4,3\) However, to achieve a quantum limited sensitivity, a further improvement of the noise matching of the SQUID control unit in the frequency range of interest is needed.

References

EVENT RATE, PARAMETERS ESTIMATION AND SIGNAL TO NOISE RATIO FOR COALESCING BINARIES AND SPHERICAL DETECTORS OF LARGE BANDWIDTH

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ABSTRACT

A brief introduction to the GRAIL project is given and its status. Transducers of large bandwidth are planned. If a bandwidth of $130 - 200 \text{ Hz}$ could be achieved, spherical detectors would become valid tools for analysis of coalescing binaries, and requiring novel applications of data analysis techniques. The number of observed cycles of a coalescing binary, the signal to noise ratio with matched filtering, the event rate, and the accuracies in the determination of the chirp mass, time of arrival and the amplitude are calculated herein, showing the relation to the bandwidth and the resonant frequency. Further, the probability of detection and false alarm improve when considering both windows at fundamental and first harmonic frequencies.

1. Introduction

Spherical detectors were lately reconsidered for their potential. GRAIL is a Dutch project that involves various groups from the Universities of Leiden, Amsterdam, Twente, Eindhoven and the NIKHEF Institute. More than twenty scientists are working on GRAIL in the feasibility phase, funded by FOM (Fundamentele Onderzoek der Materie). International cooperation is running with scientists of the major international groups. Considerations are now focused on the opportunity of building a prototype, possibly of $1.5 \text{ m}$ or other size. The final design for the large antenna appears to finalize towards the following features:

- $3 \text{ m}$ diameter of the sphere;
- $100 \text{ tons}$;
- large bandwidth of at least $130 \text{ Hz}$;
- resonant frequency not higher than $750 \text{ Hz}$;
- noise quantum limit of $4 \times 10^{-24} \text{ Hz}^{-1/2}$;
- operating temperature $10 \text{ mK}$;
- $Q$ factor about $10^6$;
- damping of spurious vibrations to a level of $5 \times 10^{-22} \text{ m}$.

The feasibility study has concentrated on the sphere material, the cryogenics, the suppression of vibrations, the suspensions, the vibration mode analysis, the transducers and the SQUIDS, the cosmic rays and finally on the analysis of the sources. The sphere material might be a Cu-Al alloy with a third non-magnetic element. Other solutions include Cu-Sn, Cu-Be and steel with high Cr content. Intensive
measurements of Q factors and heat capacity were performed. Computational, experimental and test set-ups have been realized for the suppression of vibrations, the sphere suspension and the vibration analysis. Inductive SQUIDS operating at 100 mK have been baselined. The background noise from cosmic rays may be so relevant that the large antenna need to be shielded by artificial means or set in a mountain, as shown by computations. A benchmark test is being prepared which will involve irradiation of a 150 mm sphere by the MEA accelerator and recording of the excited vibrations.

1.1. Transducers

For the transducers, two options are under consideration. The former is a chain consisting of a number of coupled oscillators. At Twente University the performance of multimode transducers has been simulated for systems with two, three, four and five masses interconnected with springs. It appears that one opportunity would be given by a three-mode transducer, in which the intermediate mass is of 130 kg and the final of 500 g for a bandwidth of 130 Hz. A larger bandwidth with ad-hoc arrangements is under consideration.

The research groups at Twente and NIKHEF also consider the employment of one resonant tapered body in which the mass is reduced continuously. This latter option presents the advantage to transport the vibrational energy in the fastest possible way.

2. Motivation

For this work, numerical values have been derived from the GRAIL project, but in order for the conclusions to be of general validity, we have parameterized bandwidth and resonant frequency. The relevance of coalescing binaries for spherical detectors of large bandwidth was first pointed out in terms of:

i) observeability of a number of cycles,
ii) applicability of matched filtering technique to the extraction of the signal from the noise,
iii) use of the coincidences between the first harmonic and the fundamental frequency windows, to increase the probability of detection, technique of double passage,
iv) autonomy of the detector in the determination of source location and its distance, and consequently, as originally suggested for laser interferometers, featuring of coalescing binaries as standard candles and their role in the determination of the Hubble constant.

Part of the implications of iv), although solely for narrow band detectors and using thus the first harmonic, were later analyzed. Therein, it is shown that measurements of the gravitational wave signal from a binary by a single spherical antenna in two frequency bandwidths, corresponding to the fundamental and the
first harmonic frequencies, can determine the chirp mass, the orbital orientation, and the position of the binary in the sky, apart from antipodal ambiguity.

In this work we concentrate upon points i), ii), iii), and investigate the detectability of the gravitational wave signal from a binary with matched filters. We also determine the event rate and examine the accuracy of the determination of the parameters of the binary, chirp mass, amplitude, time and phase of arrival. The first two parameters were found using the first harmonic and without estimation of accuracy. The full version of this work has shown in greater detail the novel concept of resonant antenna acting as large band instrument.

### 3. Analysis

#### 3.1. Method and Assumptions

We stress that the mechanical bandwidth of the resonator is very narrow, since related to the high quality factor, but for the calculation of the signal to noise ratio what matters is the bandwidth of the transducer, see e.g. The bandwidth is given by \( B = 2f_r \sqrt{\mu} \) where \( \mu \) is dependent from the ratio of masses of the transducers and the sphere.

The signal to noise ratio for a narrow band spherical resonator has been calculated, but in case of a large band resonator it is known that the optimal signal to noise ratio is given by matched filtering:

\[
\left( \frac{S}{N} \right)_{mf} = 4 \int_0^\infty \frac{|\tilde{h}|^2}{S_h(f)} df
\]

(1)

where \( \tilde{h} \) is the Fourier transform of the signal and \( S_h(f) \) is the one-sided spectral density of the noise.

For sinusoidal signals, matched filtering improves the signal to noise ratio with respect to bursts of the same amplitude roughly by a factor equal to the square root of the number of cycles in the signal within the bandwidth of the detector.

When the waveform of the signal is known in terms of a number of undetermined parameters, the method is to maximize the correlation with respect to these parameters. The parameters of the template that give the maximum correlation are the estimators of the parameters of the signal. For high signal to noise ratio, the covariance matrix of the estimators can be approximated by the inverse of the Fisher information matrix.

In the Peters-Mathews model, the number of gravitational wave cycles \( N_{gw} \) from a frequency \( f_{in} \), lowest detector frequency, to a frequency \( f_{out} \), highest detector frequency, is given by

\[ a \quad \text{Quadrupole, adiabatic motion, circularized orbit, slow motion, weak field and point masses approximations are used in this model.} \\
\[ b \quad \text{The units are chosen such that } G = c = 1. \]
\[ N_{gw} = \frac{\Delta \phi_{gw}}{2\pi} = \int_{t_{in}}^{t_{out}} f dt = \int_{f_{in}}^{f_{out}} \frac{f}{df} df = \frac{1}{32\pi^{8/3}} \frac{1}{M^{5/3}} \left( \frac{1}{f_{in}^{5/3}} - \frac{1}{f_{out}^{5/3}} \right) \] (2)

where the chirp mass \( M \) is \((\mu M^2/3)^{3/5}\), \( \mu \) and \( M \) being the reduced mass and the total mass of the binary respectively.

The number of cycles increases as the chirp mass decreases. Since the amplitude of the signal increases as \( M^{5/3} \), \((S/N)_{mf}\) increases as \( \sqrt{N} \) with respect to \( S/N \) of bursts of the same amplitude, (2) indicates that the number of cycles increases when the difference \( f_{in} - f_{out} \) increases and when the initial frequency \( f_{in} \) decreases. For highest signal to noise ratio it is best to have the largest bandwidth and highest sensitivity at lowest frequencies as possible. These conclusions apply for the case of an inspiralling binary to any detector.

We assume a resonant frequency \( f_r \) between 600 and 1000 Hz and a bandwidth \( B \) equal to \( f_r/3^{11} \) or \( f_r/6 \), as the GRAIL project is planning large bandwidth transducers.

For the noise the following simplified model of the noise is assumed:

\[
S_h(f) = \begin{cases} 
\infty, & f \leq f_r - B/2, \\
S_0, & f_r - B/2 \leq f \leq f_r + B/2, \\
\infty, & f \geq f_r + B/2 \end{cases}
\] (3)

where \( S_0 = (4 \times 10^{-24}Hz^{-1/2})^2 \) corresponds to the noise at the quantum limit of GRAIL.

For the \((S/N)_{mf}\) and rms error of the estimators of the parameters, the Fourier transform of the chirp signal, in the stationary phase approximation, is given by:

\[
\tilde{h} = \tilde{A} f^{-7/6} \exp\{2\pi ft_a + \phi_a + \pi/4 + \frac{5}{48}a(f; f_a)k\} \] (4)

where the amplitude parameter \( \tilde{A} \), the function \( a(f; f_a) \), and the mass parameter \( k \) are given by:

\[
\tilde{A} = \left( \frac{5}{24} \right)^{1/2} \frac{1}{\pi^{2/3}} \frac{M^{5/6}}{R} \] (5)

\[
a(f; f_a) = \frac{9}{40} \frac{1}{(\pi f)^{5/3}} + \frac{3}{8} \frac{\pi f}{(\pi f_a)^{8/3}} - \frac{3}{5} \frac{1}{(\pi f_a)^{5/3}} \] (6)

\[
k = \frac{1}{M^{5/3}} \] (7)

The parameter \( t_a \) is the time of arrival that can be freely chosen; \( f_a = f(t_a) \) and \( \phi_a \) are the corresponding frequency and phase respectively. Thus the gravitational wave signal from a binary in quadrupole approximation can be characterized by 4
parameters: \( \tilde{A}, \mathcal{M}, t_a, \phi_a \). In this approximation the \((S/N)_{mf}\) and the correlation function \( C \) are given by:

\[
\left( \frac{S}{N} \right)^2_{mf} = 4 \tilde{A}^2 \int_0^\infty \frac{df}{f^{7/3} S_h(f)}
\]

\[
C = 4 \tilde{A} \tilde{A}_T \int_0^\infty \cos \left[ 2\pi f (t_a - t_T) + (\phi_a - \phi_{aT}) + \frac{5}{48} a(f; f_a)(k - k_T) \right] \frac{df}{f^{7/3} S_h(f)}
\]

where the subscript \( T \) denotes the parameters of the matched filter (template).

3.2. Number of Cycles, Signal to Noise, Event Rate and Binary Parameters

For a typical neutron stars binary \( c \) with members of 1.4 solar masses, we have calculated the number of observable cycles and the signal to noise ratio, plotted versus different bandwidths and resonant frequencies.\(^6\)

The \((S/N)_{mf}\), assuming the distance to the binary of 45 Mpc, is 7.4 (\( B = 250 \) Hz and \( f_r = 750 \) Hz) or 5.2 (\( B = 125 \) Hz and \( f_r = 750 \) Hz) for operations at quantum limit. For operations at 10 quantum units, the \((S/N)_{mf}\) would be equal to unity at 33 and 24 Mpc, respectively for \( B = 250 \) Hz and \( B = 125 \) Hz.

The relative error in the determination of the amplitude of the signal is simply given by:

\[
\frac{\sigma_{\tilde{A}}}{\tilde{A}} = \frac{1}{\left( \frac{S}{N} \right)_{mf}}
\]

where \( \sigma_{\tilde{A}} \) is the rms error in the amplitude obtained from the Fisher information matrix.

The rms errors in chirp mass and the time of coalescence have also been plotted.\(^6\)

For a resonant frequency of 800 Hz and corresponding bandwidth of 270 Hz the \((S/N)_{mf}\) is 7, the chirp mass is determined with 17% accuracy and time of arrival can be determined to an impressive accuracy of 0.3 ms. The amplitude can be determined to around 15% accuracy.\(^d\)

The event rate for coalescing binaries is subject of very different estimates. An estimate\(^1\) is one per year to a distance of 45 Mpc, as arguments based on progenitor evolution scenarios suggest. Instead an estimate, a safe lower bound, based on the number of compact binaries known in our galaxy and extrapolated to the rest of the Universe shows that there should be one neutron star compact binary coalescence per year out to the distance of 200 Mpc\(^1\) as it appears confirmed by later work.\(^1\)

\(^a\)For black hole binaries only negligible fractions of a cycle can be seen in the spectrum considered herein.

\(^d\)Post-Newtonian parameters cannot be estimated accurately unless the binary is very close. Otherwise, only the chirp mass can be determined to a satisfactory accuracy with a Newtonian or post-Newtonian model.
If we assume that a typical threshold for detection \( d_T \) is 5, (making use of the coincidence at the harmonic frequency), and assuming a typical signal to noise ratio equal to 7, the detector will see events to a distance roughly \( 7/5 \times 45 \text{ Mpc} \). Thus the event rate \( E_r \) can be expressed as:

\[
E_r \simeq 2.74 \times \left( \frac{5}{d_T} \right)^3 \left[ \frac{(S/N)}{7} \right]^3 \left( \frac{N_{45}}{1} \right) \text{ per year,}
\]

(11)

Table 1 shows some detection event rates per year for an astrophysical event rate of one per year at 45 Mpc, for different values of noise, resonant frequency and bandwidth \( (d_t = 5) \).

<table>
<thead>
<tr>
<th>Astrophysical event rate = ( 1/y ) @ 45 Mpc</th>
<th>( f_r = 600 \text{ Hz} )</th>
<th>( f_r = 700 \text{ Hz} )</th>
<th>( f_r = 800 \text{ Hz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{S_0} = 4 \times 10^{-23} \text{ Hz}^{-1/2} ), ( B = f_r/3 )</td>
<td>4.7</td>
<td>3.6</td>
<td>3</td>
</tr>
<tr>
<td>( \sqrt{S_0} = 4 \times 10^{-24} \text{ Hz}^{-1/2} ), ( B = f_r/6 )</td>
<td>1.6</td>
<td>1.3</td>
<td>0.9</td>
</tr>
<tr>
<td>( \sqrt{S_0} = 10^{-23} \text{ Hz}^{-1/2} ), ( B = f_r/3 )</td>
<td>0.3</td>
<td>0.23</td>
<td>0.19</td>
</tr>
<tr>
<td>( \sqrt{S_0} = 10^{-23} \text{ Hz}^{-1/2} ), ( B = f_r/6 )</td>
<td>0.1</td>
<td>0.08</td>
<td>0.06</td>
</tr>
</tbody>
</table>

Table 1 Detection event rate per year, for an astrophysical event rate = \( 1/y \) @ 45 Mpc.

We note that the event rate changes significantly, especially when the noise is not at the quantum limit as it varies from e.g. one event every 3.3 years \( (B = 200 \text{ Hz}, f_r = 600 \text{ Hz}) \) to one event every 12.5 years \( (B = 115 \text{ Hz}, f_r = 700 \text{ Hz}) \).

Table 2 shows some detection event rates per year for an astrophysical event rate of one per year at 200 Mpc, for different values of noise, resonant frequency and bandwidth \( (d_t = 5) \).

<table>
<thead>
<tr>
<th>Astrophysical event rate = ( 1/y ) @ 200 Mpc</th>
<th>( f_r = 600 \text{ Hz} )</th>
<th>( f_r = 700 \text{ Hz} )</th>
<th>( f_r = 800 \text{ Hz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sqrt{S_0} = 4 \times 10^{-23} \text{ Hz}^{-1/2} ), ( B = f_r/3 )</td>
<td>0.0535</td>
<td>0.041</td>
<td>0.0342</td>
</tr>
<tr>
<td>( \sqrt{S_0} = 4 \times 10^{-23} \text{ Hz}^{-1/2} ), ( B = f_r/6 )</td>
<td>0.0182</td>
<td>0.0148</td>
<td>0.0102</td>
</tr>
<tr>
<td>( \sqrt{S_0} = 10^{-23} \text{ Hz}^{-1/2} ), ( B = f_r/3 )</td>
<td>0.0034</td>
<td>0.0026</td>
<td>0.0021</td>
</tr>
<tr>
<td>( \sqrt{S_0} = 10^{-23} \text{ Hz}^{-1/2} ), ( B = f_r/6 )</td>
<td>0.0011</td>
<td>0.0009</td>
<td>0.0007</td>
</tr>
</tbody>
</table>

Table 2 Detection event rate per year for an astrophysical event rate = \( 1/y \) @ 200 Mpc.

For a worse astrophysical rate of one event per year at 200 Mpc, the detection event rate, at the quantum limit, varies from e.g. one event every 18.5 years \( (B = 200 \text{ Hz}, f_r = 600 \text{ Hz}) \) to one event every 70 years \( (B = 115 \text{ Hz}, f_r = 700 \text{ Hz}) \) and in the worst case, table 2, 1400 years are overpassed. We must stress that lowering the threshold \( d_t \) would increment the detection event rate up to a factor 25.

If for coalescing binaries, matched filtering would not be adopted, and instead one would hope for a signal stronger than the noise and thus recognition by simply looking at the incoming data, the detection event rate would worsen of a factor.
proportional to $\sqrt{N^3}$, $N$ number of cycles. At the quantum limit the event rate decreases from 3.6 events per year ($B = 230\ Hz$, $f_r = 700\ Hz$) with matched filtering to one event every 6.5 years without matched filtering. These values are justified by the amplitude of the signal which is comparable to the noise level; thus a time series search, for which the source, supposed to be strong enough to stand out above the noise, is identified by simple threshold crossing criteria, is not effective. As further example, fig. 2 of\textsuperscript{5} reports a $S/N$, without matched filtering, equal to unity for a distance around 120 Mpc for a 3 meters sphere of Cu-Al and resonant at 725 Hz; with matched filtering the unitary value of signal to noise ratio at 725 Hz is achieved at about 240 Mpc for a bandwidth of 120 Hz, and at about 330 Mpc for a bandwidth of 240 Hz. In simple words matched filtering increases the explored universe by a factor 8 or 20 in this example.

More optimistic astrophysical event rates have appeared, e.g. 1 event per year at the distance of 15 Mpc.\textsuperscript{15} In such a case, the detection event rate would become 27 times larger the values of table 1 and thus up to 2.4 events per week ($d_t = 5$)!. Finally, we stress that a network of spherical detectors would improve the detection event rate considerably.

3.3. Use of the First Harmonic

Using the sensitivity at the first harmonic of the same detector, due to a cross section smaller only of a factor 2.61 with respect to the cross section at the fundamental,\textsuperscript{17} and/or using the fundamental frequency of a second smaller sphere which fundamental frequency is the same as the first harmonic frequency of the first sphere, at least two benefits are identified:

a) determination of the chirp mass, source direction and distance, and orbit orientation as calculated for narrow band resonators\textsuperscript{5 e}. The state of the art does not exclude a first harmonic frequency below 1.25 kHz (Dewey frequency\textsuperscript{18} for a chirp mass of 1.22$M_\odot$), where about one source cycle of a typical neutron star binary could be observed, provided that the Peters-Mathews model still somewhat holds, and thus matched filtering could be still applied. Nevertheless, we prefer to be conservative and assume that at the first harmonic a large band burst signal of unknown structure will be observed.

b) increment of probability of successful detection. A spherical detector of low resonant frequency would observe a sinusoidal like signal lasting up to $10 - 20\ ms$, and composed of a number of cycles for neutron stars binaries, followed in a very

\textsuperscript{e}We point out that the determination of the chirp mass, the distance and other results based, instead on records of first harmonic resonant frequency\textsuperscript{5} cannot be considered totally reliable since based on i) an yet undemonstrated, and likely undemonstrable, validity of the Peters-Mathews model at the very last stages of the evolution, ii) empiricity of the Dewey criterium.\textsuperscript{18} Finally, since for typical neutron stars binary of 1.22$M_\odot$ chirp mass, the Dewey criterium implies that for a resonant, or indeed harmonic frequency above 1.25 kHz, the analysis detection scheme becomes inaccurate, we remark the inapplicability of the Dewey criterium in spherical detectors of Cu-Al smaller than 3.5 m diameter.\textsuperscript{5} For heavier binaries the situation worsens.
short time interval by a burst at the first harmonic. We called this feature "double passage" and remark that the probability of detection increases by observing coincidences at different frequencies, while the probability of rejection of a true signal decreases. The probability that the total output energy of a spherical detector exceeds the threshold energy $E_0$ in the absence of signal is $p_e$. For a single detector, the probability at fundamental and first harmonic is $(p_e)_f(p_e)_1$, less than $(p_e)_f$, while for two spheres, one large and one small, where the first harmonic of the large sphere coincides with the fundamental of the small is $(p_e)_f(p_e)_1^2$, even smaller. Consequently for a given acceptable false alarm probability, the threshold for detection will be reduced and thus the probability of detection of a signal with a given strength will be increased.

4. Conclusions

We have demonstrated that a considerable gain in the signal to noise ratio of spherical antennae of large bandwidth can be achieved by using matched filtering techniques. Further, the observation of cycles of coalescing binaries and the extracted astrophysical information pose spherical detectors as advanced tools for relativistic astrophysics and gravitation theory. For the detection of coalescing binaries spherical detectors should have the lowest resonant frequency to optimize signal to noise ratio and thus the event rate and the accuracy of the estimation of the parameters. Further, the first harmonic increments the probability of successful detection. The detection event rates would further improve for a lower detection threshold, set by the potential of coincidences with the first harmonic, of a factor to the third power (for $d_t = 2$, instead of $d_t = 5$, the improvement is about 15 times), thereby allowing operations above the quantum limit and smaller bandwidth $f_r/6$. If the antenna was operated with a factor 10 above the quantum limit $4 \times 10^{-24} Hz^{-1/2}$, a very large bandwidth become obligatory to have reasonable detection rates for the case of one astrophysical event per year at 200 $Mpc$.

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$^{\dagger}$A $1.4M_\odot$ binary sweeps from 60 $Hz$ to 140 $Hz$ in 10 s, from 140 $Hz$ to 330 $Hz$ in 1 s and from 330 $Hz$ to 1 $KHz$ in 100 ms. In case the detector is peaked at 660 $Hz$, the sweeping lasts about 15 $ms$ from 585 $Hz$ to 735 $Hz$. 226
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ABSTRACT

The large optical components (the test masses, beamsplitters, and recycling mirrors) represent one of the most challenging aspects for large gravitational wave interferometers. The requirements for the LIGO optical components have been derived using a computer model of the interferometer which uses an FFT-based optical propagation code. This model includes the surface figure of all optical components, the homogeneity of the substrates, an allowance for losses due to scattering and absorption in the optical coatings, and the carrier and sideband modulation/detection technique. To meet these requirements, LIGO has undertaken a program to work with industry to evaluate and improve current fabrication capabilities. Full-size LIGO test masses have been polished and measured for microroughness and surface figure. Evaluation of these optics show that it is possible to polish and measure optic substrates with surface figures accurate to $< 1 \text{ nm}$ over spatial scales from 0.2 mm to 10 cm. To measure coating uniformity, LIGO has developed a technique using measurements of the reflectivity of specially-designed two-layer coatings to extract the thicknesses of the individual layers with a precision of $\sim 0.02\%$ (rms). This paper summarizes the requirements for LIGO optics that have been derived, results from the polishing development, and preliminary data on the large-scale uniformity of ion-beam-sputtered coatings.

1. Introduction

The large optical components represent one of the most challenging aspects for large gravitational wave interferometers. The large optical components ("Core Optics") in a LIGO interferometer\(^1\) (see Figure 1) consist of four test masses (two end mirrors and two input mirrors), a beamsplitter, and a recycling mirror. The total number required for the three initial LIGO interferometers is 20, 6 each for the Washington and Louisianna 4 km interferometers and 8 for the Washington 2 km interferometer (the 2 km interferometer also includes 2 folding mirrors which must meet requirements similar to those of the recycling mirror). In addition, a number of spares are required to insure against possible damage during the fabrication and installation processes. Because of the long time required for their fabrication, these spares must be procured along with the main optics.

The LIGO Core Optics will be made from high purity fused silica. They will...
be 25 cm in diameter \( \times \) 10 cm thick (except the beamsplitter which will be \( \sim \) 4 cm thick). Beams fill some of the optics, with approximately 1 part per million (ppm) of the total intensity lost outside the mirror. All optics have enhanced reflectivity 1064 nm coatings on one surface and anti-reflective (AR) coatings on the second surface. Their principal performance requirements include:

- \( < 50 \text{ ppm loss per surface} \)  \( \Rightarrow \) Limit loss of resonant stored energy: minimize shot noise
- Surface figure errors to scatter negligible power from TEM\(_{00}\)  \( \Rightarrow \) Best dark fringe
- High mechanical \( Q \)  \( (Q > \text{few} \times 10^6) \)  \( \Rightarrow \) Minimize thermal noise
- Low bulk \( (< 5 \text{ ppm/cm}) \) and coating \( (< 2 \text{ ppm}) \) absorption  \( \Rightarrow \) Limit effect of thermal lensing on power and dark fringe contrast

2. Defining the Optics Requirements

The primary tool for investigating the effects of different optical parameters on the LIGO interferometer sensitivity is a computer model of a full recycled Michelson interferometer with Fabry-Perot arms.\(^2\)\(^-\)\(^5\) This computer model is based on original code provided to LIGO by Jean-Yves Vinet and Patrice Hello of the VIRGO Project.\(^6\) This computer model uses an FFT-based optical propagation code. It includes the surface figure of all optical components (either real or simulated maps of the surface errors) and the optical pathlength difference (OPD) maps of substrates for the input mirror and beamsplitter. It solves for both the carrier and the modulation sidebands, and combines them to realistically model the demodulated
Table 1: partial listing of requirements for the LIGO Core Optics.

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Test Mass End</th>
<th>Test Mass Input</th>
<th>Beam splitter</th>
<th>Recycling Mirror</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of optic (cm)</td>
<td>25</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Thickness of optic (cm)</td>
<td>10</td>
<td>10</td>
<td>4</td>
<td>10</td>
</tr>
<tr>
<td>1 ppm intensity dia. (cm)</td>
<td>24</td>
<td>19.1</td>
<td>30.2a</td>
<td>19.2</td>
</tr>
<tr>
<td>Lowest internal mode (kHz)</td>
<td>6.79</td>
<td>6.79</td>
<td>3.58</td>
<td>6.79</td>
</tr>
<tr>
<td>Mass of optic (kg)</td>
<td>10.7</td>
<td>10.7</td>
<td>4.2</td>
<td>10.7</td>
</tr>
<tr>
<td>Nominal surface 1 radius of curvature (m) and g factor</td>
<td>7400</td>
<td>14571</td>
<td>inf</td>
<td>9998</td>
</tr>
<tr>
<td>g2 = 0.46</td>
<td>g1 = 0.72</td>
<td></td>
<td>g = 0.98</td>
<td></td>
</tr>
<tr>
<td>Tolerance on radius of curvature (m)</td>
<td>absolute:</td>
<td>-1000,</td>
<td>&gt; -720 km</td>
<td>-100,</td>
</tr>
<tr>
<td>matching:</td>
<td>±220</td>
<td>+145</td>
<td>convex,</td>
<td>+500</td>
</tr>
<tr>
<td></td>
<td>±111</td>
<td></td>
<td>&gt; 200 km</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>concave</td>
<td></td>
</tr>
</tbody>
</table>

*aFor these 45° angle of incidence optics, this is the smallest diameter circle centered on the optic face which is everywhere outside of the 1 ppm intensity field.*

gravitational wave signal. Important features of the code include its adaptation to a supercomputer\(^4\) and development of a fast convergence algorithm,\(^3\) enabling full recycled interferometers to be modeled rapidly. It has been tested in a variety of limiting cases against a semianalytic modal model that was developed independently.

This model has been used to develop a set of requirements for the LIGO Core Optics. These requirements include the size of the optics (to ensure that diffractive losses are not too large), surface figure, scatter losses, and tolerances on radii of curvature. Some of these requirements are summarized in Table 1.

3. Optics Development Program (“Pathfinder”)

To ensure that LIGO can obtain suitable optics for its initial detectors, an optics development program (called “Pathfinder”) has been underway for some time. The purposes of this program were to evaluate the state of the art in optical fabrication and metrology, to initiate work to further the state of the art where needed, and to identify companies with the ability to fabricate the LIGO Core Optics. The main steps in this program were:

- Purchase and evaluate fused silica blanks (5/94)
- Best effort polishing of substrates (8/95-4/96)
- Independent substrate metrology (4/96-8/96)
- Coating uniformity development (7/95-ongoing)
Coated optic metrology (expected in early 1997)

Industrial partners were engaged in all phases of this effort. Data from the Pathfinder are analyzed in the LIGO computer model to assess the performance of the optics against the requirements.

4. Substrate Material Results

The Pathfinder program began in 1994 with the ordering of a number of large fused silica blanks to be fabricated into finished components. These blanks were specified to have bulk index of refraction variations \( \delta n \leq 5 \times 10^{-7} \) through the 10 cm thickness. The order was placed with Corning and other specifications were consistent with their specifications for OAA Grade 7940 fused silica. Optical homogeneity maps were provided with each Pathfinder fused silica blank and these were evaluated using FFT model. These analyses indicate sufficient homogeneity that optical distortions due to transmission thorough the beamsplitter and input mirror substrates would not degrade the optical performance of the initial LIGO interferometers.

Mechanical \( Q \)'s for the five lowest-frequency internal vibrational modes were measured on one of the Pathfinder substrates after polishing.\(^7\) For these modes, the average \( Q \) was \( > 5 \times 10^6 \). These values meet the requirements for the initial LIGO interferometers.

In late 1995, the LIGO project made a decision to switch from using visible Argon ion lasers to using Nd:YAG lasers operating at 1064 nm. This change made the Corning fused silica unsuitable in one way. The large dependence of the index of refraction with temperature \( \frac{dn}{dT} \) in fused silica requires very low absorption to avoid thermal lensing of transmitted beams in the beamsplitter and input mirrors. Measurements by the VIRGO project\(^8\) have shown a correlation between 1064 nm absorption and OH concentration. Typical high purity fused silica contains 500-1000 parts per million (ppm) OH giving 10-20 ppm/cm absorption, which is too high for the input mirrors and beamsplitters. Fortunately, the VIRGO project has determined that Heraeus has a process which yields fused silica with \( \sim 200 \) ppm OH (\( \sim 5 \) ppm/cm absorption). In the initial LIGO interferometers, the low absorption of the Heraeus substrate material is only critical for input mirrors and beamsplitters where the laser light is increased by the recycling cavity.

Orders for approximately 40 blanks were placed in 1996, with delivery scheduled for 1997. Heraeus was selected for input mirrors and beamsplitters and Corning for all others. The total number of blanks ordered allows for both spare finished optics and for any problems during the fabrication process.

5. Core Optics Polishing Demonstration

The Core Optics polishing demonstration was carried out through best effort polishing and metrology of full-size substrates. Three companies participated in this effort: Commonwealth Scientific and Industrial Research Organization (CSIRO),
General Optics (GO), and Hughes-Danbury Optical Systems (HDOS). Each of these companies polished full size substrates with characteristics typical of the LIGO Core Optics. The specification called for the optics to be polished concave on one side with a radius of curvature of 6000 m and flat on other side.

The requirements for these polishing efforts were derived using the FFT optical model for the LIGO interferometers and tailored to the known capabilities of different classes of polishers. For CSIRO and HDOS, the primary goal was to demonstrate "mid scale" surface figure errors (after removing focus error and astigmatism) $< 0.8$ nm rms over the central 8 cm diameter; a second requirement was to achieve simultaneously a microroughness $< 0.4$ nm rms. For GO, the requirements were a surface figure error $< \lambda/20$ with a microroughness of $< 0.1$ nm.

To provide a consistent comparison of the figure errors in the polished substrates, independent metrology was performed by Chris Evans, Robert Parks, and Paul Sullivan at the National Institute of Standards and Technology (NIST). The technique used involved multiply redundant measurements on an existing 633 nm phase shifting interferometer, analyzed to give absolute metrology at the subnanometer level. With care, measurements at $< 1$ nm level proved possible. Reproducible features were seen and consistent intercomparisons demonstrated at this level. The instrument used gave information on spatial scales from its full aperture (15 cm) down to 3mm. Figure 2 shows power spectral densities of the figure errors for the GO and CSIRO substrates derived from these measurements. The HDOS results were overall comparable to those for CSIRO.
Table 2: Comparison of microroughness of polished substrates as measured by REO (using a Micromap instrument with a 20X objective).

<table>
<thead>
<tr>
<th>Polisher</th>
<th>Serial Number/ Surface</th>
<th>Microroughness (Å) (Ave. 5 Locations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSIRO</td>
<td>006/Curved</td>
<td>3.6</td>
</tr>
<tr>
<td></td>
<td>006/Flat</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>002/Curved</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>002/Flat</td>
<td>3.1</td>
</tr>
<tr>
<td>GO</td>
<td>005/Curved</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>005/Flat</td>
<td>0.88</td>
</tr>
</tbody>
</table>

The most important conclusion from Figure 2 is that polished surfaces with rms deviation (after removing focus and astigmatism) < 1 nm over ~ 20 cm diameter are achievable! In some cases, apparent deviations ~ 0.5 nm were measured.

To evaluate the microroughness of the various substrates, comparative surface roughness measurements were made at Research Electro-Optics (REO). Table 2 shows the result of these measurements. Microroughness contributes to large-angle scattering, and thus is particularly important for the test masses where the light intensity is greatest. Again, the HDOS results were comparable to those for CSIRO.

Based on the Pathfinder results and on competitive proposals, GO and CSIRO have been selected to polish the LIGO Core Optics.

6. Coating Uniformity Development

In developing coating uniformity, the LIGO project has collaborated with Research Electro Optics (REO). The goal of this effort is to scale REO’s low-loss ion-beam-sputtered coating technology to LIGO diameters. Preliminary work has focussed on developing the techniques needed to quickly measure coating thickness variations over both long and short spatial scales and optimizing the coating process.

The technique developed for measuring coating uniformity involves mapping the reflectivity of specially-designed two-layer AR coatings (see Figure 3). Near the reflectivity minimum, the reflectivity depends on the interference of the fields from the three interfaces in a two layer stack and is thus a strong function of the thickness of the two layers. By mapping the reflectivity at different angles of incidence and polarizations, one can derive maps of individual coating layers by fitting the observed reflectance maps through a least-squares minimization process. A standard set of measurements consists of six maps at 2 polarizations and 3 different angles of incidence.

The AR coating design which was selected for initial testing is the design shown in the top right of Figure 3. This coating is primarily sensitive to the thickness of
the Ta$_2$O$_5$ layer. A 24 cm diameter test piece was coated with this design. Reflectance measurements were made with the apparatus shown in Figure 4. Typical reflectance scan data show good reproducibility ($< 0.2\%$); because of the steep dependence of reflectance on layer thickness this permits the thickness variations of the Ta$_2$O$_5$ layer to be determined with a precision of $\sim 0.02\%$. One advantage of this technique is that it is insensitive to the surface figure of the underlying substrate, thus permitting multiple iterations in the coating chamber with easily obtainable, inexpensive substrates.

After a complete set of measurements is performed, individual maps of thicknesses for the two layers are determined by a least squares minimization process. This minimization takes into account known instrumental effects and uncertainties. For the initial test coating, maps of the thickness of the SiO$_2$ and Ta$_2$O$_5$ layers are shown in Figure 5.

To evaluate this coating against our requirements, we fit the maps in Figure 5 with Zernike polynomials up to tenth order. Residual deviations from the fits are consistent with measurement errors indicating that the fine structure observed in Figure 5 is dominated by noise in the measurements and therefore not a real property of the coating. These layer maps are then stacked in a coherent way to synthesize a predicted phase map for a HR coating. (The coherent stacking assumes that the observed structure is systematic and represents a worst case assumption about how the individual layer stack.) The predicted map produced in this fashion is shown in Figure 6.

The map in Figure 6 was then tested in the FFT optical model to assess its suitability in a LIGO interferometer. The shot-noise limited sensitivity for an interferometer with this surface profile used for end mirrors (and a similar one used
Figure 4: A schematic diagram of the apparatus used to test coating uniformity. A chopped Argon ion laser is used to illuminate the AR test piece which is mounted on the sample stage. Photodetectors D2 and D3 are used to measure the incident and reflected power respectively. The signals are synchronously detected and recorded by a computer which controls the motion of the motorized stage to map the reflectivity automatically. A quarter wave plate and polarizer are used to control the polarization of the incident light and the angle incidence can be adjusted by mirror M2.

Figure 5: Maps of the thickness of the SiO$_2$ and Ta$_2$O$_5$ layers of the test coating. The smoother appearance of the Ta$_2$O$_5$ map is due in part to the fact that this map was made for a Ta$_2$O$_5$-sensitive coating, and thus noise in the measurement affects the derived Ta$_2$O$_5$ thickness less than that of the SiO$_2$. 

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Figure 6: A simulated map of a 40 layer high reflector (HR) stack constructed from the individual layer maps from Figure 5. The graph at the right shows cuts through the map at two different angles.

for input mirrors) is compared with a standard configuration in which all of the mirrors have surface figures derived from maps of bare polished substrates in Table 3. Two measures of interferometer performance are compared in the table: the recycling factor and the shot noise limit to sensitivity at 100 Hz. These runs show some loss in sensitivity due to the coating non-uniformity, but the relatively modest degradation indicates that the coating non-uniformities are not too far from the required level.

The next steps in developing improved coating uniformity are beginning immediately. REO is making adjustments to their coating chamber and masking to reduce curvature. Once these are completed, they will make two new test AR coating runs, one identical to the coating tested above and one designed for high sensitivity to the SiO₂ thickness. If these are satisfactory then the Pathfinder optics will be coated with HR coatings (at 633 nm). These will then be measured at NIST to confirm scaling from single layers to HR coatings.

Note: After this paper was presented at the Workshop, another iteration of the coating uniformity development was completed by REO, and the preliminary analysis of test data indicate that the uniformity has been improved by approximately a factor of 5 over the central 20 cm diameter region, and that the SiO₂ layer is similar, but not identical, to the Ta₂O₅ layer.

7. Summary

The Pathfinder program to develop the optics for LIGO is nearing its conclusion. It has enabled us to put in place the tools and techniques (both experimental and analytical) to evaluate optics against the LIGO requirements. The capabilities of industry to manufacture substrate material and to polish the substrates appear
Table 3: Very preliminary estimates of the effect of coating nonuniformities on interferometer sensitivity.

<table>
<thead>
<tr>
<th>Run Conditions</th>
<th>Surface Figure (Arms)</th>
<th>Recycling Factor</th>
<th>$h(100Hz)$ ($\times 10^{-23}Hz^{1/2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Configuration: Measured substrate OPD’s Surface phase maps based on polished substrates</td>
<td>0.8</td>
<td>52</td>
<td>1.39</td>
</tr>
<tr>
<td>Standard Configuration, except 40 Layer HR substituted on End Mirror</td>
<td>3.8 (ETM)</td>
<td>17</td>
<td>2.14</td>
</tr>
<tr>
<td>Standard Configuration, except 16 Layer HR substituted on Input Mirror</td>
<td>1.9 (ITM)</td>
<td>33</td>
<td>1.73</td>
</tr>
<tr>
<td>Standard Configuration, except End and Input Mirror substituted</td>
<td>3.8 (ETM) 1.9 (ITM)</td>
<td>15</td>
<td>2.52</td>
</tr>
</tbody>
</table>

to be adequate for initial LIGO interferometers. The main ongoing activity is the development of coating uniformity. Preliminary coating uniformity data are promising, and further improvements and testing are expected within the next few months. Procurement of the LIGO substrates and their polishing are underway.

Acknowledgements

We thank our colleagues on the LIGO Project who have contributed in many ways to the work described here. We also thank Jean-Yves Vinet and Patrice Hello for the code which formed the basis of the FFT optical model of the LIGO interferometers. This work is supported by the National Science Foundation under Cooperative Agreement PHY-9210038.

References


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OPTICAL SYSTEM OF TAMA
Norikatsu MIO* and the TAMA collaboration
*Department of Applied Physics, University of Tokyo, 7-3-1 Hongo Bunkyo-ku, Tokyo 113, JAPAN

ABSTRACT
A 300-m interferometer, called TAMA300 is being constructed in Mitaka campus of National Astronomical Observatory. The optical system of TAMA300 is based on the standard design, so-called Fabry-Perot-Michelson configuration with the frontal modulation scheme. As a light source of the interferometer, we will use a laser-diode-pumped Nd:YAG laser whose output power is about 10W. The power recycling system will be installed in the advanced stage to reach the aimed sensitivity of TAMA project. We report here the overview of TAMA300 optical system and the recent status of the ongoing programs developing the elements required for TAMA300.

1. Introduction
A 300-m interferometer, called TAMA300, is being constructed in Mitaka campus of National Astronomical Observatory (NAO). The optical system is responsible for realizing the expected shot noise: to sustain high-power laser light with very small losses, to remove common noises such as a frequency noise of the laser, to realize stable operation with a definite recycling gain and so on. Since the effective power of 30W is necessary to reach the goal sensitivity of TAMA project, we expect 3-W light power at the main interferometer input and the recycling gain of 10.

In the following, we report on the overview of the interferometer and the recent status of the ongoing programs developing the elements required for TAMA300.

2. Overview of the Interferometer
The optical system of TAMA300 is schematically shown in Fig. 1. Two optical cavities of 300-m length form the orthogonal arms of a Michelson interferometer. Each arm cavity consists of a flat mirror (98.8% reflectance) and a concave mirror (450-m radius of curvature and 99.99% reflectance); the finesse of the cavity is about 500 to obtain an enough storage time for gravitational waves of a few hundreds hertz frequency. Since the cavity is shorter than km-scale interferometers such as LIGO and VIRGO, the finesse should be rather high; this fact imposes rather severe requirements on optical elements even though the circulating power is not so high. The recycling mirror may be installed in the final stage of the project; the detailed design of the recycling mirror has not been fixed yet.

All of mirrors and a beamsplitter are fabricated from monolithic synthetic fused silica substrates with super-polishing and ion-beam-sputtering-coating techniques. The allowed loss is about 100ppm to obtain the aimed recycling gain of 10 for the mirrors used in the main cavities.

In order to obtain control signals, we will use the frontal modulation method originally proposed by Schnupp. Thus, there is a small path difference between
Figure 1: The optical system of TAMA300 interferometer. Two optical cavities of 300-m length form the orthogonal arms of a Michelson interferometer. Each arm cavity consists of a flat mirror and a concave mirror; the finesse of the cavity is about 500. In order to obtain control signals, the frontal modulation method is used. As a light source, a laser-diode pumped Nd:YAG laser of 10-W output power at 1064-nm wavelength is used. Between the main interferometer and the light source, there is a mode cleaner forming a triangular ring cavity; its length is chosen so that the cavity can transmit the carrier wave and the sideband which is generated by an electro-optical modulator provided in front of the mode cleaner.

two arms. The asymmetry in lengths between the beamsplitter and the front mirrors $\Delta l$ is changeable from 0m to 1m so as to obtain an optimal modulation depth which depends on the characteristics of the interferometer. The length of the recycling cavity is almost same as the quarter wave length of the radio frequency signal used for the phase modulation.

As a light source, we will use a laser-diode pumped Nd:YAG laser of 10-W output power at 1064-nm wavelength; the single-mode and single-frequency oscillation can be realized by injection locking technique. The detail of the laser is described in the later section.

Between the main interferometer and the light source, there is a mode cleaner forming a triangular ring cavity; its finesse is designed to be about 2000. The length ($\sim$ 10m) is chosen so that the cavity can transmit the carrier wave and the sideband which is generated by an electro-optical phase modulator provided in front of the mode cleaner; the sideband is used to control the interferometer. The frequency of the phase modulation is now planed to be 15.25MHz.

As for a photodetector, we will use an array of 16 InGaAs-type photodiodes
Table 1: Parameters of the mirrors of the interferometer.

<table>
<thead>
<tr>
<th></th>
<th>Flat-Concave Cavity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat Mirror</td>
<td>Reflectance 98.8%</td>
</tr>
<tr>
<td>Concave Mirror</td>
<td>Curvature Radius 450m</td>
</tr>
<tr>
<td></td>
<td>Reflectance 99.99%</td>
</tr>
<tr>
<td>Dimension</td>
<td>$d \ 10\text{cm} \times t\ 6\text{cm}$</td>
</tr>
<tr>
<td>Mode Cleaner</td>
<td>Triangle Ring Cavity</td>
</tr>
<tr>
<td>Flat Mirror ($\times 2$)</td>
<td>Reflectance 99.82%</td>
</tr>
<tr>
<td>Dimension</td>
<td>$d \ 10\text{cm} \times t\ 3\text{cm}$</td>
</tr>
<tr>
<td>Concave Mirror</td>
<td>Curvature Radius 15m</td>
</tr>
<tr>
<td></td>
<td>Reflectance 99.99%</td>
</tr>
<tr>
<td>Dimension</td>
<td>$d \ 10\text{cm} \times t\ 6\text{cm}$</td>
</tr>
<tr>
<td>Recycling Mirror</td>
<td>Curvature Radius 9km</td>
</tr>
<tr>
<td>Size and Reflectance</td>
<td>Under consideration</td>
</tr>
<tr>
<td>Coating</td>
<td>IBS Machine at NAO</td>
</tr>
</tbody>
</table>

($d$: diameter, $t$: thickness)

(1-mm acceptance diameter) for high-power laser light; we expect that the residual power at the dark port of the beamsplitter is about 500mW. We will use a mechanical shutter in order to avoid the excess power when the dark fringe is not maintained by some accidental unlock of the control system.

According to the schedule of TAMA project, all of the system must be completed until the end of March 1998.

3. Current Status

We are now preparing optical elements for TAMA300. The design of the mirrors and the beamsplitter have been finalized. The laser oscillator has been also completed. We are also testing and designing the photodetector system. In the following, we describe their details.

3.1. Mirror

The parameters of these mirrors are listed in Table 1. For the mirrors used in the main cavities, the ratio between the diameter and the thickness was chosen to optimize the thermal noise due to internal vibration modes.

The mirrors, except the recycling mirror, and the beamsplitter will be fabricated (both polishing and coating) by Japan Aviation Electronics Industry Ltd. (JAE). The quality of the coating by this company has been tested and confirmed by the University of Electro-Communications (UEC)$^{3,4}$ for small mirrors. Now, the mirrors
of the 20-m prototype interferometer at NAO have been changed to those made by JAE; the performance of the mirrors is under testing.

The mirrors for the mode cleaner of TAMA300 have been delivered to NAO from JAE. Figure 2 shows the surface status of one of the flat mirrors (10-cm diameter), measured by Zygo MarkIII interferometer. Both the photographs were taken for the same surface. The difference is the measurement area; the left one covers all over the surface and shows that the distortion expressed in peak-to-valley (PV) is $0.037\lambda$ ($\lambda = 633$nm). If the measurement area is limited to $9\text{cm} \times 9\text{cm}$ (the right side), the distortion level is reduced to $0.019\lambda$ (PV). The distortion expressed in RMS is $0.003\lambda$ for both the measurements. Since the guaranteed flatness of the reference used in the interferometer was $\lambda/20$, the measured distortion might originate from that of the reference. The mirrors for the main interferometer are now being developed in JAE.

As for the recycling mirror, we have decided to use the ion-beam-sputtering coating machine installed in NAO, which has enough capability to make it. However, we are still discussing how to polish the mirror and evaluate its surface figure.

The substrate for all of the optical elements is high-homogeneity synthetic fused silica made by Shin-etsu Silica, called Sprasil P10. The absorption of this silica was measured by the French group and found to be about 3ppm/cm; this value is enough for TAMA300.

3.2. Laser

We have obtained the laser which can be used for TAMA300 at the end of October 1996; the laser was developed by Kubota group at SONY research center. The laser system is shown in Fig. 3. As mentioned above, the laser can be operated in a single-mode and single-frequency by injection locking. A commercial laser of 700-mW output is used as the master laser. The slave laser consists of a bow-tie ring cavity and two Nd:YAG rods; each rod is pumped by a fiber-coupled laser diode.
Figure 3: Laser system for TAMA300. The laser system is operated in a single-mode and single-frequency by injection locking. A commercial laser of 700-mW output is used as the master laser. The slave laser consists of a bow-tie ring cavity and two Nd:YAG rods which are pumped by fiber-coupled laser diodes with the end-pumping scheme. To keep the locked state, the cavity length is controlled by a voice coil motor which is devolved by SONY group specially for controlling an optical path length.

The output power of the laser was about 10W at 23-W pumping power. The operation of the laser is remarkably stable because of the elaborated mechanical design of the cavity and the wide-band servo-control system.

The control systems for the frequency and intensity stabilization are being developed at UEC and University of Tokyo. However, the requirements for these control systems are still under consideration.

3.3. Photodetector

We are testing two types of InGaAs photodiodes made by Hamamatsu (G5832-01) and EG&G (C30641). According to their specification data sheets, the high-frequency response of 1-mm diameter diodes is much better than 3-mm ones, while there is no significant difference in the maximum photo-current. Thus, we will
use the smaller one though the saturation effect due to the small spot size on the photodiode has been reported.\textsuperscript{8}

Figure 4: DC and AC (at 15MHz) responses of photodiodes made by Hamamatsu (G5832-01) and EG&G (C30641). When the bias voltage is not applied to the photodiode, the high-frequency response saturates at rather small light power which is within the linear response region for DC signal. The higher bias voltage is applied, the better response is obtained. However, C30641 does not work with 15-V bias when the incident power is larger than 50mW; the dissipation in the photodiode might limit the acceptable power in this case.

Figure 4 show the DC and AC (at 15MHz) performance of the photodiodes. As can be seen, the appropriate bias voltage is important to obtain the enough AC response for rather high power light. When the bias voltage was not applied to the photodiode, the high-frequency response saturated at rather small light power which was within the linear response region for DC signal.\textsuperscript{8} The higher bias voltage was applied, the better response was obtained. However, C30641 did not work with 15-V bias when the incident power was larger than 50mW; the dissipation which is given by the product of the bias voltage and the photo-current might limit the acceptable power in this case. Thus, there is an optimal value for the bias voltage.
As for a pre-amplifier, we are designing a low-noise circuit making use of a very-low-noise and wide-band operational amplifier MAX4106 produced by MAXIM. Since the photo current is large, the load resistor must be small. As a result, the thermal noise of the resistor becomes very small. To obtain the noise floor set by this thermal noise, the electric noise of the amplifier must be very low. The voltage noise of this OP amplifier is 0.75nV/√Hz, which corresponds to the thermal noise of 34-ohm resistor and low enough for our purpose. The final design of the circuit is under consideration.

4. Summary

The optical system of TAMA300 is described here. The elements of the interferometer are being developed in collaboration with the research groups of Japanese companies. Since the quality of the elements which we have obtained is quite satisfactory, we have now firm prospects of completing TAMA300.

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References

5. A. Brillet, private communication.
CONTROL SYSTEM OF TAMA300 INTERFEROMETER

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ABSTRACT

In order to maintain a good signal-to-noise ratio of an interferometric gravitational wave detector, several degrees of freedom must be controlled. In this paper, the length-control servo system of the TAMA300 interferometer is discussed.

1. Introduction

In order to maintain a good signal-to-noise ratio of an interferometric gravitational wave detector, all of the cavities must be kept on resonance with the incident laser field, and the fringe of the Michelson interferometer must be kept dark at the anti-symmetric port. In a ground-based detector, such as TAMA300, there are large external disturbances, such as seismic vibration and drift of the ground. Though the interferometer is well-isolated from any noise sources in the observation band, the mirrors of the interferometer are always driven by large disturbances in the low-frequency range below several hertz. Thus, several control systems are indispensable to maintain the operation point of the interferometer. Any deviation of the optical-path length from the operational point is extracted from the interferometer, then fed back to the position of the mirrors and canceled. However, since the feedback system has finite noise, the control system introduces additional noise into the interferometer. The noise added by the control system must be smaller than the intrinsic noise of TAMA300, which is limited by the thermal noise (of the internal- and the pendulum-mode) and the photon shot noise.

In this paper, the relation between the sensitivity of the interferometer and the dynamic range of the control system is discussed, together with the typical amplitude of seismic motion and drift. Also, the signal-extraction system and the control topology of the TAMA300 interferometer are briefly reviewed.

2. External Disturbances

The TAMA300 interferometer is located at the campus of National Astronomical Observatory (NAO), Mitaka, Japan. To design the control system of the interferometer, the amplitude of the external disturbances at the site must be studied. One of the sources of disturbance is seismic motion of the ground. The amplitude of the seismic noise at the NAO campus was measured,\(^1\) which agrees well with the equation

\[
\tilde{x}_{\text{sei}}(f) \simeq 10^{-7} \left( \frac{1}{f} \right)^2 \text{m/}\sqrt{\text{Hz}}
\]  

Gravitational Wave Detection
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from 1Hz to 100Hz, where $\bar{x}^{sei}$ is the displacement spectrum of the ground at frequency $f$. There is a broad peak at around 0.2Hz; the frequency range below 0.1Hz has not been measured. We assume that the spectrum is flat below 0.1Hz. Integrating this noise from DC to 100Hz, the seismic motion is obtained as

$$\left[ \int \left| \bar{x}^{sei}(f) \right|^2 \, df \right]^{1/2} \sim 4\mu m$$

in the root-mean-square (RMS) amplitude. Since the arm Fabry-Perot cavity comprises two mirrors, the seismic disturbance in one arm is equivalent to

$$x^{sei}_{rms} \sim 4\sqrt{2} \mu m \sim 6\mu m$$

in RMS amplitude, or $x^{sei}_{pp} \sim 2\sqrt{2}x^{sei}_{rms} \sim 10\mu m$ in peak-to-peak (pp).

Another large disturbance is the drift of the baseline due to drift of the ground and the vacuum enclosure of the arms. Each arm of the TAMA300 interferometer is housed in an underground tunnel; the temperature variation in the tunnel was measured to be within 1K per day. Assuming that the thermal-expansion rate of the baseline is on the order of $10^{-6}/K$, the drift of the 300-meter arm is estimated to be

$$x^{drift}_{pp} \sim 300\mu m/day$$

in the peak-to-peak amplitude. Though the drift of the baseline has not yet been measured at the TAMA site, it has been measured at the campus of the Institute of Space and Astronautical Science (ISAS), which is about 20 kilometers away from NAO, by using the absolute control signal of the 100-meter delay-line prototype interferometer (TENKO100). In a measurement lasting for over five hours, the drift rate was about $30\mu m/hour$ at its maximum. Since this drift is thought to be a daily one, the drift of the baseline is expected to be less than $300 \sim 400\mu m/day$ in peak-to-peak for a 100-meter baseline. Considering the fact that the arm chambers of the TENKO100 interferometer are exposed in the open air, one can expect that the drift at the TAMA site is much smaller than that at the TENKO site. Therefore, we believe that the estimation of the drift in the TAMA site represented by Eq. 4 is a modest one.

3. Dynamic Range of the Control System for TAMA300

In TAMA300, the magnet-and-coil actuators will be used to feed back the error signal to the position of the mirror. A model of the actuator and its driver is shown in Figure 1. The coil of the actuator is driven by a current-source which is proportional to the input voltage, rather than by a voltage-amplifier, to avoid any decrease in the quality factor of the pendulum due to the finite output impedance of the voltage-amplifier. The noise performance of the driver is represented by three parameters: the current noise ($i_{n1}$ and $i_{n2}$) which flows in the coil, and the voltage noise ($v_n$) in the read-out port of the feedback signal. Although both $i_{n1}$ and $i_{n2}$
flow in the coil, only $i_n$ is sensed by $R$. We assume that there is no correlation between $i_n$, $i_{n2}$ and $v_n$. The coupling constant $(a)$ of the actuator is defined by

$$F = ai,$$  \hfill (5)

where $i$ and $F$ represent the current in the coil and the force applied to the mirror, respectively. The maximum output current of the current source represented by $i_{\text{max}}$ is related to the maximum displacement of the mirror by the driving force of the actuator. The Fourier spectrum of the fluctuation of the length of the cavity represented by $\tilde{x}$ is related to the external disturbance ($\tilde{x}_{\text{dist}}$) and the force ($\tilde{F}$) applied to the mirror by the actuator as

$$\tilde{x}(f) = H_{\text{grd}}(f)\tilde{x}_{\text{dist}}(f) + H_{\text{force}}(f)\tilde{F}(f)$$  \hfill (6)

at frequency $f$, where $H_{\text{grd}}$ and $H_{\text{force}}$ are the transfer functions from the ground-motion to and from the force on the mirror to the position of the mirror, respectively.

The noise of the actuator, together with the coupling constant, determines the equivalent displacement noise of the mirror. The equivalent displacement noise of the mirror produced by $i_n$, $i_{n2}$ and $v_n$ has different forms, because these noise sources are introduced to the interferometer at different points. A simple calculation shows that the equivalent displacement noise in the feedback signal is represented by

$$\tilde{x}_n(f) = a |H_{\text{force}}(f)| \sqrt{\frac{1}{G(f)} \tilde{i}^2_{n1}(f) + \tilde{i}^2_{n2}(f) + \left| \frac{1 + G(f)}{G(f)} \right|^2 \frac{\tilde{v}_n(f)}{R^2}},$$  \hfill (7)
where \( G(f) \) is the open loop transfer function of the servo loop at frequency \( f \). In the observation band, the transfer function \( (H_{\text{force}}) \) is well-approximated by

\[
H_{\text{force}}(f) \approx -\frac{1}{m\omega^2}, \tag{8}
\]

since the resonant frequency of the suspension system is much lower than the observation band. The equivalent noise of the mirrors in the feedback signal produced by the noise in the actuator circuit must be smaller than the intrinsic noise of the interferometer \( (x_{\text{int}}) \) in the observation band. Combining \( x_{\text{int}} \) with Eqs. 6 and 8, the following inequality in the observation band is obtained:

\[
\alpha x_{\text{int}} \geq \frac{a}{m\omega^2} \sqrt{\left| \frac{1}{G(f)} \right|^2 \tilde{v}_{n1}^2(f) + \tilde{v}_{n2}^2(f) + \left| \frac{1 + G(f)}{G(f)} \right|^2 \tilde{v}_n^2(f) }, \tag{9}
\]

where \( \alpha \) is the safety factor. This gives the relation between the acceptable upper limit of the coupling constant \( (a) \) and the noise in the actuator circuit.

Next, we assume that the maximum current from the current source is limited mainly by the voltage of the power supply \( (v_{cc}) \) of the driver circuit and the currentsensing resistor \( (R) \)

\[
i_{\max} \approx \frac{v_{cc}}{R}. \tag{10}
\]

In a simple pendulum suspension, \( H_{\text{force}} \) is approximately equal to \( H_{\text{grd}}/m\omega_0^2 \), where \( m \) is the mass of the mirror and \( \omega_0 \) is the resonant angular frequency of the suspension. By using Eqs. 6 and 10, the following equation is obtained:

\[
\tilde{x}(f) = H_{\text{grd}}(f) \left[ \tilde{x}_{\text{dist}}(f) + \frac{a}{m\omega_0^2} \tilde{i}(\omega) \right]. \tag{11}
\]

Suppose that the disturbance has to be completely canceled by the feedback system. This is a good approximation because the acceptable deviation of the position of the mirrors from the operation point is much smaller than the external disturbance. The current in the coil is proportional to the disturbance in this case,

\[
\tilde{x}_{\text{dist}} = -\frac{ai}{m\omega_0^2}, \tag{12}
\]

for any frequency. Therefore, the RMS amplitude of the current is written as

\[
\frac{a}{m\omega_0^2} i_{\text{rms}} = x_{\text{dist}}^{\text{rms}}, \tag{13}
\]

where \( x_{\text{dist}}^{\text{rms}} \) is the RMS amplitude of the disturbance. The RMS current and the maximum current of the current source are related to each other by the inequality \( \sqrt{2}i_{\text{rms}} \leq i_{\max} \). Combining this inequality with Eqs. 10 and 13, a requirement for
the coupling constant of the actuator \( (a) \), the resistance \( (R) \) and the power voltage \( (v_{cc}) \) is obtained as

\[
\beta \frac{a v_{cc}}{m \omega_0^2 R} \geq \sqrt{2} x_{\text{rms}} \tag{14}
\]

where \( \beta \) is the safety factor.

Equations 9 and 14 define the requirements for the dynamic range of the control system of the interferometer. Combining Eqs. 9 and 14 yields

\[
\frac{\alpha \beta v_{cc} / \sqrt{2} R}{\sqrt{f_{\text{bw}}} \left[ \left| \frac{1}{G(f)} \right|^2 \tilde{\eta}_{n1}^2 (f) + \tilde{\eta}_{n2}^2 (f) + \left| \frac{1+G(f)}{G(f)} \right|^2 \tilde{\xi}_n^2 (f) \right] \left( \frac{4k_B T}{R} \right)^2} \geq \frac{x_{\text{rms}}}{x_{\text{int}} \sqrt{f_{\text{bw}}}} \tag{15}
\]

in the observation band, where \( f_{\text{bw}} \) is the width of the observation band and \( f_0 \) is the resonant frequency of the pendulum, respectively. The right side of the above equation is the dynamic range necessary for controlling the interferometer, while the left side is the dynamic range of the actuator system factored by the safety factor.

For a quantitative discussion, a more realistic model of the driver is necessary. Figure 2 shows a simple example of the current driver. In this circuit, no voltage noise appears, in principle. The Fourier spectrum of the current noise can be written as

\[
\tilde{\eta}_{n1} = \frac{\tilde{\nu}_{\text{op}}}{R} \tag{16}
\]

\[
\tilde{\eta}_{n2} = \frac{\tilde{\nu}_{\text{R}}}{R} = \sqrt{\frac{4k_B T}{R^2}} \tag{17}
\]

where \( \tilde{\nu}_{\text{op}}, \tilde{\nu}_{\text{R}}, k_B \) and \( T \) are the input voltage noise of the operational amp, the thermal noise of the current-sensing resistor, Boltzmann’s constant and the temperature, respectively. Let us put the following realistic parameters into Eqs. 15,
16 and 17: \[ x_{\text{int}} = 5 \times 10^{-20} \text{m} / \sqrt{\text{Hz}}, \ f_0 \sim 1 \text{Hz}, \ f = 150 \sim 450 \text{Hz}, \ m = 1 \text{kg} \] and \[ v_{\text{op}} = 1 \text{nv} / \sqrt{\text{Hz}}. \] These equations are combined to give a relation between the resistance \( R \) of the driver, the amplitude of the external disturbances and the safety factor, which is calculated as

\[
\alpha \beta \geq \frac{1}{13} \left( \frac{15\text{volt}}{v_{\text{cc}}} \right) \left( \frac{x_{\text{dist}}^{\text{rms}}}{1\mu\text{m}} \right) \sqrt{\frac{R}{100\Omega}} \sqrt{1 + 0.6 \frac{100\Omega}{R|1 + G|^2}}.
\]

(18)

Though the smaller resistance and the larger power supply give a wider safety range, a current in the coil larger than several hundred milliamperes is very hard to realize due to a technical reason.

For seismic motion, \( x_{\text{dist}}^{\text{rms}} \) is 6 \( \mu \text{m} \) (Eq. 3); therefore, a resistance of 100\( \Omega \) and a power supply of 15volt gives the safety factor of \( \alpha \beta \sim 1/2 \) when \( G \gg 1 \). The noise of the driver is about 1\( \text{nv} / \sqrt{\text{Hz}} \) in the feedback signal. The actuator of the TAMA300 interferometer must realize the same level as this performance or better.

For the drift of the ground, \( x_{\text{dist}} \) is estimated to be several hundred microns in peak-to-peak (Eq. 4). To keep the safety factor less than the unity, \( R \) must be on the order of 0.1\( \Omega \), which is completely unrealistic. Therefore, we must feed back the long-term drift of the ground to the suspension point of the mirror in TAMA300. It is impossible to compensate this motion by using the same actuator as that which directly drives the mirror. For this purpose, the motorized optical stage will be used. Since the motor will make additional noise, control of the stages is performed only when the driver of the magnet-and-coil actuator is supplying current at close to \( i_{\text{max}} \).

Figure 3 is a schematic diagram of the actuators in the TAMA300 interferometer.

4. Signal Extraction and the Control Topology of TAMA300

In this section, the signal-extraction and control topology of TAMA300 are briefly described. The TAMA300 interferometer is a Michelson interferometer with a Fabry-Perot cavity in each arm (Fabry-Perot-Michelson interferometer), with power-recycling. The interferometer is at first tested without power recycling (Phase I). The signal-extraction system of the Phase-I TAMA300 interferometer is based on the standard pre-modulation (or Schnupp modulation) technique. The TAMA300 interferometer with power-recycling (Phase II) also employs the pre-modulation configuration. Figure 4. shows the simplified topology of the signal-extraction and control of the TAMA300 interferometer. The carrier and RF sidebands of 15.25 MHz are transmitted by a ring mode cleaner and are injected into the interferometer. There is an asymmetry in the paths between the beam splitter and the front mirrors. By demodulating the photo-current at the asymmetric port, the differential motion of the cavities (represented by \( L_- \) in the figure) is extracted. From the reflected field, two degrees of freedom are extracted. In the interferometer without recycling: one is the differential motion of the Michelson.
Figure 3: Though it is possible to compensate for the seismic motion by using the magnet-and-coil actuator, the long-term drift of the ground is too large to be canceled by the same actuator system as the seismic motion. In the TAMA300 interferometer, the motorized stage will be used for this purpose.

The amplitude of the seismic motion has been measured at the TAMA site. Also, the drift of the ground has been estimated. From these values, we conclude that it is impossible to compensate both the seismic motion and the drift by using only one actuator, because of the limited dynamic range of the standard electronics circuits. For controlling the mirror against the seismic motion, the coil-and-magnet actuator is used for fast feedback to the mirrors (indicated by “M-C” in the figure), while the motorized stages are used for slow feedback to the suspension point of the mirrors (“picomotor” in the figure). There are also several slow paths for the stability of operation. The signal-extraction and control topology of the Phase-I interferometer without the mode cleaner have been successfully tested in a 3-m Fabry-Perot-Michelson interferometer at the University of Tokyo. A 10-meter mode cleaner is now being tested on site. The Phase-II topology is being tested in the 3-m interferometer, in a 20-m prototype in NAO, and in a table-top experiment.

5. Summary

The amplitude of the seismic motion has been measured at the TAMA site. Also, the drift of the ground has been estimated. From these values, we conclude that it is impossible to compensate both the seismic motion and the drift by using only one actuator, because of the limited dynamic range of the standard electronics circuits. For controlling the mirror against the seismic motion, the coil-and-magnet actuator is used for fast feedback to the mirrors (indicated by “M-C” in the figure), while the motorized stages are used for slow feedback to the suspension point of the mirrors (“picomotor” in the figure). There are also several slow paths for the stability of operation. The signal-extraction and control topology of the Phase-I interferometer without the mode cleaner have been successfully tested in a 3-m Fabry-Perot-Michelson interferometer at the University of Tokyo. A 10-meter mode cleaner is now being tested on site. The Phase-II topology is being tested in the 3-m interferometer, in a 20-m prototype in NAO, and in a table-top experiment.
Figure 4: Simplified topology of the signal-extraction and control of the TAMA300 interferometer. BS, the Beam splitter; PD, the Photo Detector; EO, the Electro-Optical modulator; ADC, the Analog-to-Digital Converter; DAC, the Digital-to-Analog Converter; M-C, the Magnet and Coil actuator.
type actuator can be used, though it is not easy to realize the requirement for the electronics. For compensating the drift of the ground, motorized stages will be used. The signal-extraction and control topology of the Phase-I interferometer without recycling have been fixed and successfully tested in the 3-m baseline interferometer at the University of Tokyo. The 10-m modecleaner of the TAMA300 interferometer is being tested on site. The Phase-II topology is being tested in the 3-m interferometer, in the 20-m prototype, and in a table-top experiment.

References

2. T. Yamazaki, private communication.
4. S. Miyoki, Development of a 100-meter Delay-Line Laser Interferometer (Doctor thesis, University of Tokyo, 1996)
ABSTRACT
The laser source for the Italian-French VIRGO is an ultra-stabilized high power cw laser. The shot noise limited sensitivity around the Fourier frequency of 200 Hz puts a limit to the laser power of about 1 kW. By using the recycling configuration and an expected recycling gain of 100, the need of laser power is relaxed to 10 W. This laser is operating in a monomode output and injection locked by a low power master oscillator.

We report on the performances obtained in the stabilization of the master laser to a reference cavity and the injection-locking technique that transfers the stability of the master laser to the slave laser, in frequency and in amplitude. We will present also the reference cavity performances in terms of noise in the detection range of 10 Hz to 10 kHz.

1. Introduction
In the sources of noise that can limit the sensitivity of an interferometric gravitational wave detector there are the ones due to the laser source. They are of fundamental type like the shot noise which originates from random processes intrinsic to the measurement, as well as of technical origin whose influence can be reduced by care and advances in the technology.

We will identify and estimate first the noise sources in order to put the requirements on the laser beam and then describe the solutions we propose in Virgo to reach the sensitivity goal. The results obtained on the laser and stabilisations development will be presented.

2. Requirements
2.1. Power
The ability to measure the optical phase difference at the output of the interferometer is limited by the quantum fluctuations of the light. The equivalent gravitational-wave strain noise is given by:

$$\tilde{h}_{sn} = \frac{\lambda}{2\pi L} \sqrt{\frac{\hbar \nu}{\eta P}} = 2.10^{-23} \sqrt{\frac{1\text{ kW}}{\eta P}} / \sqrt{\text{Hz}}$$

(1)

Around 1 kHz the expected sensitivity of $\tilde{h} = 2.10^{-23}/\sqrt{\text{Hz}}$ can be reached with an optimum length of 150 km, obtained in Virgo by folding the 3 km arms about 50 times, and illuminating the interferometer with a 1 kW power incident on the central mass.

2.2. Frequency Stability
For a given strain sensitivity goal, the frequency stability is related to the asymmetry expected in the two arms of the Michelson. There will be a balance between
the requirement on asymmetry and on frequency stability, that depends upon the achievement feasible technically in one case or the other. The relation between these parameters is:

$$\frac{\delta \nu}{\nu} = \frac{\delta h}{\beta}$$  \hspace{1cm} (2)$$

where $\beta$ is the asymmetry factor of the Michelson arms and $\frac{\delta \nu}{\nu}$ is the relative laser frequency stability achievable with a given stabilisation technique. To give an order of magnitude for the goal in laser frequency stabilization, we plot now the requirements for an asymmetry of 1% (figure 1) and we can see that the lowest level to be achieved is around $10^{-6}$Hz/$\sqrt{\text{Hz}}$.

2.3. Power Stability

The main cause of laser power noise effect is due to the finite open loop gain of the mirror position servo system yielding a residual offset from the dark fringe. A reasonable order of magnitude for this residual offset can be estimated to be $\delta L_{\text{offset}} = 10^{-11}$ m and it corresponds to a phase lag of $\delta \Phi_{\text{offset}} = \frac{2 \pi \delta L_{\text{offset}}}{\lambda} \mathcal{F} = 2.10^{-5}$ rd for a finesse $\mathcal{F}$ of 50 in the arms of the interferometer. Given that technical limitation, we require the laser power noise in the detection range to be below the ratio of phase sensitivity to phase noise. The curve $a$ of Figure 2 gives the level
of amplitude stability requirement following the level of strain sensitivity expected for Virgo. We can see that the lowest level required is \( \delta P/P \approx 10^{-8}/\sqrt{\text{Hz}} \) at a frequency 500 Hz.

The other possible cause is due to the beam impinging on a mirror causing a displacement induced by radiation pressure, which is transformed into an equivalent strain sensitivity when there is an asymmetry \( \beta \) between the arms: with an intracavity laser power of 15 kW, an asymmetry of \( 10^{-2} \), tests mass of 30 kg (typically in Virgo), we get \( \delta P/P < 7.10^{-13} F^2 h(F) \), represented by the curve \( \mathbf{b} \) of the Figure 2: this effect is less stringent than the noise mentioned above. Then the overall level of amplitude stability is dictated by the curve \( \mathbf{a} \) at all frequencies.

Figure 2: Relative power stability required for Virgo (see text).

2.4. Beam Stability

The lateral or angular jitter of the beam can couple to imperfections in the interferometer and results in a phase difference at the output interference pattern. The imperfections are of many kinds: misalignments of the recombined output wavefronts in their tilts or in their curvatures, waist mismatchings between the two arms, originating from residual misalignments of beamsplitter and/or test-mass themselves. The calculations have been done for a simple Michelson case\(^1\) and extended here for a recycled Michelson and we summarize the results to give the noise in term of linear spectral density. The coupling of the laser jitter \( \alpha(t) \) in \( \text{rd}/\sqrt{\text{Hz}} \), with the interferometer misalignments \( \Delta x \) (in m) gives a phase jitter

\[
\delta \Phi(t) = \frac{2\pi}{\lambda} \frac{1 - \sqrt{R_{\text{rec}}}}{1 + \sqrt{R_{\text{rec}}}} \Delta x \alpha(t) = 1.23 \times 10^5 \Delta x \alpha(t)
\]  

(3)
with a recycling mirror of $R_{rec} = 0.92$. We can transform this jitter into noise density and express that we want this induced phase noise to be smaller than the Virgo sensitivity, ie $\delta \Phi_{misalign}(F) < \delta \Phi_{Virgo}(F)$ and represents it vs the frequency on Figure 3.

3. Solutions Selected for Virgo

- Choice of laser: in 1986, we have decided to use high power Nd:YAG lasers to replace the Ar$^+$ lasers commonly used in the Glasgow, Garching and Caltech prototypes, this choice being dictated by the electrical-optical efficiency of this kind of lasers, and confirmed later by the possibility of laser diodes pumping. After our first injection-locking demonstration of a 18W lamp-pumped Yag laser,$^2$ we switched to diode-pumped Yag laser in 1988 by stabilizing a low power laser$^3$ to a reference cavity with a double servo-loop.

- Power: the solution proposed in Virgo is the use of power recycling technique consisting of including the whole interferometer inside a resonant cavity; the light incident on the Michelson is then multiplied by a storage factor of the cavity which can be estimated to be more than 50, putting the laser power constraints to less than 20 W. Moreover the recycling gain is inversely proportional to the total losses, so the mirrors technology will determine later on what recycling factor will be possible. Meanwhile our strategy for the laser power is the following: we will develop a laser of 10 W for the first interferometer tests and experiences its reliability. Then that experience and the mirrors state-of-art will determine what will be the final laser power required.
Injection locking: we have demonstrated injection locking on Ar\(^+\) lasers in 1984\(^4\) as well as coherent addition of these lasers\(^5\) and since then, this technique seems to be a good alternative to solve the problem of compatibility between frequency stabilising a laser and keeping its high output power. Injection locking can happen between a low power laser (called master laser) and a high power laser (called slave laser) when the frequency separation between the two is in a certain range called injection locking range which is proportional to the square root of their power ratio. Then the slave laser, which needs only to be single transverse mode, starts its oscillation on the same frequency as the master laser and gets the same stability in frequency and amplitude as the master laser in that injection locking range.

Choice of multistage frequency stabilisation: the final stability required is about \(10^7\) below the level of usual laser including Yag lasers. Knowing that it is impossible nowadays to perform servo-loops with such high gain at 500 Hz, we choose to achieve the stabilisation in 2 steps; the 1st is the prestabilisation which reduces the laser fluctuations to those of a short term rigid reference cavity; the 2nd is the final stage which lock the laser to the interferometer and brings it at the required level.

Choice of using input mode cleaner: the beam geometry required is about \(10^9\) times better than the normal jitter presented by any stable laser beam, and the best way to reach the desired beam stability in position and geometry is to pass it through a resonant cavity, named as mode cleaner. Then with the adequate characteristics, a mode cleaner can filter out the residual beam jitter and bring it down to the required level.

3.1. High Power Laser

The slave laser is a zig-zag slab laser, transversely pumped by 10 fiber-coupled 12W laser diodes chosen for their easy collimating beam and their easy maintenance possibility (a change of one diode could be done without stopping the whole laser). In order to avoid optical feedback the cavity is an X- shaped with an output coupler of 10\% transmission. Two of the mirrors are mounted on pzt transducers for frequency control, a slow one with a sensitivity of 3.6 MHz/V and a flat response up to 28 kHz, and a fast one with a sensitivity of 0.4 MHz/V and a flat response up to 200 kHz. The thermal control of the laser is done via Peltier elements which sensitivity is 500 MHz/°C and a dynamical range larger than 10 GHz. TEM\(_{00}\) mode operation of 10 W is achieved for an effective pumped power of 60 W (optical efficiency of 15\%) in an elliptical beam due to non symmetrical thermal focal length.

The strategy of our high power laser is quite clear: we use an efficient laser at 60\% of the nominal power (reached with 120 W of diode pumping) to prevent maintenance problems. This means that we can afford to have up to 4 laser diodes failed out before stopping the laser operation, plus the fact that using the laser diodes far from their nominal current will lengthen their lifetime.
3.2. *Injection Locking*

The principle is described on the optical scheme of Figure 4: the light coming from the master laser is matched and sent to the slave laser via a Faraday isolator and a phase modulator EO. To prevent long term drifts, a Pound-Drever-Hall scheme is used to maintain the slave laser in the injection locking range which is around 1 MHz; thanks to the two transducers-pzt of the slave cavity the unity gain bandwidth of the servo-loop is 100 kHz. The behaviour of the injection locking loop completed by the Pound-Drever-Hall servo can be qualitatively explained as followed, the details of the calculations can be found in the ref\(^6\).

Figure 4: Injection locking of the 10 W slave laser by a commercial miser-type master laser of 600 mW.

If \(\delta \nu_0\), \(\delta \nu_1\), \(\delta \nu_2\) are respectively the frequency fluctuations of the slave cavity, the master laser and the injected laser, the injection locking process sets a relation between them and the injection locking range \(\gamma\) by:

\[
\delta \nu_2 = \frac{\gamma}{p + \gamma} \delta \nu_1 + \frac{p}{p + \gamma} \delta \nu_0
\]

and we can see that the fluctuations of the slave cavity are filtered out at first order with a cut-off frequency of \(\gamma\) and the fluctuations of the master laser are transmitted to the slave cavity up to that same frequency. Let’s \(G\) be the gain of the PDH servo-loop, then the slave laser frequency fluctuations is now:

\[
\delta \nu_0 = \frac{G}{1 + G} \delta \nu_1 + \frac{1}{1 + G} \delta \nu_{0F}
\]

where \(\delta \nu_{0F}\) is the free-running slave cavity frequency fluctuations.

Then for frequencies below the unity gain bandwidth of the PDH servo-loop, we get:

\[
\delta \nu_2 = \delta \nu_1 + \frac{p}{p + \gamma} \frac{1}{1 + G} \delta \nu_{0F}
\]
This expression shows that the contribution of the active loop is to reduce the initial noise of the slave laser; it also gives the residual noise equation between the two lasers. The figure 5 shows the residual frequency noise of the injected laser vs frequency and one can see that the residual noise contribution of injection locking is negligible compared to the master laser noise (upper curve a). Some extra noise around 4 kHz remains due to the water cooling which can be reduced later on. Below 100 kHz which is the unity gain of the servo-loop, the residual noise is under the shot noise limit for 2 mW of light represented by the line c, while below 1 kHz it is limited by the noise of the amplifier used to read the signal. Anyway, we have shown that the residual noise reproduces the master laser noise up to 100 kHz and the other step is to stabilize the master laser to reach the requirements for Virgo.

3.3. Prestabilisation

To know the level required for the prestabilization, let’s examine how much stabilization we can get from the final stage of lock to the interferometer. The free-spectral-range of the arm cavities is 50 kHz and limits the bandwidth of this loop typically to 15 kHz. It is reasonable then to assume that one can get a gain of about $10^3$ at 500 Hz, and lots of loop gain at lower frequencies. Now taking into account the presence of the mode cleaner and the recycling cavities that will put first order passive filtering respectively at 500 Hz and 5 Hz, the remaining requirement for the
prestabilization is around 1 Hz.Hz\(^{-1/2}\) below 200 Hz and down to 10\(^{-3}\) Hz.Hz\(^{-1/2}\) around 500 Hz at the lowest level.

The optical scheme for prestabilization is the classical PDH scheme (Figure 6) with high frequency corrections (10 kHz–1 MHz) directed towards the phase modulator, intermediate frequencies (0.1 Hz–10 kHz) towards the laser piezoelectric transducer, which provides a dynamic range of about 450 MHz, and the very low frequency components are used for temperature tuning of the laser crystal, allowing to cover many GHz. The unity gain frequency of the complete loop is about 1 MHz, the open loop gain is higher than 10\(^4\) at 10 kHz, and it increases as 1/f\(^4\) below 30 kHz. This servo loop actually stays locked for weeks, between unlockings due to electrical or mechanical shocks. Furthermore, an automatic locking system was implemented, which always relocks the system in a few seconds. The electronics noise is about one order of magnitude lower than the shot noise, for a typical laser power of 5 mW on the (enhanced silicon) detector: a higher power results in a saturation. Given the cavity finesse of 32000, this results in an expected shot-noise level of about 10\(^{-4}\) Hz.Hz\(^{-1/2}\). The mechanical details of the reference cavity are described in ref\(^7\); actually two identical cavities are used the first one for the laser lock and the second one serves as frequency discriminant.

Figure 7 shows the spectral density of frequency fluctuations, in the frequency range of interest for Virgo, for the free running laser (upper curve, measured with an additional low finesse cavity), as measured on the reference cavity error signal (lower curve), and as measured with the second cavity (middle curve). The free running laser spectral density can be approximated by \(\delta \nu(f) \approx \frac{7.10^{-3}}{f}\) Hz.Hz\(^{-1/2}\), which is excellent, but worse than the specification by about 2 orders of magnitude. The reference cavity error signal shows that the feedback loop is working well: the noise level is lower than 10\(^{-4}\) Hz.Hz\(^{-1/2}\), and the loop gain is higher than 10\(^4\) at all frequencies below 10 kHz. The middle curve gives a (pessimistic) measurement of the stabilized laser frequency noise. Its main topics are understood: the lowest noise level, achieved at 20 kHz, is close to the shot noise level \(\delta \nu(f) = 2.10^{-4}\) Hz.Hz\(^{-1/2}\), or 6.10\(^{-19}\) Hz\(^{-1/2}\)). The bump at 5 kHz is associated with a knee in the transfer function of the servo loop. The many peaks between 4 Hz and 200 Hz correspond to mechanical resonances in the cavities mechanical supports, and in the optical table supporting the experiment, excited by seismic and by acoustic noise. The general 1/f tendency which can be observed on the middle curve is likely a remnant of the low frequency temperature drifts of the measurement cavity and of the laser: it comes from the Fourier transform of a linear frequency drift over a finite time. In the end, these results lie between one and three orders of magnitude below the specification (dashed line on Figure 7), and they probably correspond to the best spectral purity ever observed with any kind of oscillator. In the ref\(^7\) the Allan variance curves of the cavity noise are also shown in comparison with other oscillators.
Figure 6: Pound-Drever-Hall technique to prestabilize the master laser on reference cavity FP1 and use of FP2 as frequency discriminant.

Figure 7: Linear spectral density of noise: upper curve is the free-running laser, bottom curve is the error signal, middle curve is the FP noise, dashed line is the Virgo requirements.
3.4. **Power Stabilization**

The power stabilisation system for the Virgo slave laser consists of two separate servo loops effective in two different regimes, hereunder labelled the Primary and Secondary Servo Loops; they feedback directly on the laser diodes currents to control the slave laser. The photodiode used for the measurement is placed after the mode cleaner in order to benefit of the filtering properties.

The Primary Servo Loop reduces the amplitude noise of the light in the region of interest of the Virgo experiment, that is 10Hz–10kHz, down to or below the specified noise level shown in Fig 2. It also is responsible for maintaining the average power level of the slave to that specified by the operator. It works in the frequency region from DC up to approximately 50kHz, the latter being an intermediate point above the region of interest of the Virgo experiment and below the relaxation oscillation of the slave laser occurring at 100kHz. It is at this point where the Secondary Servo Loop takes over, and continues out to several hundred kHz.

The Figure 8 shows the error signal spectral density where we can see that the electronic noise is below the shot noise for 26 mW ($4 \times 10^{-9}$ Hz$^{-1/2}$), the straight line shows the requirement to be fulfilled for Virgo. It remains now to measure in good conditions, with a second photodiode the real power noise of the laser after stabilization.

![Figure 8: Spectral density of noise in the power stabilization.](image_url)

3.5. **Prestabilized Injection-Locked Laser**

The global frequency stabilization will be realized by adding to the injection locking scheme above, the reference cavity interrogated by a small fraction of light
derived from the slave laser. This experiment is underway and we have studied the behaviour of the two loops: a scheme is proposed in order to keep the global loop stable in presence of transients.

4. Conclusion

We have studied the steps and the tools necessary for the goal of a 10 W slave laser frequency prestabilized on a rigid cavity, as well as the power stabilization of the slave laser. The assembling of the whole experiment will happen in the next months during the final development of the laser and input bench for Virgo.

References

5. C.N.Man and A.Brillet, 91984 Cleo Proceed. paper TuB36.
RESONANT SIDEBAND EXTRACTION FOR DETECTORS WITH FABRY-PEROT ARMS

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ABSTRACT
Resonant Sideband Extraction is a new optical configuration for laser-interferometric gravitational wave detectors with Fabry-Perot cavities in the arms. It reduces the thermal load on the beamsplitter and the coupling mirrors of the cavities and also allows one to adapt the frequency response of the detector to a variety of requirements. A table-top experiment has shown good agreement of the measured transfer function with theoretical predictions. Remaining discrepancies in the data helped us to identify one important condition for the applicability of RSE.

1. Introduction
Construction work is proceeding for several large laser interferometric gravitational wave detectors (e.g. LIGO in the U.S., VIRGO and GEO600 in Europe, TAMA300 in Japan and others).

All of them will employ power recycling in order to increase the light power circulating in the interferometer. The power gain achievable will, however, be limited by imperfect contrast as well as losses in the substrates of the beamsplitter (and those of the arm cavity coupling mirrors). Furthermore, the high-power beam may cause absorption-induced thermal lensing and birefringence in the substrates.

By using high-finesse Fabry-Perot cavities in the arms, the high-power beam is confined to within these arm cavities and is not transmitted through any substrate. It was believed, however, that the required detector bandwidth would limit the finesse allowed in the arms. Taking a 300 m-cavity as an example, a detector bandwidth of 500 Hz limits the finesse to 500.

Resonant Sideband Extraction (RSE) was proposed as a new optical configuration to overcome this limitation. In particular, the detector bandwidth can be made broader than the arm-cavity bandwidth, thereby permitting the use of high-finesse cavities in the arms. There are non-detuned and detuned modes of operation, where ‘tuned’ here refers to making the signal extraction cavity resonant for the carrier, which yields the broadest possible detector bandwidth. The ‘detuned’ modes, on the other hand, permit a sensitivity peak to be obtained at an adjustable frequency.

The outline of this paper is as follows: In the next section, we describe the principle and frequency response of RSE. Our experimental setup is described in...
Section 3. Two methods to lock the position of the Signal Extraction mirror are discussed in Section 4. Finally the results are discussed in Section 5.

2. **Principle of Resonant Sideband Extraction**

Figure 1: Optical layout for Resonant Sideband Extraction. Power recycling (with mirror $M_{PR}$) is an option not affecting the principle of RSE. The compensation plate may be included for reasons of symmetry between the two arms.

Figure 1 shows the basic optical layout for RSE. It resembles that of signal recycling with Fabry-Perot cavities in the arms, but here a different sub-wavelength positioning of the signal extraction mirror ($M_S$ in Figure 1) produces a different effect, as described below.

In the following we need to make some simplifying assumptions, i.e. identical polarization and transverse mode structure for all interfering beams, a 50/50 beamsplitter and both arm cavities to be identical and in resonance with the ‘carrier’. Furthermore, we assume the interferometer to be operated at a dark fringe (at the detection port), i.e. all carrier light returning from the arms is interfering constructively towards the laser (or the power recycling mirror).

Note that the amplitude reflectivity of the Michelson interferometer as a whole seen from either port is identical (if the beamsplitter’s transmittance equals its reflectivity), whatever the arms are. In particular when the Michelson interferometer is operated at the dark fringe, any light injected from the detection port is reflected back towards the signal extraction mirror.

A suitably oriented and polarized gravitational wave produces a phase modulation of opposite sign in the two arms. The phase modulation sidebands interfere
constructively towards the signal extraction mirror $M_S$. After being reflected by $M_S$, they re-enter the interferometer, which in turn reflects them back to $M_S$. The signal sidebands are thus stored in a ‘split’ cavity\(^{10}\) composed of the Michelson interferometer and $M_S$. Under the above assumptions, this split cavity is equivalent to a \textit{three-mirror-cavity} (Figure 2) formed by $M_S$, $M_C$ and $M_R$, where the two identical arm cavities have been ‘folded’ together to form a single cavity consisting of $M_C$ and $M_R$.

Figure 2: Three-mirror cavity storing the signal sidebands. Here ‘in’ refers to the production of sidebands in the arms by the gravitational wave, and ‘out’ indicates the signal finally leaving the interferometer.

One possible interpretation of this three-mirror-cavity is to consider the cavity as being composed of $M_R$ and a \textit{compound mirror} formed by $M_S$ and $M_C$. The compound mirror (which we also call the ‘signal extraction cavity’) has frequency-dependent transmittance and reflectivity. In \textit{signal recycling}, this cavity would be tuned to ‘anti-resonance’ (i.e. centered between two successive resonances) so as to obtain a \textit{lower} transmittance than that of $M_C$ alone, resulting in an \textit{increased} storage time for the signal.

The purpose of RSE, on the other hand, is to \textit{reduce} the storage time for the signal, in order to allow a long storage time for the carrier without sacrificing detector bandwidth. This can be achieved by tuning the signal extraction cavity to ‘resonance’ so that its transmittance for the signal frequencies of interest is \textit{higher} than that of $M_C$ alone. For these frequencies the storage time in the three-mirror-cavity is shorter than that in the unmodified arm cavity. In the non-detuned case, this reduction of the storage time simply results in an increased bandwidth.

3. Experiment

In our table-top experiment\(^{11}\) (Figure 3), the arm cavities were 40 cm long and had a finesse of approximately 3000. The interferometer was illuminated with approximately 300 mW of single-mode light at 514.5 nm from an Ar\(^+\)-laser. The laser was locked to one arm cavity by a Pound-Drever system\(^{12}\) with a control bandwidth of 120 kHz. Total cavity losses (excluding the transmission of $M_C$) were measured as 350 ppm, yielding a power reflectivity of the cavities of about 50\% at resonance. With two mode-matching lenses, more than 90\% of the incoming power was coupled into the fundamental mode of the arm cavities.

Another Pound-Drever loop locked the length of the second cavity to the laser by
adjusting the cavity’s length with two PZT’s. This loop had a unity gain frequency of approximately 40 kHz. The signal extraction mirror M_S had a power reflectivity of 82% and was placed 40 cm away from the flat mirrors M_{C1} and M_{C2}.

The Michelson interferometer was locked to a dark fringe by means of an ‘internal’ phase modulation at 42 kHz applied between the beamsplitter and the second cavity by dithering a Brewster plate. The light power returning towards the laser was demodulated at 42 kHz, and the error signal thus obtained was fed back to the Brewster plate through an appropriate filter. The unity-gain frequency of this loop was approximately 700 Hz. With both arm cavities locked, the interference minimum was approximately 1% of the power at the maximum.

We used external modulation to detect the signal sidebands in our experiment. External modulation uses an additional Mach-Zehnder type interferometer where a phase-modulated reference beam is brought to interference with the signal beam. As compared with Schnupp modulation (pre-modulation) there is one additional degree of freedom which is the phase between signal beam and reference beam, that needs to be controlled and that influences the frequency response\textsuperscript{9,11}. In our experiment, the 42 kHz phase modulation was used at the same time to lock the
Mach-Zehnder phase. This was done by continually maximizing the amplitude of the 42 kHz signal at the output.

Test signals up to 500 kHz were fed into the interferometer at the PZT holding M\text{R2}. This phase modulation in only one arm can be separated into two components of equal magnitude: a common mode component directed towards the laser, and a differential component, which represents the signal of interest.

Figure 4: Calculated frequency response of our table-top interferometer as a function of the tuning \(\delta\) of the signal extraction mirror M\text{S}.

Figure 4 shows the computed frequency response of our table-top interferometer. We have plotted the responses for the non-detuned cases of RSE and signal recycling (\(\delta = 0\) and \(\delta = \pi\), respectively), for two detuned operating points used in our experiment, and the response without any mirror M\text{S} for comparison. In order to eliminate the effect of PZT resonances we compared the response with M\text{S} locked to its proper position to the response with M\text{S} removed.

4. Controlling the Signal Extraction Mirror

Two different schemes to lock the position of M\text{S} (using another PZT) were investigated. Both schemes use a 180-kHz calibration signal (with an amplitude corresponding to less than 1 pm movement), which was also fed to M\text{R2}.

4.1. First Scheme

The first scheme (switch position 1 in Figure 3) was aiming for a peak sensitivity at 180 kHz. The amplitude of the 180-kHz calibration signal at the output was detected by synchronous demodulation. In order to continually maximize the 180 kHz
amplitude, the position of $M_S$ was dithered with 400 Hz. The 180-kHz-amplitude was synchronously demodulated once more at 400 Hz to obtain an error signal to be fed back to $M_S$. For real gravitational wave detectors, this scheme will not be the best choice, since one presumably will not want to introduce test signals just at the sensitivity peak, and because the frequency of that peak needs to be dithered to obtain an error signal. It was however good enough to verify experimentally the calculated frequency response of RSE (curves labelled 1 in Figure 7).

4.2. Second Scheme

This first scheme was also useful to acquire lock of the different coupled loops and to adjust the electronics for the second scheme (switch position 2 in Figure 3). Here $M_S$ is no longer dithered, but instead controlled in such a way as to maintain the amplitude of the 180-kHz signal at the output at a predefined constant level $U$, similar to a concept initially proposed by Strain\textsuperscript{13} for the use with signal recycling. Figure 5 shows the amplitude and phase of the 180-kHz signal at the output as a function of the signal extraction mirror tuning $\delta$.

Figure 5: Magnitude and phase of 180kHz calibration signal at the output.

The 180-kHz amplitude is again obtained by synchronous demodulation, now, however, with a different and adjustable demodulation phase $\Delta \phi$. Figure 6 shows the amplitude obtainable by demodulation with different values of the phase $\Delta \phi$. If the level $U$ is chosen somewhere between the extreme amplitude values, the error signal obtained by subtracting the instantaneous amplitude from $U$ has the proper sign on both sides of the desired operating point. There are two degrees of freedom
which allow to select the desired tuning $\delta$ (and hence the detector’s frequency response) in a wide range.

Figure 6: Demodulated amplitude of 180 kHz calibration signal at the output for some values of demodulation phase $\Delta \phi$.

5. Experimental Results and Conclusions

Figure 7 shows the measured results. The ratios of the interferometer’s measured transfer functions with and without mirror $M_S$ are shown for the two different schemes of controlling $M_S$. The computed ratios are shown for comparison. The curves labelled 1 refer to the first scheme (setting the peak sensitivity to 180 kHz by dithering $M_S$).

The curves labelled 2 were obtained by locking $M_S$ with the second scheme (maintaining the 180-kHz output signal at a constant level). A sensitivity peak at approximately 250 kHz was obtained, corresponding to $\delta = 0.433$ rad. A constant of 1.8 dB had to be added to the theoretical curve for a best fit, which can be understood by the misalignment of the Mach-Zehnder interferometer introduced by removing $M_S$ together with the lens used to compensate the lensing effect of the curved mirror substrate. Around 420 kHz the PZT response was so small that both signals were buried in the noise. In further experiments (not shown), the sensitivity peak could be shifted to as high as 400 kHz, more than twice the calibration signal frequency.

At frequencies around and above the peak, theory and experiment agree reasonably well. The discrepancies at lower frequencies were found to result from an interaction with the Pound-Drever systems used to keep the two arm cavities res-
onant. With $M_S$ in place, the two cavities are no longer independent. The laser frequency is stabilized to the first cavity with a unity-gain frequency of approximately 120 kHz (without $M_S$). The gain of this loop is modified by $M_S$, because the detuned signal extraction cavity causes additional optical feedback and introduces extra phase shifts in the loop. This was verified by changing the overall loop gain of the frequency stabilization and observing a shift in frequency and height of the 120-kHz peak in Figure 7.

Another effect is caused by the asymmetry between the two Pound-Drever systems. As described above, the second arm cavity is kept resonant with a unity-gain frequency of only up to 40 kHz, considerably lower than the 120 kHz of the first cavity loop. The test signal being fed into the second cavity appears, after reflection by $M_S$, as frequency deviations on the photodetectors of the Pound-Drever loops. Here they are amplified and fed back to the laser frequency and the length of the second cavity, respectively. At those frequencies where some loop gain remains, the theory described above is no longer valid, especially when the gains for the two arms are not identical.

Both these effects arise from limitations in our specific experimental setup. In the planned large-scale detectors the laser frequency will be stabilized either to an independent reference or to the power-recycling cavity instead of a single arm cavity. The auxiliary loops used to lock the arm cavities will anyway need to have a unity-gain frequency below the observation bandwidth.

If any of the schemes of power recycling, signal recycling or RSE are to be
incorporated, only a small fraction of the light can be used for these auxiliary loops, which therefore will have a poor signal-to-noise ratio. Feeding back such noisy signals to control the lengths of the arm cavities would then obscure the gravitational wave signal, if the unity gain frequencies of the auxiliary loops were too high.

The interaction of the various control loops with each other and with the signal sidebands is a complex problem that requires further investigation, for any detector configuration. The necessity in RSE to lock long, high-finesse arm cavities with only a limited control bandwidth may prove a technical challenge. During lock acquisition, however, a broader control bandwidth may be permissible.

References

4. K. Tsubono et al., ibid., 112–114.

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POWER RECYCLING EXPERIMENT

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ABSTRACT

In a future stage of TAMA300 interferometer, power recycling will be applied in order to obtain better S/N. The installation of power recycling will, however, bring some technical difficulties, for instance, locking acquisition, or sensitivity fluctuation caused by optics contamination. We will report the results of a recycling experiment for a suspended Michelson interferometer in which simultaneous locking acquisition has worked well for two degrees of freedom to be controlled. We will mention about some diagnostics technique which is useful for empirical tuning of the servo filters.

1. Introduction

A scheme of power recycling has been incorporated in several recent plans of ground-based interferometric gravitational wave detectors. In TAMA300 interferometer we will utilize the power recycling with the gain of about 10. The principle of power recycling has been demonstrated in several prototype interferometers. On the other hand, power recycling may bring some technical difficulties, for instance, locking acquisition, separation of control signals and thermal distortion. In order to study the property of the locking acquisition, we construct a table-top interferometer with suspended mirrors. This system has only two degrees of freedom and they are originally easy to separate so that the problem of signal separation will not arise. Thus we can focus development of multi-variable servo systems which has strong gain interaction. We also report about a qualitative diagnostic method for misalignment and servo performance.

2. Experimental Setup

The optical and control system is shown in Figure 1. A stabilized He-Ne laser is used as a light source. The light is introduced to an interferometer through Faraday isolators (FI) and matching lenses. A Michelson interferometer consists of two arm mirrors (mirror A and B) and a beam splitter (BS). Between the light source and the interferometer we put a recycling mirror whose intensity transmission and reflection are 0.72% and 98.9%. This interferometer has two degrees of freedom in displacement along the optical axes; one is differential displacement of two arm lengths and the other is common displacement which is equivalent to displacement of the recycling mirror. To obtain the error signals, we use 15kHz (OSC1) and 10kHz (OSC2) synchronous detections. We apply mechanical modulations to a recycling mirror at 15kHz, and to arm mirrors differentially at 10kHz. The signal of differential displacement can be obtained by demodulating a photocurrent at antisymmetric port (PD2) while the common displacement is obtained from the reflected light (PD1) which is separated by Faraday rotator (FR) and polarizing beam splitter (PBS). These demodulated signals do not have cross-talk ideally but
Figure 1: Experimental setup.
have strong gain interaction. This makes it difficult to design servo controllers. The detail of servo filters are described in the next section. All of the mirrors and the beam splitter are independently suspended as a double pendulum. Each final mass has four magnetic actuators; two of them are for servo control and other ones are used for mechanical modulation. Note that the beam splitter has no actuators.

We can monitor the intensity ratio of internal light leaked through the arm mirrors (PD3 and PD4). Typical value of observed recycling gain is about 50.

3. Property of the Servo Filters

In the acquiring sequence of operating point, the differential control filter must have sufficient phase margin under the large gain fluctuation. We design a filter which keeps stability over 60dB gain variation. Open-loop Bode/Nichols diagram of differential control is shown in Figure 2. The measured and calculated values have good agreement. A filter for differential loop (servo filter 2 in Figure 1) itself

Figure 2: Bode and Nichols plot of open-loop transfer function for differential displacement control. The solid lines and the points show calculated and measured values respectively.
has two zeros (2.9Hz and 97Hz) and three poles (32Hz, 1.1kHz and 1.1kHz again). Note that we have to take account of the frequency response of bandpass filter between the photo detector and the demodulator (BPF2 in Figure 1). This correction is indispensable because the frequency bandwidth of BPF2 is close to unity gain frequency of the control loop in our situation. In TAMA300 interferometer, we will obtain main signal with much higher modulation frequency thus we will be able to avoid these type of complexity. Although this system does not introduce the nonlinear servo technique, such like an automatic gain control, simultaneous acquisition of two degrees of freedom occurred frequently.

4. Correlation Plot of DC Photocurrent

In a recycled Michelson interferometer, it is not so easy to confirm that the servo system is working to realize a correct operating point, since the dc photocurrent at antisymmetric port (PD2 in Figure 1) is not neither minimum nor maximum when the interferometer has maximum sensitivity. To monitoring the operating point, we have found that it is useful to draw a correlation diagram of dc photocurrents at PD1 and PD2 (denoted as $I_1$ and $I_2$). Suppose the situation in which the servo filters are turned off and common and differential displacements ($l_c$ and $l_d$) are caused randomly. According to displacement of mirrors, the point ($I_1$, $I_2$) fills a region enclosed by a strait line and a conic indicated as a hatched region in Figure 3(a).

Figure 3: (a) A correlation plot of photocurrent. The dc photocurrents at PD1 and PD2 are denoted as $I_1$ and $I_2$. A variable $I_0$ means photocurrent corresponding to the incident light. (b) The coupling dependence of envelope around the resonant point.

(a) (b)
Figure 4: An examples of the correlation plot of photocurrent. The measured points are shown by dots. Dashed lines represent the contours of total loss in the interferometer. (a) A correlation plot with all servo filters working. (b) A magnified plot around the resonant point.

The boundary has two edges corresponding to the resonant and antiresonant states. An examples of such a diagram is shown in Figure 4. The solid lines in Figure 4 are boundaries calculated with optical parameters of mirrors determined by another measurement. We can see measured points are localized near the resonant point when servo loops work correctly. After some calculations of tilt perturbation, we have found that both the common phase change and the symmetric tilt of the arm mirrors increase almost only $I_1$, while both the differential displacement and antisymmetric tilt cause increment of $I_2$. In any case of displacement and tilt of mirrors, the resonant edge of correlation plot moved so as to decrease $1 - I_1 - I_2$, the total loss of an interferometer. Therefore a real-time view of this diagram on an oscilloscope is quite helpful when we trim the servo gain and align the mirrors.

We can also determine whether the recycling cavity is over or under coupled. On the correlation plane, the deviation of a point $(I_1, I_2)$ from the resonance condition is approximately proportional to the square of length displacement $\delta l_c$ and $\delta l_d$. If the recycling cavity is under coupled then

$$\left( \frac{\partial^2 I_1}{\partial l_d^2} \right)_{\delta l_c=0} > 0.$$

Otherwise, the cavity is over coupled. The sign of this derivative is determined qualitatively as in Figure 3 (b). From Figure 4 (b), we can see that the cavity is under coupled in our experiment. This method can be applicable to the interferometer which incorporates RF modulation, or which has arm cavities, with some modifications. If we extract above derivative with a synchronous detection, we may be able to monitor the contamination of optics.
5. Conclusion

We have demonstrated the simultaneous locking acquisitions of differential and common displacements of suspended Michelson interferometer under the large gain variance caused by power recycling. Observed value of the recycling gain is about 50 typically.

A correlation diagram of photocurrent can be used to diagnose the servo performance and to support the initial alignment of mirrors. We have find that it is possible to determine the sign of coupling for the reflectivity of recycling mirror. In our case, the recycling cavity is under coupling.

Acknowledgments

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References

ABSTRACT

The LIGO (Laser Interferometer Gravitational-Wave Observatory) 40m interferometer at Caltech has recently made two major advances which were necessary milestones to ensure the LIGO interferometer design: incorporating the recombination configuration and testing a new suspension system. The 40m recombined optical configuration was the first operation of a suspended-mass Fabry Perot interferometer in which signals carried by the optically recombined beams are used to detect and control all the important test mass displacements. The experimental results were found to be in generally good agreement with the theoretical analysis of the performance expected from such an interferometer. A prototype of the new suspension system, which has the same basic design as the planned LIGO suspension, was also installed and characterized in the 40m interferometer for locking performance and sensitivity. The features and performance of the new suspension design were found to be desirable for the LIGO suspension.

1. Introduction

The Laser Interferometer Gravitational-Wave Observatory (LIGO) Project is developing facilities aimed at detecting and studying gravitational waves from astrophysical sources. The LIGO detectors will use highly sensitive laser interferometry to measure the relative motions of four test masses placed at the ends of two perpendicular 4 km long arms. One of the principal research tools used in this effort is the 40 meter interferometer at Caltech. It is used to test many of the features of the planned 4 km long detectors. The 40m interferometer has recently made two major advances which were necessary milestones to ensure the LIGO interferometer design: incorporating the recombination configuration and testing a new suspension system. This report discusses these two advances and also the future work in the 40 m interferometer.

2. Recombined Interferometer

2.1. Motivation and Significance

The optical topology chosen for the initial LIGO interferometer is an asymmetric power recycled Michelson with Fabry-Perot arm cavities with the asymmetry signal extraction scheme. The 40m interferometer had been operated as a locked Fabry-Perot interferometer since its original commissioning until 1995. In this configuration, as shown in Figure 1, the light returning from the two arms was independently sensed by two photodiodes. The frequency of the light was stabilized with the first arm as a reference, and the second arm length was measured using this stabilized light. The modification of the 40 m interferometer to a recom-
combined optical topology using the asymmetry signal extraction scheme is, therefore, a necessary intermediate step toward the full recycled optical topology in the 40 m interferometer.

2.2. **System Description**

A diagram of the recombined optical and control topology with asymmetric scheme is shown in Figure 2. The recombined beam at the symmetric port is demodulated with the in-phase modulating signal (I) or the quadrature-phase signal (Q) and produces signals, \( v_1 \) or \( v_2 \), respectively. The recombined beam at the asymmetric port is demodulated with the quadrature-phase signal and produces a signal, \( v_3 \). This scheme is valid due to the asymmetry in the Michelson arm lengths. The signal, \( v_1 \) is used to control the frequency of the light to the common mode arm cavity length. The signal, \( v_2 \) is fed back to the beam splitter control system to control the differential Michelson length. The signal, \( v_3 \) is used to control the differential arm cavity length. All three signals depend primarily on the degree of freedom they control.

2.3. **Performance**

Sensitivity

Figure 3 shows the sensitivity of the 40 m interferometer with the recombined configuration together with the predicted shot noise curves. The shot noise was estimated by two methods: calculation and empirical measurement. The shot noise was calculated using the measured parameters such as modulation depth, reflectivity and transmission of the mirror (See Table 1). The empirical measurement of the shot noise was accomplished by blocking the laser light and shining incandescent light on the antisymmetric photodiode. The interferometer readout signal was then calibrated with the effect of the loop gain properly accounted for. The calculated shot noise agrees with the empirical measurement within the uncertainties of the
parameters in the calculation. The effect of shot noise on the symmetric photodiode was verified to be negligible by both theory and experiment. However, the interferometer sensitivity does not seem to be limited by shot noise at any frequency, although the frequency dependence of the interferometer noise above 500 Hz is very similar to that of shot noise. This discrepancy was confirmed to be true by attenuating the light at the antisymmetric port and observing much smaller degradation in noise than expected if the sensitivity were limited by shot noise. All the known noise sources were carefully estimated, but nothing explained the noise above 500 Hz. This is one outstanding unresolved issue in the recombined interferometer experiment.
Table 1: Optical parameters for the recombined interferometer.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmissions</td>
<td></td>
</tr>
<tr>
<td>Beamsplitter</td>
<td>0.45</td>
</tr>
<tr>
<td>Input mirror</td>
<td>280 - 300 ppm</td>
</tr>
<tr>
<td>End mirror</td>
<td>12 ppm</td>
</tr>
<tr>
<td>Loss</td>
<td></td>
</tr>
<tr>
<td>Input mirror</td>
<td>110 ppm</td>
</tr>
<tr>
<td>End mirror</td>
<td>56 ppm</td>
</tr>
<tr>
<td>Asymmetry</td>
<td>50.8 cm</td>
</tr>
<tr>
<td>Modulation frequency</td>
<td>12.33 MHz</td>
</tr>
<tr>
<td>Modulation index</td>
<td>0.7 - 1.49</td>
</tr>
<tr>
<td>Contrast defect, 1-C</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Lock Acquisition

The largest concern before the operation of the recombined interferometer was
the undefined mechanism for the lock acquisition. The fundamental problem is
that the Fabry-Perot cavities using RF reflection locking techniques only provide a
signal which is linearly proportional to their length for very small deviations from
resonance. It is even more difficult in a suspended interferometer with high finesse.
The lock acquisition sequence observed was that the beam splitter acquired lock first
because of its broad range of linear operation, and then the common mode control
system acquired lock to hold one of the arm cavities on resonance, and finally the
remaining arm was locked by the differential mode control system after a while. It
was a surprise that the resonance of the arm brought by the common mode control
system was not disrupted by the intermittent chirp which was a contribution of the
out-of-lock arm while it was passing through resonances. This sequence was similar
to the situation we had with the locked Fabry-Perot interferometer.

Phase Reversal

A problem with the lock acquisition sequence is that the signal to control the
beamsplitter reverses sign in going from the case of one arm and the beamsplitter
in lock to the entire interferometer in lock. This is because the overcoupled Fabry-
perot arm cavities switch the phase of the reflected carrier light by 180 degrees
going from out of lock to in lock, as shown in Figure 4. A practical method to
avoid this problem is to increase the modulation depth, which changes the sign of
the beamsplitter error signal for the all-lock state with the sign maintained for the
one-arm-lock state. Table 2 shows that increasing the modulation depth changes
the $v_2$ signal on the differential Michelson phase $\phi_-$, whereas the other diagonal
components, the signal $v_1$ on the common mode arm cavity phase $\Phi_+$ and $v_3$ on
the differential mode arm cavity phase $\Phi_-$, are not affected at all. This is due to a
beating between the first and second order sidebands, neither of which experience
Table 2: Extracted signal sensitivity at different modulation depth.

<table>
<thead>
<tr>
<th>Phase</th>
<th>$\partial v_1$</th>
<th>$\partial v_2$</th>
<th>$\partial v_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\partial \Phi_+$</td>
<td>-7.8</td>
<td>-7.8</td>
<td>0</td>
</tr>
<tr>
<td>$\partial \Phi_-$</td>
<td>$2.3 \times 10^{-4}$</td>
<td>$2.3 \times 10^{-4}$</td>
<td>$-1.3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\partial \Phi_-$</td>
<td>2.5</td>
<td>2.5</td>
<td>$-2.5 \times 10^{-9}$</td>
</tr>
</tbody>
</table>

a phase reversal when the arm cavities go into resonance. This method worked reliably and it didn't degrade the sensitivity of the 40 m interferometer, because it was not limited by the shot noise. Another method we tried was to incorporate an automatic switch of the polarity change of the beamsplitter control system triggered by the locking of the both arm cavities. This system also worked, although not as reliably as the other method.

![Phaser diagram of the reflected carrier light to the symmetric port.](image)

3. New Suspension System

3.1. Motivation and Significance

The suspension system is one of the essential subsystem of the LIGO interferometer. The main function of the suspension system is to isolate a test mass from ground motion, to damp the motion of the test mass, and to provide inputs for length and alignment control signal. The design effort of the LIGO suspension started with establishing the requirements of the suspension system in 1995. Meanwhile the 40 m interferometer had been using the suspension system of the first

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This work was carried out by Seiji Kawamura and the LIGO team.
generation since the 40m interferometer was born. In 1991 the control system of the suspension was replaced to improve the sensitivity of the interferometer drastically at low frequencies, but the mechanical system still remained the same. We had found many undesirable features on this first-generation suspension system, and finally in 1993 a new suspension system of the second generation was designed for the small optics, such as the beam splitter and the mode cleaner mirrors. This type of the suspension system was adopted for the phase noise interferometer in MIT, too. Since then, we have found some unsatisfactory features on this system when we closely looked at the system, considering the requirements for the LIGO suspension. In 1995 we designed the suspension system of the third generation to demonstrate concepts of the LIGO suspension design; this third-generation suspension system has all the improvement incorporated based on our experience and the LIGO requirements.

3.2. System Description

The schematic view of the mechanical system and the control system of the 40 m test mass suspension is shown in Figure 5 and Figure 6, respectively. The test mass is suspended by a single loop suspension wire from the suspension block on the top plate of the suspension support structure. The wire standoffs and the guide rods are used to balance the test mass. Six magnet/standoff assemblies are glued to the test mass and five sensor/actuator heads are mounted on the head holders. The suspension support structure is strengthened by the stiffening bars. The test mass is protected by the safety cage and the safety bar which contain the safety stops. The position and angle of the test mass are detected by the edge sensor which consist of a pair of LED and photodiode. Either this signal or a signal from the 40m optical lever sensor is filtered, amplified and fed back to the coil to damp the test mass motion. A bias signal and an interferometer length control signal are injected in the control loop.

Figure 5: Schematic view of the mechanical system of the 40 m test mass suspension.
3.3. Performance

Single Loop Wire

The new suspension system employs a single loop wire instead of double loops. The single loop wire scheme simplifies the mechanical design of the suspension significantly. It is, thus, less subject to practical thermal losses due to complicated mechanics which may cause undesirable thermal noise in some modes. The single loop suspension is also free from any high-frequency resonances which arose from the complicated mechanism of the double loop wire suspension. The first-generation suspension system used a control block from which the test mass was suspended by two loops of wire. Figure 7 shows the schematic view of the old suspension. This system had a pitch resonance frequency of around 100 Hz above which the test mass and the control block moves in pitch with anti-phase. The Q factor of the pitch resonance was measured to be 2,000 - 3,000, from which the thermal noise of this pitch mode was predicted to be close to the noise spectrum of the interferometer around 100 Hz. Figure 8 shows the displacement sensitivity of the 40 m interferometer as of Oct. 28, 94 together with the predicted noise of various sources. It can be seen that at least two noise peaks at 80 Hz and 110 Hz are very likely due to the pitch thermal noise.

Since only one of the four old suspensions was replaced with the new suspension this time, we could not observe any significant improvement around 100 Hz in the noise spectrum after the replacement. However, it is expected that replacement of the remaining old suspensions with the new suspensions in near future will improve the noise performance around 100 Hz.

It was a concern that balancing the test mass in pitch might be difficult using only one loop of wire. As shown in Figure 9, guide rods and wire standoffs were used to balance the test mass. A small aluminum guide rod was first glued to the mass. A larger aluminum rod was then placed below the guide rod between the test mass and the wire. The wire standoff has a groove on it, on which the wire rests. The
Figure 7: Schematic view of the old 40m test mass suspension.

Figure 8: Displacement sensitivity of the 40m interferometer and noise prediction of various sources.
test mass was balanced in pitch adjusting the position of the wire standoff along the guide rod using a piezoelectric buzzer. This procedure allowed balancing the test mass in pitch within 0.5 mrad from the vertical direction dictated by gravity.

Figure 9: Guide rod and wire standoff.

Edge Sensor without Vane

The new suspension system employs a simple edge sensor without vane. As shown in Figure 10, an LED and photodiode pair senses the shadow of the magnet. The magnet does not have any vane on it; the magnet itself acts as a vane. This design ensures minimum degradation of the thermal noise of the test mass internal mode due to additional attachments to the mass. This ”no vane” scheme is also good to keep the resonance frequency of the magnet/standoff assembly high.

Figure 10: Edge sensor without a vane in the new suspension system.

The second-generation suspension system used a slot sensor instead of an edge sensor. Figure 11 shows the slot sensor with a big vane with a slot through which
the LED light shines the split photodiode. The differential signal of the split photodiode gives the position signal of the test mass. The essential point behind this design is that the LED intensity can be stabilized using the sum output of the split photodiode. However, it requires the vane to be too large, which is not only bad for thermal noise of the test mass internal mode but also undesirable in terms of mechanical features. The resonance frequency of the vane/magnet/standoff assembly was found to be as low as 1 kHz, which could interfere with a control system with a unity gain frequency of around 1 kHz.

Figure 11: Slot sensor and a large vane in the second-generation suspension system.

We measured the lowest resonance frequency of the magnet/standoff assembly without a vane to be 7.7 kHz; this is high enough not to interfere with the control system. The biggest concern of this design was that large cross-coupling of sensing between each degree of freedom due to no vane might cause instability in the control system. It turned out that by carefully aligning the relative position of the magnet to the sensor within 500 mm or so, the control system can be made stable and robust. Another concern was the stability of the sensor; no active intensity stabilization is available for the edge sensor. It was found that the stability of the LED power was well below the shot noise level of the sensor above 50 Hz, when the current to the LED was passively stabilized.

4. Future Work

4.1. Recycling

The program to convert the 40 m interferometer to a recycled configuration has started recently (summer in 1996). The vertex masses have already been replaced with ones of higher transmission. Reconfiguration of the vacuum envelope to accommodate the recycling mirror is underway. The recycled interferometer will provide exciting opportunities to answer questions which are to be solved before the 

\textsuperscript{c}This work is being carried out by Jennifer Logan and the LIGO team.
LIGO design is finalized. Understanding the lock acquisition process in the recycled configuration is necessary to proceed to the LIGO interferometer. A suspended recycled interferometer will also provide the chance to investigate noise performance with the same configuration as LIGO.

4.2. Beamsplitter Suspension
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The prototype of the suspension system for the LIGO small optics, such as the mode cleaner mirrors, has already been built. It will be tested as the beamsplitter suspension in the 40 m interferometer. The installation of the new beamsplitter suspension will take place during the reconfiguration of the vacuum envelope for the recycled interferometer. This system has a mechanical design similar to the second-generation suspension with features of the new 40m test mass suspension. It has a VME-based control system, so that the gain, matrix coefficient, and polarity switch, etc. are all controlled on the computer screen instead of by turning knobs.

5. Summary

The 40m recombined optical configuration was the first operation of a suspended-mass Fabry Perot interferometer in which signals carried by the optically recombined beams were used to detect and control all the important test mass displacements. The experimental results were found to be in generally good agreement with the theoretical analysis of the performance expected from such an interferometer. The lock acquisition was found to be quite easier than expected. The phase reversal of the beamsplitter control signal, which had not appreciated before the experiment, was coped with cleverly. The experiment of the recycled interferometer has been started, and will provide useful information for the LIGO design.

A prototype of the new suspension system, which has the same basic design as the planned LIGO suspension, was also installed and characterized in the 40m interferometer for locking performance and sensitivity. The features and performance of the new suspension design, such as a single loop wire and simple edge sensor, were found to be desirable for the LIGO suspension. The prototype of the LIGO small optics suspension will be installed and tested as a beamsplitter suspension in the 40 m interferometer. This will give useful feedback to finalize the LIGO small optics suspension.

Acknowledgments

I thank the entire LIGO team, because this work was done by the LIGO team led by Barry Barish. Torrey Lyons was the main contributor for the recombination work, and Jenny Logan is a task leader for the recycling experiment. This work was supported by the National Science Foundation cooperative agreement number PHY-9210038.

\( ^d \)This work is being carried out by Seiji Kawamura and the LIGO team.
References
PROGRESS IN DEVELOPMENT OF SOME NEW TECHNIQUES FOR LASER INTERFEROMETER GRAVITATIONAL WAVE DETECTORS

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ABSTRACT
A new experimental project aimed largely at exploring and developing concepts for extending the performance of gravitational wave laser interferometers has recently been initiated at Caltech. One area is extension of operation to lower frequencies. The practical viability of some proposed test mass suspension and seismic isolation systems incorporating magnetic levitation techniques is being experimentally investigated. Some more general concepts for interferometric systems for low frequency gravitational wave detection, with potential applicability in large scale interferometer facilities currently under construction, are being developed. Interferometer designs incorporating diffractive coupling to allow use of lower-noise test mass materials, and increased light power for high frequency operation, are also being investigated. Progress in the development of some of the above techniques and concepts is discussed, and experience and results from current experimental work described.

1. Introduction
A new project to explore and develop various concepts and techniques for gravitational wave detection, and other measurements in gravitational physics, is now being built up at Caltech. A prime objective is to develop techniques which will make possible improved detection capabilities in large-scale gravitational wave facilities. At this stage in the project we are just beginning to build up a prototype interferometer test system, but it may be useful to outline some of the concepts on which we have already begun experimental work. In the original development of laser interferometers for gravitational wave detection it was natural to concentrate on optical sensing techniques first, for these seemed to present the most serious problems. In the early work in which the author was involved at Glasgow and subsequently at Caltech the simplest test mass suspension and isolation methods were used - the suspensions being simple pendulums with steel wires, and seismic isolation being by lead or stainless steel and elastomer stacks, which had proved convenient and practicable in resonant bar detectors. With techniques such as these, the overall interferometer performance achieved in a few groups eventually reached a level which could justify the expansion to larger-scale facilities. Further development work on optical sensing is still important, but in the early stages of the present concept-exploring project it seemed useful to begin preliminary work in some slightly different areas.

The overall sensitivity of present ground-based laser interferometer gravity-wave detector systems is expected to be mainly limited at high frequencies, above about 1 kHz, by photon shot noise; at medium frequencies, between around 1 kHz and
100 Hz, by thermal noise in test masses and suspensions, and at lower frequencies by seismic noise transmitted by suspensions, with an anticipated underlying gravity-gradient background from moving material and objects in the neighborhood. Improvement of the performance in any part of the frequency spectrum would be beneficial. However, there can be particular importance in extending the range of sensitive operation at both high frequencies, where detection of signals from both supernova events and pulsars could be enhanced; and at low frequencies where detection of signals from coalescence of neutron-star and black-hole binaries could be tracked for longer periods, and there are more potential pulsar sources. It is planned to explore concepts for extending performance in both regions.

At high frequencies, detector sensitivity is expected to be limited mainly by photon shot noise, and this may be improved by increasing the light flux in the interferometer. One practical limit to power can be heating effects in test masses and mirror substrates, and we have earlier outlined some ideas for use of diffractive coupling into the main optical cavities of the interferometer to obviate the need for transmission of light through any substrate material, and allow use of higher light flux. Experimental investigation of this technique is planned. However, preliminary experiments relating to the low-frequency performance of interferometers have already begun, and we will outline some of these here.

2. Problems of Low-Frequency Ground-Based Gravity-Wave Detectors

The main practical limits to the low-frequency performance of existing interferometers arise from the need to suspend the test masses against gravity, and from the associated problems of isolating the suspension systems from seismic and other ground motions, which are typically many orders of magnitude larger than the displacements expected from gravitational waves. Seismic isolation systems are often made up of a stack of cascaded mechanical oscillator assemblies, each in principle approximately equivalent to a mass supported by a flexible spring. The natural resonance frequency $f_0$ of each stage is usually in the range 1 to 10 Hz, and if the mechanical quality factor $Q$ of the stage is high, then the amplitude of an input motion at a frequency $f$, large compared with $f_0$, is attenuated by a factor of approximately $(f/f_0)^2$. Thus, the isolation given by a single stage falls off as the frequency becomes lower, and for a number of cascaded stages the fall-off is even more rapid. The difficulties are enhanced by the fact that ground motions themselves increase rapidly at low frequencies.

Significant work to develop more effective isolation systems is being carried out by several groups. Many ingenious methods for constructing passive mechanical isolation stages with low resonance frequencies have been proposed and tested. These include, in the VIRGO Project and elsewhere, springs of cantilever and other forms, permanent magnet assemblies to reduce restoring forces from springs, and inverted pendulums; a cross-coupled pendulum system known as an X-pendulum for the TAMA Project; a folded normal/inverse pendulum system in Western Australia, and others. Several active systems using accelerometers to sense motions and
counter them are also under development. Many of these systems are likely to be successful, but practical problems such as internal resonances and other phenomena cause the designs to become complex. It therefore seemed useful to explore some other approaches. Two general methods are being investigated here: (a) use of magnetic levitation for seismic isolation and test mass suspension, and (b) techniques for coupling the suspension systems at each end of an interferometer arm, to make residual seismic motion largely common-mode and capable of rejection.

3. Magnetic Levitation Concepts for Seismic Isolation and Suspension

The general idea of using magnetic levitation in gravitational wave detectors is not new - it was used in early work with several cryogenic bar detectors, which were designed for operation at low temperatures for other reasons such as use of superconducting SQUID sensors and reduction of thermal noise. However, the large weights of the bars, typically several tons, made the superconducting techniques inconvenient, and these techniques were gradually superseded by simpler mechanical suspensions. In the present work we are currently investigating room-temperature systems based on permanent magnets, in an effort to make designs which may be relatively convenient for use with laser interferometers.

3.1. Potential Advantages and Difficulties of Magnetic Isolation Systems

A simple magnetic support system can in some ways be regarded as equivalent to a spring-mass isolator, and the isolation would not be expected to differ from that of an ideal spring-mass system with the same effective resonant frequency. An important advantage, however, is that the magnetic field can provide a close approximation to an ideal weightless spring, so secondary spring resonances which degrade mechanical systems do not occur. Another major advantage is the possibility of designing magnetic systems with low resonance frequencies together with the convenience of obtaining isolation in all degrees of freedom in a single stage. There are also some obvious disadvantages to be considered. A magnetic system is potentially vulnerable to noise from interaction with changing external magnetic fields, and it is important to keep this small. Thermal noise may arise from interaction between magnetic fields with Johnson currents in resistive conductors including any conductivity of the magnetic material itself. Mechanical damping in the magnets may also give thermal noise. Noise arising from Barkhausen effect in the magnetic material has to be considered. And, as a static system of permanent magnets alone is in principle unstable, a servo system or other method is required to keep the system stable. One objective of the present research is to find and explore possible ways of avoiding these difficulties. Magnetic levitation methods are being considered both for seismic isolation stages, and for the final suspension stage of a test mass itself. The requirements for these two applications are different, since for isolation at stages where seismic noise is large, thermal noise may be relatively insignificant; while low noise is a major factor for the suspension of a test mass itself.
3.2. Seismic Isolation Stages: Some Magnetic Configurations Investigated

Several different magnetic configurations have been investigated in preliminary work. A simple one is indicated in principle in Figure 1, where a fixed dipole magnet with its axis vertical attracts a similarly-oriented magnet forming part of a test mass, at a position where the magnetic force precisely cancels its weight. This arrangement can be made stable by a servo system, not shown here, which senses the vertical position of the levitated magnet, and suitably adjusts the lifting field by a small trim coil around the fixed magnet. Such a system can typically have resonance frequencies of order a few Hertz for linear motion in any direction.

Lower resonance frequencies can be obtained in a system in which the supporting vertical field gradient varies less rapidly with position, and a simple way of achieving this is with a pair of lifting magnets, as illustrated in Figure 2. At the position of the levitated magnet there can be a large field gradient with a relatively small mean field. In experiments and modeling of this configuration, we have found that there can be an instability in which a finite horizontal displacement of the levitated magnet from the symmetrical position leads to its rotation and further displacement, even when its vertical position is stabilized. This instability can be avoided by connecting together three or more systems of this type, and this arrangement is practical in a seismic isolation stage. A system of this type used in preliminary experiments is illustrated schematically in Figure 3. Here Hall-effect sensors located in positions where the fields from the fixed and levitated magnets tend to cancel were used to monitor vertical position. A small test system of this type, with a payload of 1 kg, had typical horizontal oscillation periods of order 3 seconds, with shorter periods for vertical motion. The magnets used were electrically conducting rare earth magnets, giving eddy current damping which can be useful in an isolation stage. This general arrangement may be scaled to support large masses. Small changes in load may be accommodated by adjusting the vertical positions of the fixed support magnets,
keeping the power dissipation in the trim coils small. For a low-noise application in a final suspension of a test mass this system is less suitable, and a levitated system with simpler geometry giving fewer internal resonances is desirable.

3.3. A Magnetic Test Mass Suspension

For the suspension of a test mass itself, the requirement of low noise puts several difficult constraints on the design of a magnetic suspension. Consideration of these, and experience with the systems outlined above, led us to a different type of configuration. The basic concept here is to arrange as far as possible that all field gradients due to the support magnet system, including those required to provide the lifting force, are in directions orthogonal to the sensitive direction of test mass motion. This may be achieved to a satisfactory approximation by making the fixed magnet system elongated in the direction of the light beam in the relevant interferometer arm, with all dimensions in that direction much larger than dimensions normal to it. A simple example of this concept is illustrated in Figure 4, where a single long fixed magnet magnetized transverse to its length, supports a smaller levitated magnet forming part of the test mass. Stability can be achieved using a height monitoring servo, arranged in this case to operate independently of the position of the levitated magnet in the beam direction.

If the system were so long compared with its transverse dimensions that end effects are negligible, then the period of oscillation of the levitated mass in the beam direction would tend to infinity. Also, in this ideal case, Johnson currents in the levitated magnet coupling to the field of the fixed magnet would not lead directly to forces in the sensitive direction, although there are possible mechanisms for indirect coupling. Johnson noise in the fixed long support magnet could couple directly to the test mass, however, so it is important that this magnet be electrically non-conducting. In practical tests of this configuration the long support magnets have been of non-conducting ceramic magnetic material, and typically of length
30 cm and transverse dimensions of approximately 1 cm. It has been found that non-uniformity of the magnetization of the magnets we have used has been more important than end effects, and it has been practicable to choose magnets giving natural periods of oscillation of order 10 seconds without applying any compensating fields.

A serious source of noise could arise from coupling to changing external field gradients, and to reduce this we have proposed use of two or more magnets on the test mass, arranged so that the total dipole moment and possibly higher order moments are sufficiently cancelled. A practical configuration used in recent experiments is shown in Figure 5, where there are two magnets on the levitated mass and a corresponding pair of elongated support magnets above these. In this preliminary work the height of the test mass has been sensed by light beams traveling parallel to the main laser beam direction, from one or two small light-emitting diodes. The unobstructed part of the beams is sensed by photodiodes which control currents in long narrow coils around the support magnets, as shown. A double sensing system is illustrated, since this has been used in some experiments to provide active damping of transverse rocking modes of the test mass, but a single sensing system has also been used.

3.4. Experimental Findings

Preliminary experiments are in progress with suspensions of this general type, using test masses up to 0.5 Kg. In tests with a single-magnet version it was found practical to achieve periods of the pendulum-mode oscillation of up to 20 seconds by using a small auxiliary magnet to partially compensate the dominant nonuniformity of the support magnet.

The relaxation times for the longitudinal "pendulum" mode of oscillation observed in various levitated test mass systems of the types outlined here have typically been in the range from 8 to 18 hours. There are several different mechanisms which could lead to such relaxation times in experiments like these, including recoil effects in the support structure quite unconnected with the magnetic suspension itself. At this preliminary stage these possible effects have not yet been investigated, and these typical relaxation times are reported merely to indicate some experimental upper limit to the damping by the magnetic systems.

The laboratory in which these experiments are being done shows some large variations in temperature, and we have found that resultant changes in the field strength of the permanent magnets causes changes in the height of the equilibrium position for the test mass. The height-stabilizing servo system has been arranged so that it maintains the average current in the trim coils integrated over several hundred seconds close to zero, by adjusting the effective height setting. The average power dissipation in the trim coils can be small, typically of order 50 microwatts.
3.5. Proposed Techniques for Reducing Internal Thermal Noise in the Levitated System

Thermal noise from internal vibrational modes of a test mass system can be a significant noise source in a gravitational wave interferometer. It is not yet clear if suitable magnetic materials are available with internal mechanical damping as small as that of possible test mass substrate materials, such as fused silica or sapphire. We have proposed, and are considering, ways of achieving a low-noise system even in the presence of magnets having relatively high mechanical damping. In one arrangement, the magnets are mounted on a section of the low-loss test mass material which is isolated from the main part of the mass by suitable flexures. One possible way of doing this is indicated in the enlarged view of part of the test mass system given in Figure 6. Here the test mass material is shaped and etched so that a small platform is formed for the magnets, connected to the main part of the test mass by a thin blade which acts as a low-loss flexure, and partially isolates the main part of the test mass from mechanical damping in the magnet material. There will be one or more sharp resonances corresponding to bending and extension modes of the flexure, but with suitable choice of the position of the effective center of the monitoring light beam it is expected that the component of the thermal noise sensed by the beam can be kept relatively small. Further work is required to check the effectiveness of this and other possible arrangements, such as the variant illustrated in Figure 7 where the blade is replaced by a pair of small posts to reduce coupling of transverse strains.

In a high-performance interferometer it may be necessary to apply small auxiliary forces and torques to a test mass for precise positioning and orientation. In Figure 6 are indicated pairs of small fixed coils, located on either side of the levitated magnets, which have been used in some tests to provide auxiliary field gradients for such purposes, without requiring any additions to the test mass itself.

Fig. 6

Fig. 7.
4. Concepts for Extending Low-Frequency Interferometer Operation with Coupled Suspension Systems

We have suggested earlier the idea of coupling together interferometrically the suspension points of the usual pendulum suspensions at the end of each arm in a gravity-wave detector, so that residual seismic noise which penetrates the isolation stacks is rendered largely common-mode, and may be rejected in first order if the suspensions are identical. If the performance of the monitoring interferometer is similar to the main test-mass interferometer system, it becomes in principle possible to make measurements down to frequencies below the resonance frequency of the suspensions. With long-period suspensions such as those described above, any tilting of the ground, or of early stages in the isolation system, can move the effective suspension point of the equivalent pendulum system. In these circumstances it can be useful to couple the tilts of the suspension systems, as well as their relative horizontal positions. This may be done with a pair of auxiliary interferometric monitors, as indicated in Figure 8. Here an application to magnetic suspensions, of the type shown in Figure 5, is illustrated, although the technique is applicable for pendulum suspension systems also. Techniques of this kind can provide a useful and relatively simple way of enhancing seismic isolation at frequencies where the usual isolation techniques are not very effective. With this type of interferometer system it may be practicable to make measurements at very low frequencies, where changing gravity gradients are the dominant noise source.

5. Acknowledgements

I would like to acknowledge the valuable assistance of S. J. Augst, who built and tested most of the levitation systems outlined here. I would also like to thank E. W. Cowan for very helpful discussions and computer modeling; and J. L. Hall and C. W. Peck for much encouragement and stimulation. The experimental work was supported by the California Institute of Technology.
References


5. There are some similarities to a compound pendulum test mass system, such as studied by L. Ju and D.G. Blair, *Meas. Sci. Technol.*, **5**, 1053-1060 (1994).

6. Small posts were used and found to be effective for reducing thermal noise from control magnets attached to a test mass in the Caltech 40-m prototype interferometer by A.D. Gillespie: Ph.D. Thesis ”Thermal Noise in the Initial LIGO Interferometers,” California Institute of Technology, 1995.
JAPANESE GRAVITATIONAL WAVE OBSERVATORY (JGWO)

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and

Interested Members of the TAMA group
National Astronomical Observatory, Laboratory for High Energy Physics, School of Science in the University of Tokyo, School of Engineering in the University of Tokyo, Earthquake Research Institute in the University of Tokyo, Yukawa Institute for Theoretical Physics in Kyoto University, Institute for Laser Science in University of Electro-Communications, The Institute of Space and Astronautical Science

ABSTRACT
We are planning a km-scale interferometer that will be a scaled-up version of the TAMA detector in Japan. We have done a feasibility study for a detector with higher sensitivity than any currently being constructed in the world. Parameters needed to attain this have been determined and will make possible detecting events in galaxies 60Mpc away. We plan to begin this project right after the TAMA completion in 1999.

1. Introduction

Direct detection of gravitational waves is significant for two reasons. One is that it will be the first experimental test of Einstein’s theory of general relativity for strong gravitational fields\(^1\) and the other is that it will provide a new tool for observing astronomical objects that cannot be seen by any other means. No one doubts the value of detecting gravitational waves. However, there are technical gaps between the present state of the art and that necessary to realize a detector of such high sensitivity.

A single gravitational wave detector can allow estimation of the approximate distance to the source for the chirping signal from a coalescence of binary neutron stars. This is one distinctive feature of gravitational wave detection. Three ideal detectors could determine the source direction,\(^2\) although allowing for detector noise, at least four detectors with equivalent sensitivity will be needed. The detectors of LIGO\(^3\) and VIRGO\(^4\) will likely be among these four, and GEO\(^5\) and TAMA may also be included. TAMA aims for a displacement sensitivity equivalent to that of LIGO or VIRGO. It will be able to catch an event in Andromeda with S/N=10.\(^6\) However, the TAMA sensitivity is not enough to obtain astronomically useful data. Therefore, we propose here a version of TAMA scaled up by one order, whose sensitivity should be better than that of LIGO or VIRGO by nearly one order. This project will be started immediately after the completion of TAMA in 1999.
2. Plan of JGWO

2.1. Target of This Project

For resonant gravitational wave antennas, burst waves have been the main target of observation due to the narrow frequency bandwidth of the antenna. Thus it is necessary to make simultaneous observations with identical antennae or other observational tools to discriminate a signal pulse from random pulse noises. However, the observation by a laser interferometric detector does not necessarily need other observational tools to make coincidental discrimination because it can observe the highly characteristic waveform of a neutron star binary coalescence. Once a direct detection is made, the “pure physics” objective of the project will be largely achieved. At this stage, cooperative observation will likely be coordinated using a network of gravitational detectors with comparable sensitivity throughout the world. I do not know exactly when we can expect this but I can surely say that it is not now.

Since we have not succeeded in detecting gravitational waves yet, it is natural to aim for the first detection. The mission of this project is this first detection of gravitational waves. To realize the first detection, the sensitivity goals of present projects throughout the world are unlikely to be enough. We need to cover sufficiently many remote galaxies to give an expected event rate of one per year. Since the rate of binary neutron star binary coalescence is once per \(10^6\) years per galaxy, the detector needs to be sensitive to events out to 60 Mpc. This is roughly three times the distance to the Virgo cluster, which means three times improvement of the target sensitivity of the first LIGO detector. At this sensitivity we can expect at least one event every year.

2.2. Parameters of the Detector

In this project, we have given the first priority to developing sensitivity raising techniques. We hope to attain the target sensitivity by incremental improvements to the techniques of TAMA, but we are also considering an optional cryogenic mirror to ensure we meet the specification. The target sensitivity is three times that of the first LIGO. Parameters to make this possible are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline length</td>
<td>3 km</td>
</tr>
<tr>
<td>Laser power</td>
<td>100 W</td>
</tr>
<tr>
<td>Recycling gain</td>
<td>50 (Effective power 2.5 kW)</td>
</tr>
<tr>
<td>Mirror Loss</td>
<td>1 ppm</td>
</tr>
<tr>
<td>Sapphire Mirror</td>
<td>30 cm, (Q = 10^7), 54 kg</td>
</tr>
<tr>
<td>Suspension</td>
<td>(Q = 10^7)</td>
</tr>
</tbody>
</table>

We suppose here that the techniques presently under development for TAMA will be perfected by the starting time of this project and that we include the optional cryogenic mirror to guarantee the sensitivity in the mid-frequency region (around
100 Hz). Figure 1 compares the sensitivity curve with that of the initial LIGO detector.

3. Interferometer Design

The optical design basically follows that of TAMA, with appropriate allowance for the larger baseline. The biggest difference will be the light diffraction effect inside the vacuum ducts. Construction cost and problems with vacuum technology prevent us from maintaining the same geometrical ratio of diameter to length as that of TAMA, which is 750. Almost all techniques developed for TAMA will be improved and applied to JGWO as well.

4. Features of Noise Reduction Technique

Assuming the successful completion of TAMA, we will add the following techniques to enhance the sensitivity.

- **Photon shot noise**
  We will increase the laser power, reduce the mirror losses and increase the internal quality of the optical elements. TAMA uses a 10 W injection locking solid state laser system developed by SONY corporation. At present the limit of the output power of a single laser system seems to be only a little more than 20 W. Coherent addition of such lasers will be adopted to obtain the target power of 100 W.

- **Mirror thermal noise**
  We will adopt sapphire mirrors, which have a high Q even at room temperature, and can be used to even greater advantage in a cryogenic system. Since a mirror made of fused silica has loss peak at about 30 K, which is worse by two orders, we cannot expect noise improvement at low temperature so long as we use fused silica material. Heat production in the mirror will be removed by wires of high heat conductivity. Aluminum of 99.999% or greater purity is the most promising material for the wire. However, such pure metal has relatively low mechanical Q. We plan to develop a composite material of pure metal and high Q material to combine both characteristics.

- **Suspension thermal noise**
  Mirrors will be suspended by wires with high mechanical Q. For the room temperature system, fused silica threads will be bonded directly to the mirrors. We need to make the mechanical system of the suspension simpler than TAMA. Advanced calculations of the thermal vibration of mirrors suggests more displacement noise than had been thought. This is because high frequency resonant modes of a mirror have been neglected so far. Material with much higher Q at room temperature must be sought. Otherwise, we will need to cool the mirror down to cryogenic temperature.
Figure 1: Sensitivity curve of JGWO. The sensitivity of the initial LIGO is also shown for reference.
• **Vibration isolation system**

The rubber used in the isolation stacks for TAMA will be replaced by damping metal, which has been developed in sheet form for application such as quiet home washing machines. While seal by a bellow for rubber is being researched for TAMA, a new damper is desirable because the seal mechanism introduces many possible vacuum leaks and length instability due to ambient temperature change. An X-pendulum horizontal isolation system has been developed and worked as expected. It turned out to be reliable despite a number of structural resonances above several Hz. Although the target observational frequency band is located around 300 Hz in TAMA, this system of low frequency isolation will be adopted to reduce feedback load in mirror control suspension system. The effectiveness will be tested in TAMA development. If the isolation in low frequency is greatly improved, extra vertical isolation will be needed to make the system effective because some vertical excitation is converted to horizontal vibration due to internal cross-coupling in the system. At low frequency frequency, a reliable passive vertical isolation system is not known. Therefore, an active vertical isolation system will be incorporated into the horizontal two-dimensional X-pendulum system.

5. **Detector Site**

We have searched in Japan for a site where the detector can be constructed and have nominated seven candidates. Two of these would be suitable for above ground construction and the others are underground. We considered geologic features, climate, convenience of commuting from the Kanto area where Tokyo lies, ownership of land, usage of land, and possible future usage of surrounding land, construction feasibility, and so on. Since land in Japan is intensively used everywhere, transferring the ownership of land is time-consuming and expensive. We will select a few candidates soon for detailed investigation and make precise estimates of the construction cost.

6. **Host Institute and Manpower**

Researchers working in gravitational physics have been scattered so far in Japan. Although TAMA construction is hosted by National Astronomical Observatory (NAO) with the cooperation of National Laboratory for High Energy Physics (KEK) and the Institute for Cosmic Ray Research (ICRR), JGWO is, at present, expected to be hosted by ICRR. The present main project of ICRR is the super-Kamiokande neutrino detector. However since ICRR was established for inter-university cooperative research programs, it is appropriate to extend the research activity to gravitational wave detection.

The present gravitational group in ICRR is not sufficiently staffed to promote this project. We hope to have at least two and half laboratories which includes a total of 13 permanent staff and 2 exchange researchers with several postdoctoral researchers by the beginning of the project.
7. Schedule

This project should be completed in five years not including time spent for design and site investigation. Depending on the total budget, applying for funds to the government will proceed more or less as in Figure 2. Other project plans are also shown because such big projects as JGWO compete with each other in budget. Note that ICRR plans to move from its present location to a north-eastern suburb of Tokyo in 2000.

Figure 2: Schedule of JGWO in years. Other large competing projects are shown.

References


8. Kazuhiro Yamamoto, Keita Kwabe, and Kimio Tsubono, *in these proceedings*
LOOP INTERFEROMETER FOR GRAVITATIONAL WAVE DETECTION

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ABSTRACT
We proposed a zero-area loop interferometer as a gravitational wave antenna in 1990 and have been working on the tabletop prototype since 1992. In this paper, we emphasize the merits of our loop interferometer over conventional Michelson interferometers. We also report the experimental developments including stable photon recycling. Finally, we propose the ultimate recycling scheme which makes full use of the white-light nature of the loop interferometer.

1. Introduction
The major efforts in gravitational wave detection have so far converged on Fabry-Pérot Michelson interferometers with power recycling which require single-mode frequency-stabilized laser with very narrow linewidth. Unfortunately, this requirement contradicts with the desire for higher power operation of laser for better sensitivity.

In this paper, we propose a potential alternative. We proposed a zero-area loop interferometer for gravitational wave detection in 1990\(^1\) to solve the weakness of Michelson interferometers and have been working on the tabletop experiments since 1992.\(^2\) Recently, the group at Stanford University reported the same scheme.\(^3\) First, we introduce our loop interferometer and emphasize its white-light operability and stability in contrast to the Michelson interferometers. Then we report the conventional recycling experiments with our loop interferometer and point out that it sacrifices most of the virtues of the loop configuration. Finally, we describe the ultimate recycling scheme\(^2\) which makes full use of the white-light operability of the loop interferometer, allows multi-mode laser operation, and maximizes the optical power available to the interferometer.

2. Loop Interferometer
We briefly describe the principle and characteristics of loop interferometers. Figure 1 (a) shows a loop interferometer often referred to as the Sagnac interferometer. The lightwave is split into two by a beamsplitter with reflectance \(R\) and transmittance \(T\). One wave travels the loop clockwise and the other does counterclockwise. They meet again at the beamsplitter to give the interference fringes. If a round-trip phase difference between counter-propagating waves is \(\Delta \phi\), the output intensity is given by

\[
I_{\text{out}} = I_{\text{in}} \left\{ (R - T)^2 + 4RT \sin^2 \frac{\Delta \phi(t)}{2} \right\}.
\]

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(a) Loop interferometer known as the Sagnac interferometer  
(b) Zero-area loop interferometer for gravitational wave detection

Figure 1: Loop interferometers

Statically the two waves travel the same optical path. Therefore the dark fringe is automatically obtained regardless of the optical frequency as shown in Figure 2, where $L_{cw}$ and $L_{ccw}$ are the path-lengths and $\Delta L$ is their difference. This white-light dark fringe is the most distinct feature of the loop interferometers.

Figure 2: White-light dark fringe

The difference in round-trip phase or path-length arises only from non-reciprocal or time-dependent phase shifts along the loop. When we use loop interferometer as a gravitational wave antenna, two things must be considered: 1) Sagnac effect should be eliminated so as not to detect rotation; 2) Time-dependent phase shift induced by gravitational wave should be efficiently sensed. We cancel the Sagnac effect topologically by twisting the loop and making the effective loop area zero.
We extend the loop in two orthogonal directions in the similar way as the two orthogonal arms in Michelson interferometers. We proposed the loop interferometer for gravitational wave detection as shown in Figure 1 (b) in 1990.¹

3. **Loop Versus Michelson**

In Michelson interferometers, the optical path-length of the two independent arms must be stabilized to obtain the dark fringe. This becomes very difficult when arm length is extended to kilometers to tens kilometers. It becomes even harder and virtually impossible to get white-light dark fringe by making path-length difference exactly zero. Therefore Michelson interferometers are sensitive to laser frequency fluctuations and require single-mode frequency-stabilized lasers with very narrow line-width.

In loop interferometers, white-light dark fringe is automatically obtained without any stabilization in path length. Therefore loop interferometers are insensitive to laser frequency fluctuations. This makes high power operation much easier since multi-mode operation is allowed.

![Figure 3: Response to gravitational wave](image)

Now we compare them in terms of sensitivity to gravitational wave and other sources. The arms are extended in two orthogonal directions which are alternately stretched or shrunk by gravitational wave. The round-trip phase difference caused by gravitational wave at frequency $f$ with strain amplitude $h$ is given by

$$\Delta \phi_{\text{loop}}(t) = \frac{16 \pi h L_{\text{arm}} \sin^2 \left(2 \pi f L_{\text{arm}} / c \right)}{\lambda} \cos(2 \pi ft),$$

for the loop interferometer with the arm length $L_{\text{arm}},$¹ while that for the Michelson interferometer with the same arm length is given by

$$\Delta \phi_{\text{M}}(t) = \frac{8 \pi h L_{\text{arm}} \sin \left(2 \pi f L_{\text{arm}} / c \right)}{\lambda} \cos(2 \pi ft).$$
The minimum arm length which maximizes the sensitivity to the gravitational wave at frequency $f$ is $c/4f$ for both interferometers. The maximized sensitivity of the loop interferometer is twice as large as that of the Michelson interferometer since total path length is twice as well. In the loop interferometer, one lightwave sees both arms stretched while the other sees both shrunk if the loop round-trip time $4L_{\text{arm}}/c$ of the lightwaves coincides with the period of gravitational wave $1/f$.

Figure 3 shows sensitivities of the interferometers with the same arm length as functions of gravitational wave frequency $f$. The Michelson interferometer has its maximum sensitivity at frequency zero and therefore it is very sensitive to the static path-length difference and low frequency path-length fluctuations caused by seismic effects. In contrast, the loop interferometer is completely insensitive to the static path-length difference and much less sensitive to low frequency path-length fluctuations than Michelson type. Therefore the loop interferometer is much robust and less dependent on sophisticated stabilizations.

The only drawback of the loop interferometer is that it is incompatible with Fabry-Pérot arms which can gain storage time. If we insert a Fabry-Pérot cavity in the loop, it couples two counter-propagating waves, destroys automatic dark fringe, and disables white-light operation. Therefore the loop interferometer is suitable for delay-line or huge interferometers.

4. Tabletop Prototype

![Prototype loop interferometer](image)

(a) Prototype loop interferometer  
(b) Sensitivity evaluation

Figure 4: Tabletop prototype loop interferometer

Figure 4 (a) schematically shows our tabletop prototype interferometer. We use

320
an LD pumped Nd:YAG laser with 50mW output power as a light source. Each arm is 1.5m long and the loop length is 6m. Therefore the sensitivity is optimized for 50MHz. Real interferometers for gravitational wave at around 1kHz should have the effective arm length of 75km and the loop length of 300km.

Figure 4 (b) shows how we measure the sensitivity of the interferometer. We insert a LiTaO$_3$ electro-optic phase modulator into the loop at the off-center position and emulate a gravitational wave by sinusoidal phase modulation at 50MHz. Modulation-induced signals in the photo current at the dark port are observed. Preliminary experiments which we did in very early stage showed the minimum detectable phase about 3dB above the shot-noise level.

5. Photon Recycling

In order to reduce the shot noise level below the phase difference induced by gravitational wave, we must increase the number of photons in the interferometer by photon recycling.

![Experimental setup](image)

Figure 5: Conventional photon recycling for loop interferometer

Since the loop interferometer acts as a white-light reflector with almost unity reflectivity, photon recycling can be achieved by adding a recycling mirror between
the laser and the interferometer as shown in Figure 5 (a). Then the recycling mirror and the interferometer form a high-Q Fabry-Pérot recycling cavity. The reflectivity of our interferometer is measured to be 97%. Therefore we can expect the recycling gain of 30 when we use the recycling mirror with 96% reflectivity.

The equivalent recycling cavity length is the sum of the distance between recycling mirror and the beamsplitter and the half of the loop length. The cavity length must be stabilized as the loop length tends to fluctuate. We have locked the recycling cavity resonance to the laser frequency by using Pound-Drever FM sideband method and maintained the maximum recycling gain against the loop length fluctuations.

Figure 5 (b) is an FP resonance and is an error signal detected by FM sideband method. We integrate the error signal, amplify it with power MOS FET and drive the PZT attached to the recycling mirror. Figure 5 (c) is a result of stabilization. The recycling gain of about eleven is maintained over 500s. We use the error signal itself in addition to the integrated error signal to improve the response of the system.

Figure 6: White-light recycling

Although we have achieved stable photon recycling, we have sacrificed the simplicity and the white-light operability of the loop interferometer. We have introduced the recycling cavity to increase the effective optical power available to the interferometer. However, the use of high-Q recycling cavity requires the single-mode operation of laser, which makes the high power operation even more difficult.

The dilemma will be solved if we insert the laser medium into a recycling cavity and make it a laser oscillator itself as shown in Figure 6. In other words, we use the loop interferometer as an end mirror of the laser oscillator. Then high-intensity laser internal field is available to the loop interferometer. The multi-mode laser operation is again allowed. Such a laser with extremely long cavity may oscillate in huge number of modes and give quasi-continuum spectrum. No locking nor
stabilization is required. In this way, we can make the most of both recycling and automatic white-light dark fringe of the loop interferometer. This novel recycling scheme may be called “white-light recycling”.

We are preparing the experiment of this novel recycling scheme using Ti:Sapphire laser and would like to report the results in the near future.

6. Conclusion

We have introduced a zero-area loop interferometer for gravitational wave detection and shown that it is superior to conventional Michelson interferometer in stability, simplicity and white-light operability.

We have achieved conventional photon recycling for the loop interferometer with recycling gain around 11.

We have proposed a novel recycling scheme for the loop interferometer which enables multi-mode laser operation and eliminates the need for recycling cavity length stabilization.

Although our loop interferometer is incompatible with Fabry-Pérot arms, it may find important applications where delay-lines can be adopted or huge arms can be built like on the moon or in space. If the ultimate stability is desired in large scale interferometer, our loop interferometer will be the best choice. If the ultimate high power operation is desired, our white-light recycling will be the unique solution.

Acknowledgments

The authors would like to acknowledge Takeshi Endo and Hiroki Takesue for their contributions in the early stages of the experiments.

References

BINARY PULSARS AS DETECTORS OF ULTRA-LOW FREQUENCY GRAVITATIONAL WAVES

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Fundamental limits on the energy density $\Omega_g$ of the ultra-low frequency (ULF) primordial gravitational wave background (GWB) radiation which can be obtained from timing of binary pulsars are explored. For analytical convenience we choose the simple timing model being comprised of the binary system with pulsar on the circular orbit and a remote observer on the Earth whose motion about barycenter of the Solar system is assumed to be known with sufficient accuracy. The primordial gravitational waves bring about stochastic noise fluctuations in the times of arrival of the pulsar pulses which includes (as our analysis shows) both the non-stationary and stationary components. The later part of the noise is supposed to have spectral power $\sim \Omega_g/f^5$, where $f$ is the frequency of a gravitational wave passing by the line of sight and $\Omega_g$ is the energy density of the GWB radiation. Analytical technique of processing observational data in time domain is developed to derive a functional dependence of the pulsar timing residuals and variances of spin and orbital parameters of the binary pulsar on time. Using this technique we have proved that the procedure of fitting pulsar’s spin-down and orbital parameters acts not only as a low-frequency filter of the background noise but also cancel out the non-stationary component of the noise - the property having a high significance for better understanding the nature of noise presenting in the pulsar timing residuals. In order to keep the calculations manageable we idealize the observations by assuming that they are uniformly spaced and extend over an integral number of orbital revolutions $N$ which is taken so large that any sum over all observation points can be approximated by an integral over the observing period $T$. The integrals one meets in calculations are divergent because of existence of algebraic singularity in the spectrum of stochastic gravitational wave background as frequency approaches the point $f = 0$. Powerful method of analytical continuation of Riesz is applied for regularization of all divergent integrals in order to convert them into finite expressions. The regularization procedure enables us to show that the behavior of the spectrum of background gravitational radiation in the neighborhood of the singular point is irrelevant because of mutual cancellation of all singular expressions. We apparently demonstrate that observed secular variations both in the pulsar’s orbital period, $P_b$, and in its semi-major axis, $x$, projected onto the line of sight can be effectively used to set a limits on the energy density $\Omega_g$ of the GWB radiation in the frequency range $c/L < f < 1/T$, where $c$ is speed of light and $L$ is the distance to the pulsar. It is also shown that the upper bound on $\Omega_g \leq 0.04 h^{-2}$ at the ultra-low frequencies of $10^{-9}$ to $10^{-12}$ Hz, having been derived by previous authors, was based as a matter of fact on the misleading identification of theoretical expressions for variances of the pulsar’s

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spin frequency derivative, $\delta \dot{\nu}$, and its orbital frequency derivative, $\delta \dot{P}_b$, where dot denotes derivative with respect to time. Correct theoretical formulae for the upper limits on $\Omega_g$ based up on the variances for $\delta \dot{P}_b$ and $\delta \dot{x}$ are derived. Implication of these formulae shows that observed numerical value for variance of $\delta \dot{P}_b$ in PSR B1913+16 yields an upper limit on $\Omega_g \leq 138.2 \, h^{-2}$ and is not as restrictive as had been previously estimated. Rather surprising result is that the yet poorly known numerical value for variance of $\delta \dot{x}$ of PSR B1855+09 allows to set an upper limit on $\Omega_g$ less than $2.7 \times 10^{-4} \, h^{-2}$ in the frequency range $1.1 \times 10^{-11} < f < 4.5 \times 10^{-9}$ Hz which is already two orders of magnitude more stringent than that having been previously available but actually invalid. Thus, the precise measurement of $\dot{x}$ in wide orbit binary pulsars having large proper motion should be extremely useful in the exploration of properties of the ultra-low frequency gravitational radiation and provide much better limit on the cosmological parameter $\Omega_g$. This limit will improve proportionally to $T^{-3}$ until the white noise is the dominant source of observational errors. Essential feature of using $\dot{x}$ measurement for determination of upper limit on $\Omega_g$ is that it does not require to measure several other complementary relativistic effects in the orbital motion of binary pulsars except for $\dot{P}_b$ measurement. Thus, the $\dot{x}$-method of determination of upper limit on $\Omega_g$ is more economic than that based on the measurement of $\dot{P}_b$ along with two additional relativistic effects and can be applied for larger number of binary pulsars including those on the non-relativistic circular orbits.

Acknowledgements

I thank administrative and scientific staff of NAOJ (Mitaka) for long continuing support.
GAUGE DEPENDENCE OF POST-NEWTONIAN APPROXIMATION IN GENERAL RELATIVITY

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ABSTRACT
We consider to solve the Einstein equation in PPN formalism. There are many ways to estimate the higher order effects in PPN formalism, the form of Einstein equation, expansion parameters, gauge conditions, boundary conditions. This paper discusses the formal structure of metric as solution of the Einstein equation. We expand the Einstein equation to series of “algebraic” equations. Consider to solve the Einstein equation in following form

\[ nR_{\mu\nu} = -\kappa (n-4)S_{\mu\nu}, \quad R_{\mu\nu} = \sum nR_{\mu\nu}, \quad S_{\mu\nu} = \sum nR_{\mu\nu}, \quad (n = 4, 5, \ldots) . \]

For right hand side of equations, there are many expression depend on model we choose. The Energy-Momentum \( T^{\sigma\rho} \) are velocity dependent quantities, and velocities are determined by equation of motion. \( T^{\sigma\rho} \) should be expanded, but for the present, we assume that they are given. We start next power series for the \( g_{\mu\nu} \) as follow. The symbol of \( n \) in \( n g_{\mu\nu} \) indicates the power of \( 1/c, V^i/c \) or \( GM/c^2R \).

\[
\begin{align*}
g_{00} &= -1 + 2 g_{00} + 4 g_{00} + 6 g_{00} + 8 g_{00} + \ldots + 2^n g_{00} + \ldots, \\
g_{ij} &= \delta_{ij} + 2 g_{ij} + 4 g_{ij} + 6 g_{ij} + 8 g_{ij} + \ldots + 2^n g_{ij} + \ldots, \\
g_{0i} &= 3 g_{i0} + 5 g_{i0} + 7 g_{i0} + 9 g_{i0} + \ldots + 2^{n+1} g_{i0} + \ldots.
\end{align*}
\]

Then, formally, \( \Gamma^{\mu}_{\nu\lambda} \) are expanded as follow,

\[
\begin{align*}
2n\Gamma^0_{00} &= -\frac{1}{2}(2n-2)g_{00,0} + \ldots, \\
2n\Gamma^0_{0i} &= -\frac{1}{2}(2n)g_{00,0} + \ldots, \\
2n\Gamma^0_{ij} &= -\frac{1}{2}(2n-1)g_{0i,j} + (2n-1)g_{0j,i} + \ldots, \\
2n\Gamma^i_{0i} &= -\frac{1}{2}g_{0i,0} + \ldots, \\
2n\Gamma^i_{0j} &= -\frac{1}{2}g_{0j,0} + \ldots, \\
2n\Gamma^i_{ij} &= -\frac{1}{2}g_{ij,0} + \ldots.
\end{align*}
\]

And, Ricci tensors are expanded also, for example \( R_{00} \),

\[
nR_{00} = n^{-2}g^{i}_{0i} - n^{-1}g^{i}_{00,i} + n^{-m}g^{i}_{0i,m} - n^{-m}g^{i}_{00,m}g^{i}_{0i} + n^{-m}g^{j}_{0i}m^{i}_{0j} - n^{-m}g^{j}_{0j}m^{i}_{0i}.
\]

The highest connection in \( nR_{00} \) comes from \( n^{-1}g^{i}_{00,i} \). The “algebraic” series are

\[
n - thPN : 2n-2R_{00} = -\kappa 2n-2S_{00}, \quad 2n+2R_{ij} = -\kappa 2n-2S_{ij}, \\
(n + 1/2) - thPN : 2n+3R_{00} = -\kappa 2n-1S_{00}.
\]

In general, \( n \)-th PNA means for \( 2n g_{00} \) and \( 2n g_{ij} \), and \( (n + 1/2) \)-th PNA means for \( (2n+1)g_{00} \). Choice of the gauge: We use gauge condition simply a kind of subsidiary

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condition for solving the equation for $m g_{\mu \nu}$ to simplify the solutions, at present. There are many possibilities to gauge fixing: (a) harmonic condition; (b) EHI condition, but there is no principle which one one should choose. Gauge dependence of solutions for $m g_{\mu \nu}$ appears at first in $3/2$ - PN approx. ($3 g_{00}$). Therefore explicit expression for higher terms ($m g_{\mu \nu}, m \Gamma_{\mu \nu}^\lambda, m R_{\mu \nu}, m T_{\mu \nu}$) depend on the choice of gauge fixing. For $n$ - $PN$ and $(n + 1/2)$ - PN approximation, the procedure of the calculations are almost same, but messy. For example, $n$ -th $PNA$, the solution for $2 n g_{i j}$ is given as follow,

$$2 n g_{i j} = \Delta^{-1} \left[ \frac{1}{2} (2 n K_{i j} + 2 n K_{j i}) - 2 n V_{i j} - 2 \kappa 2 n - 2 S_{i j} \right]$$

where $2 n K_{i}, 2 n V_{i j}$ would be very complex function of ($2 g_{00},...2^{n} g_{00},3 g_{0i},...2^{n-1} g_{0i},2 g_{ij},...2^{n-2} g_{ij}$). The right hand side of Einstein equation, as definition, are dependent on metric and matter fields $T^{\alpha \beta}$. They are velocity dependent quantities, they should be expanded as follow,

$$T^{00} = T^{00}_0 + 2 T^{00} + ..., \ T^{0i} = T^{0i}_0 + 3 T^{0i} + ..., \ T^{ij} = T^{ij}_0 + 4 T^{ij} + ...$$

The velocities are determined by equation of motion.

$$\frac{d^2 X^i}{dt^2} = -\Gamma^i_{00} - 2 \Gamma^i_{0j} \frac{dX^j}{dt} + \Gamma^0_{00} \frac{dX^i}{dt} - \Gamma^i_{jk} \frac{dX^j}{dt} \frac{dX^k}{dt} + 2 \Gamma^0_{0j} \frac{dX^j}{dt} \frac{dX^i}{dt} + \Gamma^0_{jk} \frac{dX^j}{dt} \frac{dX^k}{dt} \frac{dX^i}{dt}$$

To solve this equation, we expand $X^i$ as follow,

$$X^i = \sum m X^i, (m = -1, 0, 1, 2,...)$$

There are again ”algebraic” series for $m X^i$, which contribute to the higher order corrections for $n T^{\mu \nu}$ and for quadrupole formula of the gravitational radiation. Work on this line, not to resort the relaxed Einstein equation, is in progress. In our expansion of metric, for example $g_{00}$, there is not term $5 g_{00}$. We use loosely the symbol of $n$ in expansion, and did not specify the expansion parameters as $1/c, V^i/c$ or $GM/c^2 R$. We think this symbol means the minimum value of these parameters, then even in $2 g_{00}$, then there are effects of order $\epsilon^n$ of parameter in exact calculation depend on model $T^{\mu \nu}$.

<table>
<thead>
<tr>
<th>order of $\epsilon$ (based on $2 g_{00}$)</th>
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</thead>
<tbody>
<tr>
<td>$2 g_{00}$</td>
</tr>
<tr>
<td>$4 g_{00}$</td>
</tr>
<tr>
<td>$6 g_{00}$</td>
</tr>
<tr>
<td>$8 g_{00}$</td>
</tr>
</tbody>
</table>

All most of observed quantities are gauge and procedure dependent, one cannot distinguish which one we should choose in theoretically, we hope this (or next) generation of gravitational wave detection experiments guides the ways.
GRAVITATIONAL WAVE CHIRP DETECTION AND SOURCE PARAMETER ESTIMATION VIA TAYLORED RADON TRANSFORM

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ABSTRACT
Efficient gravitational wave chirp detection and estimation can be implemented by the combined use of Wigner-Ville time-frequency analysis and generalized Hough algorithms for curve-parameter estimation. Both procedures can be merged into a sequence of Radon transforms.

Gravitational wave chirps from coalescing binary systems should be an abundant source of rich astrophysical information.

Matched filter detectors/estimators, discussed by several Authors, are computationally expensive, and strongly dependent on reliable signal templates. In this connection, the chirp phase issue is still open, successive post-Newtonian corrections likely forming asymptotic series rather than convergent ones.

In the last few years we pursued a different approach, much better tailored to nonstationary signals, based on the combined use of Wigner-Ville (henceforth WV) time-frequency representations and generalized Hough algorithms (henceforth HA) for curve parameter estimation. We adopt the WV transform in view of its graceful time-shifting and time-windowing properties, by which the time-series can be split into successive chunks, processed separately and spliced in succession, at the expense of an irrelevant spectral resolution loss. Each chunk undergoes an instantaneous frequency line (IFL) extraction process, producing a 2D binary image, whose points are individually tagged by a significance level.

The estimated IFL for a twin NS binary at 100 Mpc observed by a LIGO-I like antenna is shown in Fig. 1 below. A number of nontrivial technical achievements are implied: i) we proved that the IFL is the support of the asymptotic principal part of the WVT of the data in the (large) parameter $cT_{orb}/\pi r_g$, whatever the PN order needed to model the source; ii) we introduced a maximum-likelihood algorithm for extracting the instantaneous frequency line, which outperforms the barycentric algorithm used by other Authors, and relies on the obtained analytic characterization of the statistics of pure-noise WVs.

The source parameters can be subsequently estimated from the resulting IFL, using a (generalized) Hough algorithm (chosen for its special robustness against outliers), provided a model for the IFL is available.

A collection of FORTRAN codes implementing the whole procedure are being distributed among the GW search Community.

L$^2$-completeness of the WV representation (Moyal theorem) allows to gauge the statistical signifi-
Figure 1: Estimated IFL (top); estimated statistical deviation (bottom). X-axis: time bin; Y-axis (left): frequency bin; Y-axis (right): local deviation from expected WV noise level.

Under the crude (but not unrealistic) assumptions that in each chunk the frequency evolution is approximately linear, viz., \( \omega = \omega_{0k} + \alpha_k(t - t_{0k}), \quad t \in [t_{0k}, t_{0k} + T_{\text{chunk}}] \), the whole process of computing the WV transform of the chunk, and estimating \( \{\omega_{0k}, \alpha_k\} \) can be implemented in a single step, as peak-detection in a Radon-transform\textsuperscript{5}, with little additional burden as compared to plain WV. The choice of the post-newtonian IFL model order, and the related parameter-estimation can be subsequently implemented using standard curve-fitting techniques. We are currently testing possible the possible advantages and drawbacks of this alternative strategy.

References


\textsuperscript{5}\textsuperscript{cance of the estimation by implementing the matched filter as a line integral of the WVT of the data along the estimated IFL\textsuperscript{5}.}
MULTISTABILITY AND CHAOS IN MULTIPENDULAR FABRY-PEROTS

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ABSTRACT
The occurrence of chaos in multipendular Fabry-Perot resonators is confirmed by numerical simulation. The probability of evolving a chaotic solution in a given time, starting from initial conditions contained in a neighbourhood of a stable equilibrium position is estimated.

Nonlinear multistability in pendular Fabry-Perot (henceforth FP) resonators, due to the nonlinear dependence of the radiation pressure on the position of the hanging mirror was predicted and observed by Dorsel et al.\textsuperscript{1} and also indicated as a potentially effective tool for precise mirror confinement\textsuperscript{2}.

The possible occurrence of chaotic regimes in \textit{simple}-pendular Fabry Perots as a further consequence of the hereditary structure of the delay-differential equation describing the one dimensional dynamics was first pointed out by Deruelle, Tourrenc and co-Workers\textsuperscript{3}.

Chaos could equally develop, in view of the sufficient dimensionality of the associated phase-space, in \textit{multi}-pendular Fabry-Perots where the electromagnetic cavity relaxation time is short compared to the pendular eigenfrequencies, and the system is described by a nonlinear non-hereditary differential system\textsuperscript{5}.

All ground-based interferometers for gravitational wave astronomy featuring multi-pendular suspensions to decouple from platform seismic noise, the question naturally arises as to whether these systems might be chaotic in a realistic range of design parameters.

We did extensive numerical simulations\textsuperscript{5}, including phase-space trajectories, Poincaré sections, power—spectra, and Lyapounov exponents, for the simplest conceivable model: a double-pendular plane-parallel mirror FP with typical design parameters summarized in Table I, below:

<table>
<thead>
<tr>
<th>Prototype 2D FP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arm lengths 1m</td>
</tr>
<tr>
<td>Suspended masses 400 Kg</td>
</tr>
<tr>
<td>End-mirror reflection coeff. .995</td>
</tr>
<tr>
<td>Light wavelength 514 nm</td>
</tr>
<tr>
<td>Laser beam power 25 W</td>
</tr>
</tbody>
</table>

- Table I -

Our results indicate that wideband chaotic vibrations do show-up, with peak-to-peak displacement amplitudes below $10^{-3}\lambda_{\text{laser}}$, within time-scales comparable to foreseen data acquisition frames\textsuperscript{5}.
This is exemplified in Fig. 1, where the cumulative distribution of the (scaled) Sinai entropy (sum over the positive Lyapounov exponents\(^4\)) \(\bar{S}\) is displayed, obtained by evolving the system from \(\bar{t} = 0\) up to \(\bar{t} = 10^3\) starting from \(10^4\) initial conditions arranged in a regular lattice of points in a phase–space neighbourhood of the ground-state equilibrium position \(*\). The reciprocal of \(\bar{S}\) provides an estimate of the (scaled) time after which information about the state of the system is lost (Pesin theorem\(^4\)) . It is tempting to speculate that chaotic vibrations might be related to the wideband excess noise observed in some real interferometers in the frequency range between the seismic and shot-noise lines, well above the brownian noise level. These results (see\(^5\) for discussion) as well as those in\(^3\) provide enough motivation for studying the whole interferometer, including its feedback control system, in a fully nonlinear-dynamics perspective.

References

*In Fig. 1 the following scaled units are used: \(\bar{t} = (g/L)^{1/2}t\), \(\xi_i = x_i/L\), \(\eta_i = \xi_i\), where \(x_i\), \(i = 1, 2\) are the pendular displacements, \(L\) the arm lengths, and \(g\) the terrestrial gravity acceleration.
THE PRINCIPLES AND DETECTION OF GRAVITATIONAL WAVES BY RESONANT ANTENNAS

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ABSTRACT

The principles and detection of gravitational waves by resonant antennas are briefly discussed. But the main purpose of this short note is to compare the two geometries of resonant antennas, the well-known cylindrical to the spherical type. Some features of a two sphere observatory are also discussed.

1. Introduction

The demonstration by Hertz of the existence of electromagnetic [EM] waves predicted by the Maxwell’s equations was indeed a remarkable discovery. Einstein’s equations of general relativity written some eighty years ago also lead us to a wave equation for gravitational waves [GW]. Despite bold pioneering efforts of eminent scientist Weber¹ and several ongoing international collaborations of excellent scientists² we still await the direct detection of GW waves. The reasons are well-known

- The gravitational coupling constant is approximately \(10^{-38}\) times weaker than the electromagnetic coupling.

- The elementary mode of vibration induced by gravitational wave is \(quadrupole\) whereas an elementary mode excited by EM wave is \(dipole\). A simple way to see why dipole radiation does not exist for gravity is because there is no negative mass and hence no gravitational dipole.

In the weak field approximation to Einstein’s equations we can treat GW waves as gravitons [spin two massless particle] propagating on \(almost\) flat space [i.e. very close to Minkowski space]. However the existence of gravitons should not be confused with the weak field limit. Their existence is implied by the radiative solutions of the Einstein’s equations. Like EM waves GW are transverse and have two states of polarization. GW waves are more complicated than EM waves because they unlike the latter self interact [or in other words contribute to their own source].

The main goal is to first detect the GW waves directly. Once this is done then one can use the gravitational radiation as a powerful new probe and address questions, such as,

- Examining interesting astrophysical systems like coalescing neutron-star binaries, black holes, supernovae, pulsars all of which are sources of gravitational radiation.

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• GW from the early universe is expected to hold clues about inflation. In particular GW excited during inflation as quantum mechanical fluctuations is a key test of inflation and also allows one to learn about specifics of the inflationary model.

• Experimental information connected with string theory [which has the promise of providing a unified theory of particles and their interactions] may be obtained. GW detection would help test general relativity against other competing theories of gravitation such as scalar-tensor theories.

Detectors for GW waves fall into two broad categories, i: Resonant Detectors and ii: laser interferometric devices. We are concerned here with Resonant Detectors in particular comparing the two geometries cylindrical [bar] with spherical. The practical cylindrical bar antenna has been with for about 30 years whereas the suggestion for the practical implementation of spherical geometry is recent. Making a spherical antenna requires more sophisticated technology in several areas such as casting, cooling, suspension, and transducer attachment. However it has some added advantages over the cylindrical counterparts for it provides:

• Enhanced sensitivity, an order of magnitude better than the corresponding cylindrical geometry.

• Omni directionality.

• Multi mode measurement capabilities.

The suggestion for using a spherical detector for gravitational wave was made as early as 1971 by Forward where it was indicated that by a suitable positioning of a set of transducers on the sphere one could determine the direction, the amplitude, and the polarization of the gravitational wave. A free sphere has five degenerate quadrupole modes of vibration that will interact strongly with a gravitational wave. Each free mode can act as a separate antenna, oriented towards a separate polarization or direction. Wagoner and Paik showed that the angle-averaged energy absorption cross section of a spherical antenna is much larger [by a factor of 60] than a cylindrical bar [length to diameter ratio of 4.2] with the same quadrupole mode frequency. So the question arises why these results were ignored? One main reason is that a simple spherical detector is not a practical detector. For one requirement of practicality is a set of secondary mechanical resonators, which act as mechanical impedance transformers between the primary vibrational modes of the antenna and the actual motion sensors, producing the essential increase in the electromechanical coupling. All successful cryogenic bar-type detectors have such resonators. This practical consideration led Johnson and Merkowitz to propose a method of positioning six radial transducers on a truncated icosahedron to construct a nearly spherical detector. They showed that a spherical detector cooled to ultralow temperature can have sensitivity comparable or even better than the first generation Laser Interferometric Gravitational Wave Observatory [LIGO] detectors.
in the frequency range around 1 kHz. A network of six cylindrical detectors with appropriate orientation can cover the whole sky isotropically [like a spherical detector] and have source direction resolution. Such a network of six colocated cylindrical detectors have has a sensitivity $\frac{1}{7}$ that of a single spherical detector made of the same material and with the same resonant frequency.

2. Resonant Antennas-Spherical and Cylindrical

In the weak field approximation we can write

$$ g_{\mu \nu} = \eta_{\mu \nu} + h_{\mu \nu} $$

where $h_{\mu \nu} \ [h_{\mu \nu} \ll 1]$ is the GW perturbation to the flat space time metric $\eta_{\mu \nu}$. As is known and discussed in previous section GW waves have two states of polarizations $[h_+ \ and \ h_\times$ with quadrupole patterns. The perturbation $h$ will cause a change $\Delta L$ of a length $L$ between two free masses, such that

$$ \Delta L = \frac{hL}{2} $$

assuming the wavelength of GW waves is much larger than the mass separation $L$.

It is well-known that the metric perturbation $h$ caused at the detector location by a GW burst generated due to conversion of mass $M_{gw}$ into gravitational radiation at a distance $R$ from the detector position is,

$$ h = \frac{1}{R\omega_0} \sqrt{\frac{8G_N M_{gw}}{c\tau_{gw}}} $$

where $\tau_{gw}$ is the duration of the GW burst, $G_N$ is Newton’s constant and $\omega_0$ is the angular resonance frequency of the antenna. It is instructive to write this in the form

$$ h \approx 1.76 \times 10^{-17} \times \frac{1}{\omega_0} \sqrt{\frac{1}{10\tau_{gw}}} \times \frac{10 \text{Mpc}}{R} \times \sqrt{\frac{M_{gw}}{M_s}} $$

Assuming that $M_{gw} \sim 10^{-2} M_s$, $\tau_{gw} \sim 10^{-3}$ and $\omega_0 = 2\pi \ 1000 \ \text{rad/s}$ in the above we find $h \approx 3 \times 10^{-18}$ [for an event at the center of our galaxy, $R = 8.5 \ \text{kpc}$] and $h \approx 2.6 \times 10^{-21}$ [for a source of GW waves located in the virgo cluster].

The energy absorbed [captured] by the bar from GW burst is related to the absorption cross section and the incident flux [by the general definition of cross section]

$$ \sigma_{abs} = \frac{\Delta E_{abs}(\omega)}{\Phi(\omega)} $$

here $\Delta E_{abs}(\omega)$ is the energy absorbed by the detector at frequency $\omega$ and $\Phi(\omega)$ is the incident flux measured in $\text{W/m}^2\text{Hz}$. For a bar antenna [at the first longitudinal resonance mode]

$$ \sigma_{abs} = \frac{8}{\pi} \frac{G_N M v^2}{c^3} \sin^4 \delta \cos^2 2\Psi $$

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where $v_s$ is the sound velocity, $M$ is mass of the antenna, $\delta$ is the angle between the bar axis and direction of source and $\Psi$ is the angle between the plane [formed by the bar axis and direction of source] and the polarization plane. The cross section increase with both $M$ and the square of the sound velocity in medium $v_s$. In the case of a spherical detector one finds for the absorption cross section is

$$\sigma_{\text{abs}} = F_{\ln} \frac{G_N M v_s^2}{c^3} \frac{\Gamma_{\ln}}{(\omega - \omega_{\ln})^2 + \Gamma_{\ln}^2 / 4} \tag{7}$$

$F_{\ln}$ is a dimensionless coefficient which is characteristic of each mode. Assuming general relativity $F_{\ln}$ is zero unless $l = 2$. $\omega_{\ln}$ and $\Gamma_{\ln}$ are respectively, the mode resonance frequency and linewidth.

To facilitate comparison between cylindrical and spherical geometries we note $F_{21} \approx 3$ which is about 15% better than the cylinder’s $8/\pi$ [Eq. 6]. If we average over polarizations and directions the sphere’s cross section is a factor 4.4 better than the cylinder [assuming same masses for both] this result is in agreement with $8, 11, 5$. An aluminum [Al5056] cylinder [optimal orientation] of length=3 m, and diameter = 0.6 m has mass of 2.3 tons, the first longitudinal mode has angular frequency of $2\pi \times 910$ rad/s and the corresponding absorption cross section is $\sigma_{\text{abs}} = 4.3 \times 10^{-21}$ cm$^2$ Hz. A sphere [omnidirectional] of same material having a diameter 3.1 m will have a mass of 42 tons. The sphere has the same fundamental frequency. The absorption cross section is $\sigma_{\text{abs}} = 9.2 \times 10^{-20}$ cm$^2$ Hz. Similar numbers are noted in $11, 5$. We note that although the length of the cylinder and the diameter of the sphere are comparable the latter is much more heavier, representing an advantage. A conservative estimate of the sensitivity of this sphere is $h \approx 7 \times 10^{-22}$. This could be compared to a sensitivity of $h \approx 4 \times 10^{-22}$ [in the same frequency range i.e. $\sim 2\pi 1000$ Hz] for the best interferometers and that is if GW arrives at optimum direction and polarization.

Higher mode cross section $5$ values can be found by computing $F_{22}, F_{23}, F_{24}, F_{25}, \ldots$. One finds that we obtain larger cross sections at higher harmonics for a spherical detector with respect to its cylindrical counterpart. This suggests that by using spherical detector as a xylophone we can scan the frequency range $1 - 5$ kHz thus allowing us to study the stochastic background of gravitational waves. We note that a single sphere is not sufficient to identify a GW event, for at least two antennas are necessary for minimum coincidence analysis. A two sphere GW observatory has one important advantage over a network of several directional antennas with different orientations. The coincidence analysis is much simpler since the same amount of energy is absorbed by each detector while for an array each member will absorb according to its orientation. A coincidence analysis between spherical detectors is given in $11$.

**Present Detectors-Experimental Considerations**

The vibrations in the bar are converted to electrical signals [by electromechanical transducers], the electrical signals are amplified and pass through filters to
optimize the signal to noise [SNR] ratio. The minimum vibration energy which can be detected [SNR=1] is [from simple analysis]

\[ E_{\text{min}} = k_B T_{\text{eff}} = \frac{k_B T}{\xi Q} + 2k_B T_n. \]  

(8)

\( T \) is the bar’s temperature, \( T_n \) is electronic noise temperature, \( \xi \) is the fractional part of energy available to the amplifier and \( Q \) is the quality factor of all the apparatus. \( T_{\text{eff}} \) is referred to as the effective noise temperature. It is clear that to compete against thermal noise \([k_B T]\) in the bar \( \xi Q \) must be as large as possible. Clearly \( T \) and \( T_n \) must be small. SQUID amplifiers can go down to the quantum limits \( T_n \sim 6 \times 10^{-8} \) K but the difficulty lies in matching them to the transducers. For a FET amplifier the best value of \( T_n \) is more like \( T_n \approx 0.1 \) K. Let us see what kind of numbers are involved if we assume a value of \( h \sim 3 \times 10^{-21} \) which we obtained for the Virgo Cluster, Eq. 4. Now \( h \) is related to the mass of detector \( M \), its length \( L \), \( \tau_{gw} \) and speed of sound in bar material by

\[ h = \frac{1}{\tau_{gw}} \times \sqrt{\frac{L}{v_s^2}} \times \sqrt{\frac{E}{M}}. \]  

(9)

Using \( h \sim 2.6 \times 10^{-21} \) in the above equation we find \( E \sim 1.5 \times 10^{-30} \) joule. Choosing this value of \( E \) as our \( E_{\text{min}} \) in 8 we find that the set \( T \sim 40 \) mK, \( \xi Q \sim 10^6 \) and \( T_n \sim 7.5 \times 10^{-7} \) satisfies equation 8. Meeting these requirements in an experiment [which is conducted over long time] is quite demanding. From these numbers we can easily appreciate the challenging task of the experimentalists trying a direct detection of gravitational waves.

Let us now look at the characteristics of some the resonant antennas [bar type] in world. We use the following notation: Name of Experiment(Location, Detector mass in kg, Temperature in Kelvin, Amplifier used, data taking, sensitivity \( h \)). Following this notation we have

CRAB(Tokyo,1200,4.2,parametric,1991,2 \times 10^{-22}[monochromatic])

EXPLORER(Rome,2300,2.5,SQUID,July 1990(1986),7 \times 10^{-19})

NAUTILUS(Rome,2300,0.1,SQUID,1994-95,3 \times 10^{-18})

ALTAIR(Rome,390,4.2,SQUID,—,—)

AURIGA(Legnaro,2300,0.1,SQUID,1995,—)

ALLEGRO(Louisiana,2300,4.2,SQUID,June 1991,7 \times 10^{-19})

NIOBE(Perth,1500,4.2,parametric,June 1993,7 \times 10^{-19})

Moscow University(Moscow,1500,290,tunnel,1993,7 \times 10^{-17}).

For ALTAIR and AURIGA the sensitivity has not been listed for they are yet to be fully tested. Sensitivity refers to a GW burst lasting 1 ms, with the exception of the Tokyo group which is searching for the continuous GW from the CRAB pulsar. The Rome antenna EXPLORER has reached sustained strain sensitivity \( h = 6 \times 10^{-19} \) for millisecond bursts over several consecutive month period.\(^{12}\) ALLEGRO and the EXPLORER were also operated in coincidence. They were aligned parallel to each other and operated for 180 days [June 24th 1991- December 16th 1991] so that the...
same gravitational wave burst would have produced in the antennas signals with the same amplitude. Preliminary analysis of the data gives a negative result.\textsuperscript{6,2} In particular no coincident events were found above 200 mK.\textsuperscript{2}

3. Conclusions

In conclusion we have examined the advantages of the spherical detector over its cylindrical counterpart and find that our preliminary study confirms the optimistic results reported in.\textsuperscript{9,11,5}

Acknowledgments

I would like to acknowledge Prof. R. N. Henriksen and Prof. K. Lake from both of whom I learnt General Relativity.

References

SHORT FABRY-PEROT CAVITY TO STUDY THE MECHANICAL THERMAL NOISE

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ABSTRACT

We studied the design of a device limited by the mechanical thermal noise over a wide range of frequencies. By using this optomechanical system we plan to investigate the off-resonance spectrum of the thermal noise and its predicted departure from the constant behavior of the viscous-like dissipation model.

1. Introduction

The problem of the direct observation of the mechanical thermal noise spectrum is a very ambitious task. In fact, although it is well known how to observe the Brownian peak due to this kind of noise in solid resonators, the off-resonance slope of the spectrum has only been observed by using organic materials with high internal friction figure. The main purposes of this study are the followings:

- direct observation of the off-resonance slope of the thermal noise, below one resonant frequency, in the range 200-500 Hz;
- study of elastic and anelastic properties of materials vs frequency;
- development of optomechanical transducers.

A short Fabry-Perot optomechanical transducer can be exploited to this purpose. The few table top experiments already performed in this field have shown that in order to compare the theoretical predictions to the observations it is crucial to deal very carefully with the mechanical design of the sensor.

2. The Optomechanical Device

In the feasibility study, our major concerns were (I) the optimization of the mechanical design and (II) the requirement of a simple optical read-out system. We studied two detection schemes in order to minimize the laser frequency noise. A very short FP cavity \((l = 5 \times 10^{-5} \text{ m})\) and and push-pull differential FP cavity \((l = 1 \times 10^{-2} \text{ m})\), can be both implemented by using a mushroom resonator in a very compact mechanical design. The resonator, a thin disk \((d = 150 \text{ mm} \times t = 5 \text{ mm})\), is clamped at its center. This solution allows to lower the whole eigenfrequency spectrum of the resonator, bringing the off-resonance thermal noise to levels which are reasonably accessible in finest interferometric table top experiments (Fig. 1). Moreover, the stress is concentrated in the center of the disk in a reduced clamping...
area. A further advantage of choosing such a design is the easier selection of the vibrational mode shapes that are negligibly affected by the recoil losses. Specifically, we have chosen the lowest mode of vibration of (0,1) in order not meet in the region of frequencies $f < f_{0,1}$ the spectral rise due to the thermal noise of other Brownian peaks. Preliminary tests of the elastic performance of coated materials and of the differential detection scheme in the push-pull configuration have been planned.

![Figure 1: Noise components occurring at the read-out of the optical push pull configuration.](image1)

![Figure 2: Basic designed features of the push-pull scheme.](image2)

**References**

ABSTRACT

In a power-recycled Fabry-Perot-Michelson interferometer it is difficult to extract the four independent signals necessary for the longitudinal control of mirrors. However, by a proper adjustment of the optical parameters, these signals can be extracted with a good separation. In this article we describe this idea as well as an experiment to evaluate it.

1. Introduction

Laser interferometric gravitational wave detectors are under construction by the LIGO, VIRGO, GEO, and TAMA projects. All of them, except for GEO600, will be Michelson interferometers in which the mirrors are replaced by Fabry-Perot arm cavities in order to enhance the effective arm length. A power-recycling technique will be applied to these interferometers to improve the sensitivity which is limited by the shot noise.

To operate a power-recycled Fabry-Perot-Michelson interferometer at the highest sensitivity, two arm cavities and the recycling cavity must be in resonance with the incident laser beam, and the interference fringe must be dark at the output port of the interferometer. To keep this operational point, it is therefore necessary to control four degrees of freedom: the changes in the arm-cavity lengths ($\delta L_1$, $\delta L_2$) and the lengths between the recycling mirror and the front mirrors ($\delta l_1$, $\delta l_2$).

2. Signal-Extraction Schemes for a Power-recycled Fabry-Perot-Michelson Interferometer

Using the technique called frontal modulation (pre-modulation), the four signals necessary to control an interferometer are extracted as linear combinations of the four degrees of freedom (figure 1): the difference and the sum of the motions of the arm cavities ($\delta L_+ = \delta L_1 - \delta L_2$, $\delta L_- = \delta L_1 + \delta L_2$), the differential motion of the front mirrors ($\delta l_+ = \delta l_1 - \delta l_2$), and the change in the recycling cavity length ($\delta l_+ = \delta l_1 + \delta l_2$). In the general configuration, the reflectivity of the recycling mirror is chosen to be equal to, or a little less than, the reflectivity of the Fabry-Perot-Michelson part of the interferometer in order to obtain a large recycling gain. However, it is difficult to extract the $\delta l_+$ signal independently of the large $\delta L_+$ signal under this condition. With the mixing between the signals, the design of the control system can be crucial to maintain the stability of the control loop and the sensitivity of the interferometer. Several methods to extract the $\delta l_+$ signal have been either proposed or tested: such techniques as the frequency-shifted sub-carrier, the mechanical modulations, and the multiple phase modulations. There is also the idea to obtain well-separated signals using linear combinations of...
the mixed signals.

On the other hand, the four signals are separated remarkably well when the reflectivity of the recycling mirror is set to maximize the gain of the sidebands, not the carrier. In other words, a good separation between the signals is obtained when the reflectivity of the whole interferometer is chosen to be zero for the sidebands, and non-zero for the carrier. Although the recycling factor decreases slightly in this case, it is still a reasonable value, 99% of the maximum value with the TAMA300 parameters. This method has the advantage of simplicity; it calls for no additional modulation, no signal-decoding system, and fewer constraints on the longitudinal control system.

3. **Experiment on a 3-m Fabry-Perot-Michelson Interferometer**

We are researching this signal-extraction method with a 3-m Fabry-Perot-Michelson interferometer at the University of Tokyo. The mirrors and beam splitter of this prototype interferometer are all suspended independently, and the main part of the interferometer is housed in a vacuum system. The length and the finesse of the arm cavities are 3m and about 230, respectively. As a light source, a laser-diode-pumped Nd:YAG laser (LIGHTWAVE, 124-1064-050) is being used. The reflectivity of the
Table 1: Sensitivity of the signals to the motions in the four degrees of freedom.

<table>
<thead>
<tr>
<th></th>
<th>$\delta L_-$</th>
<th>$\delta l_-$</th>
<th>$\delta L_+$</th>
<th>$\delta l_+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_1$</td>
<td>1</td>
<td>$6.2 \times 10^{-3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$v_2$</td>
<td>$3.6 \times 10^{-5}$</td>
<td>$5.8 \times 10^{-3}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$v_3$</td>
<td>0</td>
<td>0</td>
<td>0.51</td>
<td>$2.6 \times 10^{-2}$</td>
</tr>
<tr>
<td>$v_4$</td>
<td>0</td>
<td>0</td>
<td>$2.9 \times 10^{-4}$</td>
<td>$4.5 \times 10^{-2}$</td>
</tr>
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</table>

recycling mirror is 91%.

Table 1 gives the sensitivity of the signals to the motions in the four degrees of freedom calculated using the parameters of the 3-m Fabry-Perot-Michelson interferometer. These values have been normalized so that the sensitivity of the $v_1$ signal to $\delta L_-$ motion is equal to unity. The diagonal values are the main signals needed to control the interferometer. The off-diagonal values represent mixing of the unnecessary signals, and are less than 1% of the main control signals. In accordance with a calculation, the recycling factor is 3.5, which is 78% of the value when the reflectivity of the recycling mirror is chosen to maximize the recycling gain; the small recycling gain is sufficient because the main purpose of this experiment is to investigate the signal-separation method.

The 3-m Fabry-Perot-Michelson interferometer has already been operated under the recombination configuration without the recycling mirror, and almost all of the noise sources have been identified. The recycling mirror has been installed and we are testing the lock acquisition of power recycling with the Fabry-Perot arm cavities.

Acknowledgments

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References


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DEVELOPMENT OF HIGHLY-STABILIZED LASER FOR GRAVITATIONAL WAVE DETECTOR

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ABSTRACT
In our prototype 20-m laser interferometric gravitational wave detector of the TAMA project, the frequency of the commercial Nd:YAG laser (NPRO) is pre-stabilized to the rigid cavity. The frequency noise of this pre-stabilized laser is measured by the 4-m mode-cleaner which consists of separately suspended mirrors. Introduction of two dissimilar cavities allows to monitor the frequency noise level more accurately, and the frequency noise is measured to be $2 \times 10^{-2} \text{Hz/}\sqrt{\text{Hz}}$. Then to realize a high power laser with high frequency stability, the injection-locked laser of 2W is frequency stabilized to the rigid cavity. The error signal is fed back to only the master laser and the relative frequency noise is suppressed down to $6 \times 10^{-4} \text{Hz/}\sqrt{\text{Hz}}$ at 1 kHz.

1. Frequency Noise Measurement of a Pre-Stabilized Laser by a 4-m Mode-Cleaner

The gravitational wave is so weak that the high frequency stability is required for the light source of the laser interferometric gravitational wave detector. In the TAMA project, the commercial monolithic LD-pumped Nd:YAG laser of 500 mW (NPRO lightwave model 122-500) is used for the light source of our 20-m prototype gravitational wave detector. We have stabilized the frequency noise of this NPRO by using Pound-Drever method to a reference Fabry-Perot (FP) cavity. The reference cavity consists of two highly reflecting mirrors with a rigid spacer between them which is made of ULE glass, which is called the rigid cavity. The FSR and finesse of this cavity is 500 MHz and 2400, respectively, and this cavity is suspended with a double-pendulum suspension and is contained in a vacuum chamber to avoid disturbances from outside. The relative frequency noise of our system estimated from the feedback error signal is suppressed down to $3 \times 10^{-4} \text{Hz/}\sqrt{\text{Hz}}$ which is the shot-noise limited level below 1 kHz. Since the absolute frequency noise is the sum of the relative frequency noise and the instability of the reference cavity, the total frequency noise should be measured by using another monitoring cavity. The total frequency noise was measured to be $7 \times 10^{-3} \text{Hz/}\sqrt{\text{Hz}}$ by using another rigid FP cavity.¹ However, for evaluating the total frequency noise more accurately, the reference cavity and the monitoring cavity should be of the different type whose resonance frequencies are not correlated to each other. In the present study, the frequency of the NPRO is stabilized to the rigid cavity and the frequency noise of the NPRO is measured by a 4-m mode-cleaner which is incorporated with our 20-m prototype gravitational wave detector. The 4-m mode cleaner consists of two mir-
rors which are independently hung by double-pendulum suspension with no rigid spacer between them; which is called a separately suspended mirrors cavity. The measured frequency noise is shown in Fig. 1 as a bold line. The frequency noise of the NPRO is suppressed down to \(2 \times 10^{-2} \text{Hz/Hz} \) at 1 kHz. Below 1kHz the characteristic of the mode-cleaner noise spectrum is observed by using a frequency-stabilized laser. The frequency noise spectrum is dominated by the mode-cleaner below 100 Hz due to the pendulum resonant vibration. On the other hand, the frequency noise spectrum between 100 Hz and 500 Hz is dominated by the rigid cavity intrinsic noise because, though the relative frequency noise is more suppressed by increasing feedback gain and PD input power, the frequency noise spectrum monitored by the mode-cleaner shows no difference. The dotted line in Fig. 1 indicates the frequency noise which is stabilized to the mode-cleaner and measured by one arm of the 20-m interferometer; both cavities are of separately suspended mirrors type.\(^2\) From our result it is clear that the frequency noise that is stabilized to the rigid cavity is highly suppressed than that stabilized to a cavity with separately suspended mirrors lower than 400 Hz.

2. Frequency Stabilization of an Injection Locked Laser

The power of more than 10W is required for the light source of the gravitational wave detector for decreasing shot-noise limited level. To realize the high power laser with low frequency noise, the injection lock technique is promising. In our study, the frequency stabilization of injection-locked laser is investigated. An Nd:YAG ring laser is made as a slave laser. This laser is end-pumped by a 10-W fiber-coupled LD (SDL3450-P5) and generates the light of 2W with bi-directional oscillation. The stable single-mode operation is obtained by injecting in it the light from a 200-mW NPRO(Lightwave 122-200) which is a master laser. Stable injection-locked oscillation is maintained by the phase-locking servo which controls the length of the slave laser cavity by means of two PZTs on which mirrors are mounted. The Injected master laser power is 70 mW and the locking range of this injection-locked laser is measured to be 1.2 MHz at the output power of 2.1W. The bandwidth of the phase locking servo is 15 kHz and the residual frequency noise between the master and the slave laser is \(2 \times 10^{-2} \text{Hz/Hz} \) at 1 kHz. The frequency of the injection-locked laser is locked to a high finesse and highly stabilized rigid cavity (as mentioned in Sec.1) by Pound-Drever technique, and an error signal from the servo circuit is fed back to only the master laser. The relative frequency noise spectrum of the injection-locked laser is shown in Fig. 2 as a bold line and is suppressed down to \(6 \times 10^{-4} \text{Hz/Hz} \) at 1 kHz. The dotted line indicates the relative frequency noise of the frequency-stabilized master laser. From this results, at the range where the noise suppression gain is high, the frequency noise is suppressed as low as that of a master laser and the residual frequency noise of the injection-lock system is inclusively suppressed. On the other hand, the frequency noise above 10 kHz is sufferd from the residual frequency noise; some resonant peaks of the residual frequency noise is converted into the intensity noise of the injection-locked laser, and they result in some resonant
Figure 1: Frequency noise spectrum of the pre-stabilized laser measured by the mode-cleaner

Figure 2: Relative frequency noise spectrum of the injection-locked laser

peaks shown in the frequency noise spectrum of the injection-locked laser.

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References

DEVELOPMENT OF A MODE CLEANER FOR LASER INTERFEROMETER GRAVITATIONAL WAVE DETECTOR

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ABSTRACT
We have developed a new mode cleaner system for the 20 m prototype. A phase modulator which produce sidebands for the operation of the interferometer, distorts a wavefront of light. Therefore it should be put in front of the mode cleaner. In this case, the Free Spectral Range of the mode cleaner cavity must be equal to the modulation frequency. We report the system configuration and its noise features.

1. Introduction
A laser interferometer gravitational wave detector consists chiefly of two part; a laser and an interferometer. A mode cleaner is the interface of those. It cleans laser beam and removes beam jitter. A mode cleaner is a high finesse Fabry-Perot cavity in principle, and selects spatial mode by the difference of resonant frequencies of the modes.¹

As known well, an Electro-Optic Modulator distorts a wavefront of light. So all modulators should be put in front of a mode cleaner. But in general, modulation sidebands which are necessary to operate an interferometer cannot pass through a mode cleaner. If the modulation frequency is equal to the Free Spectral Range (FSR) of the mode cleaner cavity, the modulation sidebands can pass through it. We designed the cavity length to be 3.75 m, which corresponds to modulation frequency of 40 MHz.

2. Experiment
The configuration of the mode cleaner for the 20m prototype is shown in Figure 1. The modulation frequency for the mode cleaner servo (basically Pound-Drever method) is 18 MHz. The error signal is fed back to the cavity length control at low frequencies, and to the laser frequency stabilization at high frequencies, the cross over frequency is 30 Hz.

Although there is a little difference between the modulation frequency and the FSR of the mode cleaner cavity, the modulation sidebands can almost transmit it. But the light which pass through the mode cleaner, have an AM noise which is converted from the FM noise of a laser, because of the frequency difference.
modulation frequency must be equalized to the FSR of the mode cleaner cavity so that the AM noise is hidden below the shot noise. “FM noise” is not an absolute FM noise, but a relative FM noise for a mode cleaner cavity as a frequency reference. We add a FM signal to the incident light and measure the converted AM signal of the transmitted light. To add the FM signal, we vibrate a length of the cavity at 28 kHz which is the lowest longitudinal mode of compound mirrors. In this case, FSR also vibrates, but its effect can be ignored in this measurement. Figure 2 shows the AM signal of transmitted light at 28 kHz, as the modulation frequency is changed from 40,020,254 Hz to 40,020,288 Hz by 2 Hz. As a result, the modulation frequency can be equal to the FSR of the mode cleaner cavity within 2 Hz. But the 28 kHz AM signal doesn’t become below the noise level when 28 kHz FM signal is turned off.

Figure 3 is a noise spectrum of the demodulated signal of the transmitted light, when the modulation frequency is equal to the FSR of the mode cleaner cavity within 2 Hz. The AM noise spectrum which is calculated by FM noise of incident light is smaller than the shot noise (except for 28kHz). But the measured AM noise spectrum is about 10 dB bigger than the shot noise at 2 kHz.

3. Discussions

The excess 28 kHz AM signal may be caused by insufficient stability of the synthesizer used for the modulation (40MHz). The synthesizer needs to be stable more than $10^{-7}$ in order to equalized with the FSR of the mode cleaner cavity within 2 Hz. The synthesizer, we used, has a frequency stability of $10^{-7}$ or $10^{-8}$.
Figure 2: **28kHz Intensity Noise of transmitted light**. This graph is an amplitude of the 28kHz AM signal of transmitted light, demodulated signal (Quadra phase), when modulation frequency is changed by the synthesizer.

Figure 3: **Intensity Noise Demodulated by Inphase**. Incident power to the photo detector is 50mW. (a) Transmitted light, (b) Incident light, (c) Shot noise.

at most. On the other hand, the stability of FSR of the mode cleaner cavity is more than $10^{-10}$, because of the cavity length is locked to the laser wavelength at low frequencies. We think that it is necessary to stabilize the synthesizer with reference to the FSR. Maybe the excess noise (1kHz-20kHz) isn’t caused by FM-AM conversion noise, because that shapes of the AM noise spectrum differ from ones of the FM noise spectrum. It is possibly caused by an optical feedback. To probe into the cause of this noise is the next important work.

**References**


2. M. Musha, et al. This issue.
LOW-FREQUENCY PENDULUM USING THE ELECTROMECHANICAL ANTISPRING EFFECT

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ABSTRACT

We describe here a short pendulum (about 30 cm in length) with a resonant frequency lower than 0.5 Hz obtained by using a very simple electromechanical antispring effect. The lowest resonant frequency achieved was 74 mHz. The results are in good agreement with the theoretical prediction. This technique can be applied to the investigation of the internal friction in the pendulum system. We tested two kinds of material for the wire of the pendulum: nylon and niobium. The results indicate an interesting behavior of the dissipation mechanism.

In the field of precision measurement of small displacements, as in interferometric gravitational wave detectors, the realization of low-frequency pendulums is one key technique for the definition of the inertial frame for the test masses. In practice, it is hard to lower the resonant frequency of a pendulum, with a simple mechanical design and a realistic length, below 0.5 Hz. In fact, the control of the test masses’ position in such interferometric detectors is a crucial problem.

We present here some measurements performed using a short pendulum (about 30 cm in length) with a resonant frequency lower than 0.5 Hz. The frequency tuning of the pendulum was obtained by using a very simple electromechanical antispring effect. A tuning coil was placed under a small permanent magnet attached to the pendulum mass (Figure 1). The lowest resonant frequency achieved was 74 mHz (Figure 2).

The magnetic coupling between the pendulum mass and the tuning coil is expressed in the Fourier transform of the equation of motion by an additional term,

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proportional to:

\[ K_m = \frac{3m_z\mu_0I}{4a^3}, \]  

(1)

where \( \mu_0 \), \( m_z \), \( I \) and \( a \) are the magnetic permeability in vacuum, the vertical component of the magnetic momentum and the current flowing in the coil, and the mean radius of the coil. The sign of \( K_m \) depend on the current versus.

Then, the tuned angular frequency is

\[ \omega_o^* = 2\pi f_o^* = \omega_o^* = \left( \omega_o^2 - \frac{K_m}{M} \right)^{1/2}, \]  

(2)

where \( M \) is the pendulum’s mass.

If the dissipation losses are due only to internal, anelastic, process,\(^2\) the lag angle between the stress and the strain will affect the response of the system to an applied force according to the following equation

\[ \omega^2\theta(\omega) = \omega_o^{*2} \left( 1 + i \frac{\omega_0^2}{\omega_o^{*2}} \phi \right) \theta(\omega) + \Gamma_{ext}(\omega) \]  

(3)

where \( \Gamma_{ext}(\omega) \) and \( \theta(\omega) \) are the Fourier tranform of the forcing excitation and the angular response of the pendulum, and \( \phi \) is the overall internal friction coefficient.\(^5\)

When the pendulum is tuned at a given angular frequency \( \omega_o^* \) the quality factor of the oscillation turns out to be

\[ Q = \frac{\omega_o^{*2}}{\omega_o^2} \frac{1}{\phi} \]  

(4)

We applied the antispring technique to develop a method for the investigation of the dissipation regime which dominates the energy losses in the pendulum motion.
Two materials for the wires have been tested: niobium (0.3 mm wire) and nylon (0.1 mm single fiber). To study efficiently the problem, the system, consisting of clamps plus wires plus suspended mass, must be considered as a whole.\textsuperscript{3} In our experiment, however, the Al clamp was optimized only for the nylon wires. Two remarkable features characterized our setup and our data in such a case (Fig. 2): (A) our measurements were performed at different vacuum levels keeping the system under diffusion pumping for several days; (B) we observed an increase of the quality factors of each data set as the vacuum improves; (C) from the natural frequency ($I = 0$) change among the three sets we deduce that the length was $3-4$ mm shorter. On one side we checked if the Q increase could be related to the residual pressure,\textsuperscript{4} but the estimated effect is negligible. On the other hand, the observed change in pendulum length shows a process which certainly occurs into the nylon fiber. Therefore, we are confident we measured the internal friction property of the system which reached even unusually high Q figures for nylon material. The exponent of the Q vs frequency relations (Fig. 2) is $2.4 \pm 0.1$. This functional dependence does not agree to the predicted behavior (Eq. (4)). We plan to investigate carefully this result to address its causes and to optimize the clamps for the Nb wires as well.

References

DEVELOPMENT OF AN ANTI-VIBRATION SYSTEM WITH TWO DEGREES OF FREEDOM USING X-PENDULUM

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ABSTRACT

Two prototypes of the X-pendulum vibration isolation system. We made several experiments to evaluate the vibration isolation performance of this system. We are now analyzing the data taken in these experiments.

1. Introduction

Vibration isolation is one of the most important technologies for interferometric gravitational wave detectors such as TAMA. The vibration isolation system of TAMA consists of three stages: a vibration isolation stack, a double pendulum system, and between these, the X-pendulum vibration isolation system.

2. X-Pendulum

The X-pendulum is a type of pendulum with the distinctive feature that it uses multiple suspension wires which outline an "X" when viewed from the side. We developed this concept through two prototypes. The attraction of it is that we can get very long periods with a compact, practical size. For example, a 10 s simple pendulum would require a 25 m length wire. With the second prototype (see Figure 1), completed in August 1996, we could easily attain this level in a size of 20-30 cm. The X-pendulum system is intended to protect against seismic vibration noise at low frequencies (1 Hz to a few 10 Hz).

3. Experiments

To evaluate the vibration isolation performance, we measured the changes in displacement, due to seismic noise, between the two prototypes. For measuring the displacement we used a Michelson interferometer with a stabilized He-Ne laser, mechanical modulation and feedback control (see Figure 2). One corner cube prism with a feedback controlled magnetic actuator and the beam splitter were placed on the load table attached to one system and the second corner cube prism was placed on the other system at a distance of approximately 10 m apart. Both X-pendulums were tuned to near 5 s periods, and the displacement was measured from 0.1 Hz to 100 Hz both in the atmosphere and in the vacuum.

4. Analysis

We consider fluctuation around an optically locked point of the interferometer. So we linearize the feedback diagram. If the feedback system is in a locked state,
Figure 1: The X-pendulum vibration isolation system.

Figure 2: Configuration of the experiment.
we can write the equations for getting the change of displacement from $FBS(\omega)$.

$$
\Delta_f(\omega) = (\Delta(\omega) - \Delta_f(\omega))F1\cdot F2,
$$
$$
FBS(\omega) = (\Delta(\omega) - \Delta_f(\omega))F1,
$$

therefore

$$
\Delta(\omega) = \left(\frac{1}{F1\cdot F2}\right)F2\cdot FBS(\omega).
$$

To obtain the changes in displacement needs the open loop transfer function ($F1\cdot F2$),
transfer function of $F2$ and the output signal $FBS(\omega)$.
We have taken data both in the atmosphere and in the vacuum after the TAMA workshop. It will be disclosed soon.

References

VIBRATION ISOLATION OF TAMA SUSPENSION SYSTEM

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ABSTRACT

We describe the vibration isolation properties of the double-stage pendulum that has been developed for the mirror suspension system of TAMA300 gravitational wave detector.

1. Suspension System of TAMA300

To obtain sufficient vibration isolation, we adopt a double pendulum structure for a suspension system of TAMA300. The intermediate mass is passively damped by the strong permanent magnets to suppress the large motion of the mirror. The magnets themselves are supported flexibly in each direction not to damp the mass at the observation band, while the damping is effective below a few Hz.

2. Calculation of the Rigid-body Resonances and the Transfer Functions

To evaluate the expected performance of the suspension, the rigid-body resonances (Table 1) and the transfer functions of each degree of freedom (Figure 1) have been calculated by numerically solving the equations of motion.

In the table and the figures, X and Z mean the optical axis and the vertical direction. Y is orthogonal with X and Z. Roll, Pitch, and Yaw represent the rotation around X, Y, and Z, respectively.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Q</th>
<th>Mode</th>
<th>Frequency (Hz)</th>
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<th>Mode</th>
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<td>Yaw</td>
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<td>4.7</td>
<td>Magnet-x</td>
<td>23.3</td>
<td>300</td>
<td>Roll</td>
</tr>
</tbody>
</table>

Table 1: Calculation of rigid-body resonances.
All rigid-body resonances are below 30Hz and the transfer functions have $f^{-4}$ property, which is same as a double pendulum without damping, above the frequencies of these resonances. At the same time the quality factors (Q’s) of the coupled pendulum modes are very low because of the magnetic damping. This is the advantage of flexibly supporting of magnets.

If vertical springs are attached to the final stage of the pendulum, the thermal noise property of the pendulum become worse. Therefore, the mirror have to be suspended with wires directly. This is why the resonant frequency of the vertical motion is high and the vertical isolation is worse than the others. The difference between $z \rightarrow z$ isolation and $x \rightarrow x$ is 75 dB. If a vertical-horizontal coupling constant is worse than $-75$ dB, the vibration from the vertical seismic motion will dominate the fluctuation of the mirror.

The Q’s of the pitching mode and the yawing mode are very high. These modes will cause a large tilt of the mirror.

3. Vibration Isolation Measurements

We constructed the prototype of the TAMA suspension and have measured the transfer functions between the suspension point and the mirror (Figure 2). We have used a vibration table and PZT accelerometers for the measurements. The measurements have been done without any damping. The fluctuation of the mirror due to the sound which is coherent with the excitation limited the measurement to $-80$ dB. Therefore, the horizontal isolation ratios are correctly represented by the plot up to 10Hz, while the vertical isolation ratio up to 100Hz.

From the measurement of the mirror motion orthogonal to the excitation direction, we could estimate the coupling constant of less than $-60$ dB which was also limited by the coherent noise and the sensor noise.
Figure 2: The measured transfer functions. Left: The isolation ratios of the translational motions (solid) and the calculations (dotted). Right: The isolation ratios of the translational motions (thin lines) and their cross couplings to the optical axis (thick lines).

4. Conclusion

With respect to the calculations, all rigid-body motions were analyzed and the transfer functions were calculated. According to the result we have to lower the Q’s of the angular modes. With respect to the experiments, the transfer functions between the suspension point and the mirror were measured and agreed with the calculation up to 10Hz. However, the noise dominated the measurement below −80dB. The upper-limit of the vertical-horizontal coupling constant, which is −60dB, was obtained.

We will implement the passive damping with the flexibly supported magnets and measure the transfer functions in vacuum.

Acknowledgments

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References

AUTOMATIC ALIGNMENT CONTROL FOR THE TAMA INTERFEROMETER

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ABSTRACT
The sensitivity of a laser interferometric gravitational wave detector depends on the fluctuation of the angles between the mirrors and the input beam (the misalignment angle). We have estimated the limitation on the allowable angle of misalignment from calculations of the internal field of the power-recycled Fabry-Perot-Michelson interferometer with a misalignment. To suppress any fluctuation of the angle, we need an automatic alignment control system. In the TAMA project, the wave-front sensing technique is employed. We report on the signal-detection scheme and its servo design.

1. Introduction
The eigenmodes of a Fabry-Perot cavity and the field of the input laser beam are well approximated by the Gaussian modes. In fact, the mode of the input beam does not match the eigenmode of the cavity completely. There are two kinds of parameters which represent the difference between the two modes: one is the difference of the waist position or size, i.e., the mismatching; the other is the difference in the axes of the two modes, i.e., the misalignment. In this article, the effect of a misalignment and the automatic alignment control system for the TAMA interferometer are discussed.

2. The Effect of a Misalignment
A misalignment affects such parameters as the contrast of the interferometer, the recycling factor, and the common-mode rejection ratio (CMRR). From the requirements for these parameters, a limitation on the allowable angle of the misalignment is estimated. Assuming that we must keep a contrast larger than 99% and a recycling factor larger than 90% of the best value, the misalignment angle must be kept smaller than $5 \times 10^{-7}$ rad in the TAMA project. Basically, the fluctuation of the angle will be suppressed by the vibration-isolation system, although it has a low-frequency resonance. We thus need an automatic alignment control system for any fluctuation in the low-frequency range to maintain the sensitivity.

3. Setup of the Automatic Alignment Control for the TAMA Interferometer
There are several techniques for an automatic alignment control.$^{1-6}$ In the TAMA project, the wave-front sensing technique$^2$ is employed. Figure 1 shows the alignment control setup for the TAMA interferometer without recycling.

In this system, however, the angle fluctuation of the pick-off mirror causes a change in the optical path length. Also, the pick-off mirror may introduce an opti-
Figure 1: Automatic alignment control setup for the PhaseI TAMA interferometer without recycling.

Figure 2: No-pick-off system for the PhaseII interferometer with power recycling $a_+(a_-)$ and $\alpha_+(\alpha_-)$ represents the symmetric (antisymmetric) lateral- and tilt- misalignment, and $\alpha$ represent the misalignment of the recycling mirror, respectively.
cal asymmetry which can be a problem in the PhaseII interferometer with recycling. Therefore, we are studying a signal-extraction scheme which can obtain all information for the alignment control from the symmetric port and the antisymmetric port of the power-recycled interferometer (Figure 2).

4. The Present Status of the Experiment

We mention the present status of the experiment briefly. A tabletop experiment of the alignment control was performed using a 0.65m cavity with independently suspended mirrors. We plan to start an experiment with the 300m FP cavity (one of the arms of the TAMA interferometer) from 1997.

References

MEASUREMENT OF THE QUALITY FACTORS OF FUSED-SILICA MIRRORS

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ABSTRACT
We measured the quality factors of 11 axisymmetric vibration modes of 2 samples of fused silica. The influences of the surface roughness of the sample and the auxiliary system, such as the wires, magnets, and stand-off, were studied.

1. Introduction
The thermal noise of the mirrors is thought to be one of the fundamental limiting factors of the sensitivity of interferometric gravitational wave detectors.\(^1\) The power spectral density of the thermal motion of the mirrors is calculated by using the fluctuation-dissipation theorem; it shows that high-quality factors are required to achieve an objective sensitivity. For example, in the TAMA project, the required quality factor of the mirror is \(2 \times 10^7\) to achieve an objective sensitivity \((\hbar)\) of \(3 \times 10^{-21}\). This value is 20-times as high as the maximum quality factor of the fused-silica samples which we used in this experiment. Further experimental investigations are needed to minimize not only the internal loss of the material, itself, but also additional losses caused by the suspension system, such as wire resonance,\(^2\) magnets,\(^3\) and the method to attach the wires to the mirror.\(^4\)

We report on the Q’s of a mirror measured in a simple suspension system and then in a more realistic situation, with a magnet and stand-off.

2. Experimental Setup
We used 2 samples of fused-silica cylinders with diameters of 10cm and lengths of 6cm. To test the effect of the surface roughness, the side of one sample was polished, and that of the other was not polished. The sample was suspended in a vacuum by a single loop of tungsten wire having a diameter of 60\(\mu\)m. The mirror was excited by using a capacitive transducer. We should be careful to determine whether the excited mode is axisymmetric or not, because non-axisymmetric modes could also be excited by this excitation system. The vibration of the mirror surface is sensed by the Michelson laser interferometer. After turning off the excitation signal we measured the decay curve in order to calculate the Q.

3. Experimental Results
The measured quality factors of the 11 eigen-modes with the calculated\(^*\) and observed resonant frequencies are given in Table 1. We first compare the Q’s of two samples without a magnet and stand-off. Then, we consider the effects of a magnet

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\(^*\)according to Hutchinson’s method\(^6\)

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Figure 1: Schematic diagrams of the measurement system.

Table 1: Measured quality factors of 11 axisymmetric vibration modes.
and stand-off.

3.1. Effects of the Surface

The Q’s were measured 3 times for the polished sample and 2 times for the other. Though the Q’s varied for each measurement, the maximum values were about $1 \times 10^6$ for both samples. We couldn’t find any clear effect of surface polishing on the Q’s.

3.2. Friction between the Wire and Mirror

Looking at the Q’s measured in a simple suspension, there seems to be a dependence of the Q’s on the vibration mode. There should be a few physical phenomena which determine the quality factors, such as the wire resonance, and friction at the point where the wire touches the mirror. We examined whether the Q’s are determined by the loss caused by friction. We modelled that the loss due to friction increases in proportion to $u_z \omega$ for each mode, where $u_z$ is the displacement of the mirror in the axial direction, and $\omega$ is the resonant angular frequency. Comparing the calculated $u_z \omega$ with the observed loss (1/Q) we observed a good correlation between them.

3.3. Stand-off

Two brass cylinders with diameters of 1mm and lengths of 14mm were used as the stand-off (See Figure 1). They were glued to the mirror in order to determine the position where the wire breaks away from the mirror. The stand off doesn’t decrease the Q’s of the mirror.

3.4. Magnet

A magnet with a diameter of 1mm and a length of 10mm was glued 5mm from the edge of the mirror. It could decrease the Q’s. We should study this effect more accurately.

4. Conclusion

We saw some dependence on the Q’s of suspension system, such as friction with the wire, magnet and stand-off. However, there are still large variations of the Q values, and further investigations are needed to eliminate other physical effects, such as wire resonance.

References
1. Introduction

Thermal noise is one of the serious noise sources in interferometric gravitational wave detectors. All optical components of the detector are suspended for vibration isolation. Each vibration mode of a suspension system of an interferometer in a heat bath has kinetic energy because of the equipartition theorem. Therefore, the positions and angles of the mirrors of the interferometer fluctuate. The internal modes of the mirrors also have kinetic energy. Thus, the surfaces of the mirrors vibrate. The fluctuation of the optical-path difference caused by these thermally excited motions is called the thermal noise of interferometric gravitational wave detectors. The fluctuation-dissipation theorem predicts a relation between the fluctuation and the dissipation of the systems. The power-spectrum density of the thermal noise is calculated from the properties of the dissipation of the systems using the fluctuation-dissipation theorem.

The TAMA interferometer (with a 300m base line) is under construction at National Astronomical Observatory (NAO) in Mitaka, Tokyo. In this study, the thermal noise of the suspension systems and the mirrors in the TAMA interferometer was calculated.

2. Thermal Noise of the Suspension Systems

2.1. Suspension System

A double pendulum is employed as the suspension system in the TAMA interferometer. An intermediate mass, which is made of aluminum (1.2kg), is suspended by a bellows (6.5cm) and tungsten wires (19cm). The bellows act as soft springs. The mirror (1kg) is suspended by tungsten wires (25cm) from the intermediate mass. The motions of the intermediate mass are damped by strong permanent magnets (eddy-current damping).

The modes of the mirror, which fluctuate the optical path length, are as follows. The pitch rotation and the yaw rotation of the mirrors fluctuate. This noise is proportional to the distance between the center of the mirror and the optical axis. It is supposed that this distance is 1mm. The vertical vibration also fluctuates, because the vertical direction at the front mirror is not parallel to that at the end.
mirror due to the curvature of the Earth. The pendulum mode of the mirror and the violin modes of the wires fluctuate directly.

2.2. Dissipation and Thermal Noise

A large fluctuation force is applied to the intermediate mass, because of strong magnet damping. However, in the observation band (150Hz~450Hz), the fluctuation is filtered through the 2nd-stage pendulum. From our calculations with the parameters of the TAMA suspension system, the fluctuations of the mirror caused by the vibration of the intermediate mass are smaller than the goal of TAMA ($h = 1.7 \times 10^{-22}/\sqrt{\text{Hz}@300\text{Hz}}$). Thus, only the thermal noise caused by loss under the intermediate mass is considered, as follows: the intermediate mass is fixed, and only the part under the intermediate mass is considered.

The thermal noise of the rotations and the vertical vibration is the same as that of a single harmonic oscillator. When the Q-values are the same, as in Table 1, the amplitude of each thermal noise is a quarter of the final goal of TAMA. Since these values are not too high to achieve, the noise does not limit the sensitivity of TAMA.

When the thermal noise of the pendulum mode and the violin modes are calculated, the suspension system must be treated as a continuous system. The thermal noise of the continuous system is equivalent to a superposition of the thermal noise of the harmonic oscillators.\(^1\) Also, it is supposed that the Q-values of the violin modes are half of that of the pendulum mode\(^2\) in this calculation. When the Q-values are the same as in Table 1, this thermal noise is comparable to the goal of TAMA. This noise limits the sensitivity of TAMA.

Table 1: Lower limits of the Q-values.

<table>
<thead>
<tr>
<th>resonant freq [Hz]</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitch</td>
<td>3.27</td>
</tr>
<tr>
<td>Yaw</td>
<td>0.785</td>
</tr>
<tr>
<td>Vertical</td>
<td>17.5</td>
</tr>
<tr>
<td>Pendulum</td>
<td>1.01</td>
</tr>
<tr>
<td>n-th Violin</td>
<td>$533 \times n$</td>
</tr>
</tbody>
</table>

3. Thermal Noise of the Mirrors

The mirror of TAMA is made of fused silica. The radius of the mirror is 5cm and the height is 6cm. The beam radius at the front mirror is 8mm and at the end mirror it is 15mm. The mirror can be treated as a continuous system. This thermal noise is also a superposition of the thermal noise of the harmonic oscillators. It is necessary and difficult to compute the resonance modes of a 3-dimensional elastic body. However it is possible to estimate the thermal noise using Hutchinson’s simulation, which is a semi-analytical algorithm used to calculate the vibration
modes of an elastic cylinder very accurately. The axial-symmetric resonant modes below 600kHz have been calculated, and the thermal noise of these resonant modes summed up. From this result, if the Q-values are $10^7$, the thermal noise of the mirrors is the same as the goal of TAMA. This noise also limits the sensitivity of TAMA.

4. Summary

From the above discussions, when $Q_{\text{Pendulum}} = 5 \times 10^5$ and $Q_{\text{Mirror}} = 2 \times 10^7$, the total thermal noise is the same as that in the final goal of TAMA. Also, the thermal noise of the pendulum mode and the violin modes is equal to that of the mirrors.

![Graph of Thermal Noise](image)

Figure 1: Thermal noise of TAMA300. $Q_{\text{Pendulum}} = 5 \times 10^5$, $Q_{\text{Violin}} = 2.5 \times 10^5$, $Q_{\text{Mirror}} = 2 \times 10^7$ and $10^6$. The total thermal noise ($Q_{\text{Mirror}} = 2 \times 10^7$) is comparable to Phase II, which is the final goal of TAMA.

It is not impossible that the Q-value can reach $5 \times 10^5$ based on our experiments. However, the measured Q-values of the mirror of TAMA are $10^6$, at most. Therefore, the thermal noise of the mirrors is one of the most serious noise sources of TAMA.

The lower limit of the Q-values of the mirrors of TAMA is about 20-times larger than those of LIGO and VIRGO. There are four reasons, as follows:

1. The mirror of TAMA is smaller.
2. The ratio of the radius of the beam to the radius of the mirror is smaller than that of LIGO.
3. The goal of the displacement sensitivity is harder, because the baseline is shorter.
4. The thermal noise of the pendulum mode and the violin modes is not negligible, because the mirrors in TAMA are smaller.

References

DEVELOPMENT OF A LASER TRANSDUCER FOR A RESONANT ANTENNA

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ABSTRACT
A new type of resonant mass detector is being developed at ICRR, University of Tokyo. The detector comprises a disk-type antenna installed with a laser optical transducer. The optical system and the noise behavior of the laser transducer are described here.

1. Introduction
A disk-shape resonant antenna for gravitational wave detection is being developed at ICRR, Univ. of Tokyo. It is designed to operate at room temperature and to search for gravitational waves from sources in our galaxy. In this detector an optical transducer is used for monitoring the mechanical vibration of the antenna. An optical transducer with a Fabry-Perot cavity is a promising device for obtaining high sensitivity at low temperature, and even at room temperature.

2. Experimental Setup
The disk-type antenna is made of an Al 5052 alloy and is 2m in diameter, 20 cm thick and weighs 1,700kg. By slightly cutting four edges of the disk, the resonant frequency of the quadrupole modes splits into two; the calculated frequencies are 1,178 Hz and 1,184 Hz. We measured the motion of the latter mode, whose mechanical quality factor is $3.2 \times 10^5$ at room temperature. The laser transducer (see Figure 1) comprises a Fabry-Perot(FP) resonator installed at the cut edges of the disk. The FP cavity consists of two concave mirrors with a finesse of 3000; the distance between the mirrors is 10mm. A Nd:YAG laser (Lightwave Electronics, MISER) is used as an optical source. The output light from the laser is conducted into an optical fiber and then injected into the transducer; the incident light power into the transducer is 25 mW. The signal from the transducer is used for the feedback control of this optical system. The data are transferred through an A/D converter to a UNIX workstation for analysis.
Figure 1: Schematic view of the laser transducer mounted on the disk antenna.

Figure 2: Measured displacement noise of the laser transducer.
3. Noise Measurement

Figure 2 shows the noise spectrum of the present transducer. The displacement noise reaches to the order of $10^{-16}$ m above several hundreds of Hz; this noise level can be attributed to the frequency noise of the laser source. The Brownian motion of the antenna can be recognized at 1.18 kHz. The peak at about 750 Hz and its harmonics correspond to the spinning frequency of a turbo pump. The optical system can operate stably with a constant sensitivity unless a large and sudden change in temperature occurs. A Gaussian fitting of the amplitude distribution of the antenna motion results in the noise temperature, $T_n = 22$ K. A higher sensitivity will be obtained by frequency stabilization of the laser source, which is now in progress.

References

PRELIMINARY MEASUREMENT OF AN ANELASTICITY USING
FUSED SILICA FIBER

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ABSTRACT
We performed the measurement of the anelasticity of a fused silica using a torsion balance. Since the quality factor of a fused silica for the torsional mode was greater than $10^5$, the expected effect of the anelasticity was smaller than the error of this measurement.

1. Introduction
Since the mirrors are suspended as free masses in an interferometric gravitational wave detector, the thermal noise of pendulums and mirrors will dominate the sensitivity of the detector. Accordingly, the internal friction of materials, such as fused silica, is an important factor which will determine the thermal noise spectrum. There are two models of internal friction of matter, the anelasticity model and the velocity damping model. The velocity damping model assumes that the damping term in the equation of motion is proportional to velocity. The anelasticity is modeled by a complex spring constant $k(1 + i\phi(\omega))$ in the frequency domain representation of the equation of motion.

A number of results which support the anelasticity model with constant $\phi(\omega)$ have been reported. These results were obtained by measuring the loss $\phi(\omega)$ directly. In this work we measure the real part of the spring constant to study the anelasticity effect, where these real part and imaginary part of the complex constant have the Kramers-Kronig relations.

Another reason for interest in the anelasticity of fused silica is that constancy of the spring constant of the torsion fiber is important in the measurement of the Newtonian gravitational constant using a torsion balance. The value of the Newtonian gravitational constant $G$ recommended by CODATA was obtained by G.G. Luther and W.R. Towler.\textsuperscript{1} Their measurement used a torsion balance with a fused silica fiber. If we apply the anelasticity model to their result, the value of $G$ will increase by 150 ppm.\textsuperscript{2}

2. Experimental Setup
We used a torsion balance whose fiber was made of fused silica and whose moment of inertia could be changed. The torsion balance was set in a vacuum of about $10^{-6}$ torr. We made the fiber by fusing and stretching a fused silica rod and made both end points thicker for holding, so the losses other than the internal friction could be ignored. We set a right angle prism at the center of mass of the torsion balance and measured the torsional angle using a laser beam deflection...
3. Experimental Result

Then we calculated the period using the time-series data for the rotational swing of the torsion balance. We repeatedly changed the moment of inertia of the torsion balance and measured the variation of the period of the torsion balance from 10 to 40 mHz (Figure 1).

![Figure 1: The variation of the period.](image-url)

**Measurement of Quality Factor**

We fitted a sinusoidal function to successive 2 cycle segments of the swing data (Figure 2). From this we obtained amplitude, frequency, phase, and offset. Next we evaluated the Q of the torsion balance by plotting amplitude against time and fitting an exponential function and obtained $Q = 1.50 \times 10^5 - 8.24 \times 10^5$.

**The Effect of Anelasticity**

We calculated the spring constant $k$ versus frequency using $k = I \omega^2$, where $I$ is the moment of inertia of the torsion balance and $\omega$ is the circular frequency of the torsion (Figure 3). If $\phi(\omega)$ is independent of frequency, the spring constant $k$ is proportional to $\omega^{2\phi}$ (from the Kramers-Kronig relations). Then according to the measured $Q$ the relative variation of $k$ when we vary frequency from 0.00942 Hz to 0.03960 Hz is smaller than $\frac{\delta k}{k} = 6.1 \times 10^{-6}$. The error of $k$ of our measurement is much greater than the above. ($\delta k/k \sim 3 \times 10^{-3}$) Thus we cannot determine which model can be applied to fused silica, the velocity damping model or the anelasticity model. If the frequency of torsion balance could be made low, the effect of anelasticity would be large. If we vary the frequency from 0.0005 Hz to 0.04 Hz under $Q = 1.50 \times 10^5$, then
\[ \frac{\delta k}{k} = 1.7 \times 10^{-5}. \]

Thus we need to use a lower much frequency torsion balance and to measure the length and mass of torsion balance more precisely.

Figure 2: Time variation of torsion and its power spectrum.

Figure 3: Frequency dependence of the spring constant.

References
This workshop was intended to provide an occasion to discuss the present status of the on-going projects such as LIGO, VIRGO, GEO and TAMA, to scrutinize the technical problems encountered, and to develop strategies for coping with the technical difficulties. We are much pleased as the workshop organizer to have a lot of leading scientists in this field. I think the object has been almost fulfilled owing to your sincere cooperation. Personally, it is a pity that I myself could not have time enough to concentrate to the whole presentations in this workshop. It is therefore very hard for me to summarize the workshop, although I could feel the exciting atmosphere. I shall give only a few words on each projects.

GEO is promoted much faster than our expectation. Especially they have conducted a simple and reliable method to install vacuum system. VIRGO steadily advances both on its facility and development on laser and optics. LIGO looks leading all other projects. Basic research on interferometer noise has been done. Requirement on optics has been fixed. It is my surprise to know this in such a short time after switching Argon to YAG laser. Since TAMA people has less chance to give talks overseas, so far, this workshop served as a good chance to present the research activities as well as to see worldwide gravitational wave community and to discuss with them. Relating topics have been also impressed very much, such as LISA and Astro-D.

Organization of this kind of workshop is concerned as one of the most important activities associated with TAMA project. It is our obligation to hold an international meeting once again in 1999. It will become a larger conference. I am looking forward to hearing many reports of advancement at that time.

I would like to thank all your contributions, and I wish to meet all of you in the next meeting.
TAMA Workshop
on
Gravitational Wave Detection
Saitama, Japan
November 12–14, 1996

SCIENTIFIC PROGRAM

Organizing committee

J. Arafune
M.-K. Fujimoto (co-chair)
N. Kaifu
N. Kawashima
Y. Kozai (chairman)
K. Kuroda (co-chair)
N. Mio
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Institute of Space and Astronautical Science
National Astronomical Observatory
University of Tokyo
University of Tokyo
Kyoto University
National Laboratory for High Energy Physics
University of Tokyo
University of Electro-Communications
**PROGRAM**

*invited talks

**Morning of Tuesday (12–Nov, 9:00–12:00)**

Chaired by M-K. Fujimoto

9:00–9:10 Opening talk
Keiichi Kodaira (NAO)

9:10–9:40 Women’s social status and this foundation
Administrative staff of this facility

9:40–10:15 *Sources of gravitational waves
Takashi Nakamura (YITP, Kyoto University)

10:15–10:30 Estimation of parameters of the gravitational-wave signals: which gravitational-wave signals we can see first?
   Andrzej Krolak (Inst. Math., Polish Academy of Sciences)

10:30–10:45 Coffee Break

Chaired by K. Tsubono

10:45–11:20 *Simulation programs of LIGO
Hiroaki Yamamoto (CALTECH)

11:20–11:40 Data Acquisition System for TAMA gravitational wave interferometer
   Nobuyuki Kanda (ICRR, Univ of Tokyo)

11:40–11:50 Simulation of the Virgo interferometer
   Frederique Marion (LAPP Annecy)

11:50–12:00 A common data format for gravitational wave interferometers
   Benoit Mours (LAPP Annecy)

**Afternoon of Tuesday (12–Nov, 13:30–17:00)**

Chaired by M. A. Barton

13:30–14:05 *Prospects for low thermal noise in gravitational wave interferometers
   Peter R. Saulson (Syracuse University)

14:05–14:25 Thermal noise limit to the VIRGO sensitivity
   Luca Gammaitoni (Universita’ degli Studi di Perugia, INFN)

14:25–14:45 Quality factor measurements of full scale suspended prototypes
   Joseph Kovalik (Universita’ degli Studi di Perugia, INFN)

14:45–15:05 Design of a mirror suspension system used in the TAMA 300 m interferometer
   Akito Araya (ERI, Univ. of Tokyo)

15:05–15:25 Ambient noises of a free mass gravitational antenna
   V. N. Rudenko (Sternberg Astronomical Institute MSU, Russia)

15:25–15:45 Coffee Break

Chaired by V. N. Rudenko

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15:45–16:20 *Performance of the UWA 8 m interferometer with compound pendulum test masses and multistage cantilever vibration isolators
   David G. Blair (University of Western Australia)
16:20–16:40 Vibration isolation stack for TAMA300
   Ryutaro Takahashi (NAO)
16:40–17:00 A 2D X-pendulum vibration isolation system
   Mark A. Barton (NAO)

Morning of Wednesday (13–Nov, 9:00–12:00)

Chairied by N. Kawashima

9:00–9:35 *LISA as an ESA cornerstone mission for the detection and observation of gravitational waves
   Karsten Danzmann (University of Hannover)
9:35–10:10 *ASTROD and gravitational waves
   Wei-Tou Ni (National Tsing Hua University, Taiwan)
10:10–10:30 Fast and accurate template construction for gravitational waves from binary stars
   V. Pierro, I.M. Pinto, E. Laserra, F. Recano (Univ. of Salerno)
   A. D. Spallicci (ESTEC, Noordwijk, NL)
10:30–10:40 Coffee Break

   Chaired by N. Mio

10:40–11:15 *Recent research on the LIGO phase noise interferometer
   Michael E. Zucker (MIT)
11:15–11:40 Current status of 20 m prototype
   Masatake Ohashi (NAO)
11:40–12:00 Effort of stable/reliable operation in ISAS 100 m laser interferometer
   Nobuki Kawashima, Ei-ichi Mizuno, Shigeo Nagano, Shinji Higo and Shinji Miyoki
   (ISAS)

Afternoon of Wednesday (13–Nov, 13:30–17:00)

Chairied by Y. Kozai

13:30–14:00 *The status and plans for LIGO
   Barry Barish (CALTECH)
14:00–14:30 *Status of VIRGO project
   Alain Brillet (VIRGO-CNRS)
14:30–15:00 *GEO 600
   James Hough (Glasgow University)
15:00–15:30 TAMA project
   Kimio Tsubono (University of Tokyo) and the TAMA collaboration
15:30–15:45 Coffee Break
   (Group photo session)

   Chaired by Y. Ogawa
15:45–16:20 *Low frequency sources and detection of them with Virgo
Adalberto Giazotto (INFN, Pisa)
16:20–16:35 The ultra cryogenic detector AURIGA: status report
Massimo Cerdonio (INFN, Padova)
16:35–16:50 Perspectives of the BAE detection configuration for resonant gravitational wave antennas and design for mechanical QND experiments
Fulvio Ricci (Università of Roma I)
16:50–17:00 Event rate, parameters estimation and signal to noise ratio for coalescing binaries with large bandwidth spherical resonant detectors
A. Spallicci (ESRT center)

Morning of Thursday (14–Nov, 9:00–12:15)

Chaired by M. Ohashi

9:00–9:35 *Optics development for LIGO
Stanley Whitcomb (CALTECH)
9:35–9:55 Optical system of TAMA
Norikatsu Mio (University of Tokyo)
9:55–10:15 Control system of TAMA300 interferometer
Keita Kawabe (University of Tokyo) and the TAMA collaboration

10:15–10:30 Coffee Break

Chaired by E. Majorana

10:30–11:05 *Stabilized laser for VIRGO
Catherine N. Man (VIRGO-CNRS)
11:05–11:25 Resonant sideband extraction for interferometers with Fabry-Perot arms
Gerhard Heinzel (MPI fuer Quantenoptik)
11:25–11:45 Power recycling experiment
S. Moriwaki (University of Tokyo)
11:45–12:15 *Recent research on the LIGO 40 m interferometer
Seiji Kawamura (CALTECH)

Afternoon of Thursday (14–Nov, 13:30–17:00)

Chaired by R. Takahashi

13:30–14:00 Progress in development of some new techniques for laser interferometer gravitational wave detectors
R.W.P. Drever (CALTECH)
14:00–14:20 Japanese gravitational wave observatory
Kazuaki Kuroda (ICRR, Univ. of Tokyo) and TAMA volunteers
14:20–14:40 Loop interferometer for gravitational wave detection
Masahiro Kitagawa (Osaka University)
14:40–15:00 Measurement of birefringence of a 0.67 m suspended optical mode cleaner with finesse 1500 at resonance
Jean-Sheng Wu (National Tsing Hua University, Taiwan)

15:00–15:20 Coffee Break
Poster Session 15:20–16:30

Gravity wave detection using astronomical phenomena
Redouane Fakir (University of British Columbia)

Binary pulsars as detectors of gravitational waves
Sergei Kopeikin (NAO)

Gauge dependence of post-Newtonian approximation in G. R.
Tadayoshi Shimizu (Komazawa Junior College)

Gravitational wave chirp detection and source parameter estimation via tailored radon transform
M. Feo, V. Piero, I.M. Pinto, M. Ricciardi (Univ. of Salerno, IT)

Multistability and chaos in multipendular Fabry-Perot optical cavities
V. Pierro, I.M. Pinto (Univ. of Salerno, IT)

Quantum optics of interferometric detection of gravitational waves
S. Alam (University of Peshawar, Pakistan)

Short Fabry-Perot cavity to study the mechanical thermal noise
Ettore Majorana (INFN & ICRR, Univ. of Tokyo)

Study of power recycling of a Fabry-Perot-Michelson interferometer
Masaki Ando, Ying Li, Keita Kawabe and Kimio Tsubono (University of Tokyo)

Development of highly stabilized laser for gravity wave detectors
Mitsuru Musha (NAO)

The NAO 20 m prototype laser interferometric gravitational wave detector
Koya Suehiro (NAO)

Development of a mode cleaner for laser interferometer gravitational wave detector
Souichi Telada (NAO)

Low-frequency pendulum using the electromechanical antispring effect
Yujiro Ogawa (KEK)

Development of anti-vibration system with two degrees of freedom using X-pendulum
Takashi Uchiyama (ICRR, Univ. of Tokyo)

Vibration isolation of TAMA suspension system
Koji Arai, et al. (University of Tokyo)

Automatic alignment control of the TAMA interferometer
Kuniharu Tochikubo, Aiihiro Sasaki, Keita Kawabe and Kimio Tsubono (Univ. of Tokyo)

Measurements of quality factors of a fused silica mirror
Naoko Ohashi, Keita Kawabe and Kimio Tsubono (University of Tokyo)

Thermal noise study of the TAMA interferometer
Kazuhiko Yamamoto, Keita Kawabe and Kimio Tsubono (University of Tokyo)

Development of laser transducer for resonant antenna
Naoto Kondo, et al. (University of Tokyo)

Measurement of an anelasticity using fused silica fiber
Sumihiro Matsumura (ICRR, Univ. of Tokyo)

Summary Session 16:30–17:00

Chaired by K. Kuroda

16:30–16:50 Summary of this workshop
   M-K. Fujimoto (NAO)

16:50–17:00 Closing talk
   Yoshihide Kozai (Leader of TAMA)
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